GENERAL APTITUDE

CSIR-UGC-NET|GATE

Previous Years' Solved Papers

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PREFACE

The book "***General Aptitude***" is written with the objective of providing description of principles involved in General Aptitude including their application at a level suitable for B.Tech. and M.Sc. students. This book will be highly useful for the candidates preparing for various competitive examinations at national and state levels such as CSIR-UGC-NET/JRF, GATE, SLET, etc.

In all the papers of CSIR-UGC-NET/JRF and GATE examinations, GA questions carry 15 percent weightage of total marks.

- In every year CSIR-UGC-NET/JRF includes at least 30 marks from this section out of 200 marks
- In every year GATE paper includes at least 15 marks from this section out of 100 marks.

Apart from CSIR-UGC-NET/JRF, GATE, SLET, this book will also be very useful to candidates appearing for various Public Sector examinations i.e. NTPC, HPCL, etc.

The book contains a brief description on every topic of General Aptitude. The book is designed and prepared in such a way that it provides the required information regarding almost every topic to General aptitude with emphasis on logical reasoning graphical analysis, analytical and numerical ability, quantitative comparisons, series formation, puzzles etc.

Numerical Ability: Numerical computation, numerical estimation, numerical reasoning and data interpretation.

Verbal Ability: English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.

.Students preparing for competitive examinations are advised to go through every topic for their understanding and conceptual clarity and then get into solving the problems. For the benefit of the students, at the end of the every topic a good number of problems are also provided.

Much care has been taken to minimize the typographical errors, yet if any noticed, then your suggestions are invited with a view to improve the book in future editions.

We would like to thank **Mr. Rajeev Kumar Shukla, Puneet Trivedi** and **Mr. Manendra Kumar Sirdar** for their valuable inputs in bringing this book. Thanks also to **Mohammad Firoz** for formatting and type-setting.

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APTITUDE SECTION

CHAPTER

1

Number System

1. **(i) Natural Numbers:** Counting numbers 1, 2, 3, 4, 5 ..... are called natural numbers.

   (ii) Whole Numbers: All counting numbers together with zero from the whole numbers.

   Integers : All natural numbers, 0 and negatives of counting numbers i.e. {....., –3, –2, –1, 0, 1, 2, 3, ...} together form integers.
   (i) Positive integers : {1, 2, 3, 4, ...} is the set of all positive integers.
   (ii) Negative integers : {–1, –2, –3, –4, ...} is the set of all negative integers.

2. **Real Numbers:** All numbers that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

   Modulus of a Real Number: Modulus of a real number 'a' is defind as

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

   $|5| = 5$ and $|-5| = -(-5) = 5$.

   Prime Numbers: A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

   Prime Numbers upto 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

2. **Divisibility by 2 or 5:** A number of divisible by 2 or 5 if the last digit is divisible by 2 or 5. Example: 58694 is divisible by 2, while 86945 is not divisible by 2.

3. **Divisibility by 3 (or 9):** All such numbers the sum of whose digits are divisible by 3(or 9) are divisible by 3 (or 9).

   Example: (1) In the number 695421, the sum of digits = 27, which is divisible by 3.

   $\therefore$ 695421 is divisible by 3.

   (2) In the number 948653, the sum of digits = 35 which is not divisible by 3

   $\therefore$ 948653 is not divisible by 3.

4. **Divisibility by 4:** A number is divisible by 4 if the last 2 digits are divisible by 4.

   Example: (1) 9879376 is divisible by 4, since 76 is divisble by 4

   (2) 496138 is not divisible by 4, since 38 is not divisible by 4

5. **Divisibility by 6:** A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

6. **Divisibility by 8:** A number is divisible by 8 if the last 3 digits of the number are divisible by 8.
Example: (1) In the number 16789352, the number formed by last 3 digits namely 352 is divisible by 8.
$\therefore$ 576484 is not divisible by 8

7. **Divisibility by 11:** A number is divisible by 11 if the difference of the sum of the digits in the odd places and the sum of the digits in the even place is zero or is divisible by 11.
(i) Consider the number 29435417
(Sum of its digits at odd places) – (Sum of its digits at even places)
= (7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 – 12) = 11, which is divisible by 11.
$\therefore$ 29435417 is divisible by 11
(ii) Consider the number 57463822
(Sum of its digits at odd places) – (Sum of its digits at even places)
= (2 + 8 + 6 + 7) – (2 + 3 + 4 + 5) = (23–14) = 9, which is not divisible by 11.
$\therefore$ 57463822 is not divisible by 11

8. **Divisible by 12:** All numbers divisible by 3 and 4 are divisible by 12.

9. In 'n' consecutive whole numbers, a, a + 1 ..... a+n–1 one and only one is divisible by n.

10. A sum of 5 consecutive whole numbers will always be divisible by 5.

11. The square of an odd number when divide by 8 will always leave a remainder of 1.

12. The product of any 'r' consecutive integers (numbers) is divisible by r!.

13. If 'm' and 'n' are two integers then (m+n)! is divisible by m!n!.

14. Difference between any number and the number obtained by writing the digits in reverse order is divisible by 9.

15. Any number written in the form $10^n - 1$ is divisible by 3 and 9.

16. $\frac{a^n}{(a+1)}$ leaves a remainder of ..... a if 'n' is odd and 1 is n is even.

17. If you can express the expression in the form $\frac{(ax+1)^n}{a}$, the remainder will becomes 1 directly. In such a case, no matter how large the value of power 'n' is the remainder is 1. For instance; $\frac{(37^{12365})}{9} \xrightarrow{R} \frac{(1^{12635})}{9} \xrightarrow{R} 1$.
In such a case the value of the power does not matter.

2. Important Rule: GCD (n_1, n_2). LCM$(n_1, n_2) = n_1.n_2$. i.e. The product of the HCF and the LCM equals the product of the numbers.

3. **Rule for finding out HCF and LCM of fractions:**

(a) HCF of two or more fractions is given by: $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$

(b) LCM of two or more fractions is given by: $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Example: Find HCF and LCM of $\frac{1}{2}, \frac{1}{3}$ and $\frac{3}{7}$

$$\text{HCF} = \frac{\text{HCF of } (1,2,3)}{\text{LCM of } (2,3,7)} = \frac{1}{42}; \qquad \text{LCM} = \frac{\text{LCM of } (1,2,3)}{\text{HCF of } (2,3,7)} = \frac{6}{1} = 6$$

4. **Rules of HCF:** If the HCF of x and y is G then the HCF of

(i) $x,(x+y)$ is also G (ii) $x,(x-y)$ is also G (iii) $(x+y),(x-y)$ is also G

Calculation of units digit when a number is raised to a power:

Step-1: Find out last digit of the number and power of the number.

Step-2: Write power of the number in 4N, 4N+1, 4N+2, 4N+3 form.

Step-3: Then check the table and you will get unit digit in given expression.

Number ending with	4N+1	4N+2	4N+3	4N+0 or exactly divisible
0	0	0	0	0
1	1	1	1	1
2	2	4	8	6
3	3	9	7	1
4	4	6	4	6
5	5	5	5	5
6	6	6	6	6
7	7	9	3	1
8	8	4	2	6
9	9	1	9	1

SOLVED EXAMPLES

1. Find the unit digit of $(52)^{97}\times(43)^{72}$

Soln. $97=4\times24+1\Rightarrow 4N+1$

Last digit of $52=2$ so for 2, 4N+1 will have unit digit 2.

$72=4\times18\Rightarrow 4N$

Last digit of $43=3$ so for 3(4N+0) will have unit digit 1.

Product will have unit digit $2\times1=2$

2. Find the unit digit of $67\times35\times215\times91\times42\times53\times81$

Soln. For unit digit simply multiply

$7\times5\times5\times1\times2\times3\times1=1050$. So, unit digit will be 0.

Getting number of zeros in any expression:

When any even number is multiplied by 5, it gives a zero on right side of results. Hence we factorize the given expression and find the power of 2 and 5. The least power of 2 and 5 show the number of zero. as

$2^m\times5^n$ — m if $m<N$

— n if $n<m$

But in question of factorial, we divide the given number of 5. If result is not divisible by 5. We assume the less number which is divisible by 5. Sum of quotient is equal to number of zero.

3. $5\times10\times15\times20\times25\times30\times35\times40\times45$. Find the number of zero in given product.

Soln. $5\times5\times2\times5\times3\times5\times2^2\times5^2\times5\times2\times3\times5\times7\times2^3\times5\times3^2\times5\times5^2\times2$ Here power of 2 is less. Hence number of zero is 7.

4. Find number of zero in $\underline{|450}$ and $\underline{|183}$

(i)
5	450
5	90
5	18
	3

(ii)
5	183
5	36
5	7
	1

Hence number of zero = 90 + 18 + 3 = 111
Hence, number of zero = 36 + 7 + 1 = 44

Finding remainder: $x^n \div y$

(i) If $x > y$ then we use bianomial theorem, but $x = y + a$, then $(y+a)^n \div y$ gives the remainder a^n.

5. $7^{13} \div 6$, what will be remainder?

$$7^{13} = (6+1)^{13} \div 6 = 1^{13} = 1$$

6. $46^{15} \div 9$, what will be remainder?

$$46^{15} = (45+1)^{15} = (5\times 9+1)^{15} \div 9 = 1^{15} = 1$$

(ii) If $x < y$, then we use following method.

(a) $x^n + a^n$ is always divisble by (x+a) if n is odd
(b) $x^n - a^n$ is always divisible by (x+a) if n is even
(c) $x^n - a^n$ is always divisible by (x–a) for any value of n.
(d) $x^n + a^n$ is never divisible by (x-a)

7. When $17^{200} \div (17+1)$

Using property (b) – $\dfrac{17^{200}-1^{200}}{17+1}$ is exactly divisible.

$\Rightarrow$ $\dfrac{17^{200}-1}{18}$, Hence 1 is remainder.

Note: If (a–c) is exactly divisible by 'b' then if 'a' is divided by 'b' always gives the remainder 'c'.

8. If $67^{67} \div 68$, what will be remainder?

Soln. $68 = (67 + 1)$; $67^{67} \div (67+1)$

Using property (a), we get

$\dfrac{67^{67}+1^{67}}{67+1}$ is exactly divisible by 68.

Therefore, remainder = 68 – 1 = 67

Note: If (a+c) is exactly divisible by b, then if 'a' is divided by b, give remainder as (b–c).

9. Nitin and Pradip solved a quadratic equation. In solving it, Nitin made a mistake in the constant term and got the roots as 6 and 2, while Pradip made a mistake in the coefficient of x only and obtained the roots as –7 and –1. Find the correct roots of equation.

Soln. For Nitin, we have $\alpha + \beta = 8$ and $\alpha\beta = 12$

The equation is $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

$\therefore$ $x^2 - 8x + 12 = 0$

For Pradip, we have $\alpha + \beta = -8$ and $\alpha\beta = 7$

Therefore, the equation is $x^2 + 8x + 7 = 0$

When there is no mistake in a and b, the sum of roots must be correct.
Therefore, sum of roots = 6 + 2 = 8 and product of roots = (–7) × (–1) = 7

So, the correct equation is $x^2 - 8x + 7 = 0$

$\Rightarrow$ $x^2 - 7x - x + 7 = 0$ $\Rightarrow$ $(x-7)(x-1) = 0$ $\Rightarrow$ $x = 7$ or $x = 1$

Therefore, the roots are 7, 1.

Formulae:

1. $(a+b)(a-b)=(a^2-b^2)$

2. $(a+b)^2=(a^2+b^2+2ab)$

3. $(a-b)^2=(a^2+b^2-2ab)$

4. $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

5. $(a^3+b^3)=(a+b)(a^2-ab+b^2)$

6. $(a^3-b^3)=(a-b)(a^2+ab+b^2)$

7. $(a^3+b^3+c^3-3abc)=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

$$=\frac{1}{2}(a+b+c)\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

8. When $a+b+c=0$, then $a^3+b^3+c^3=3abc$

9. $(a+b)^3=a^3+b^3+3ab(a+b)$

10. $(a-b)^3=a^3-b^3+3ab(-a+b)$

From above result it is clear that a^n+b^n is divisible by $a+b$ when value of n is odd. But it is never divisible by $a-b$.

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EXERCISE

1. What will be the remainder when $(29)^{36}$ is divided by 28?
(a) 0 (b) 1 (c) 29 (d) 5

2. A number is multiplied by 5 and 25 is added to it. The result is divided by 5 and the original number is subtracted from the same. The remainder will be
(a) 5 (b) 1 (c) 2 (d) 3

3. The ratio of two numbes is 15:11. If their HCF is 13, then the numbers are
(a) 75, 55 (b) 45, 22 (c) 104, 44 (d) 195, 143

4. What is the smallest whole number that is exactly divisible by $1\frac{5}{28}$, $2\frac{2}{21}$ and $3\frac{1}{7}$?
(a) 132 (b) 130 (c) 138 (d) 124

5. If $\sqrt{x}-\sqrt{y}=1$ and $\sqrt{x}+\sqrt{y}=17$ and $\sqrt{xy}=?$
(a) 51 (b) 16 (c) $\sqrt{72}$ (d) 72

6. Find number of divisors of 1420
(a) 12 (b) 13 (c) 14 (d) 15

7. If 375A is divisible by 9, find the value of A
(a) 1 (b) 3 (c) 7 (d) 6

8. $\frac{(76+58)^2+(76-58)^2}{76^2+58^2}$ is equal to
(a) 2 (b) 4 (c) 134 (d) 18

9. The letters A, B, C, D, E, F and G, not necessarily in that order, stand for seven consecutive integers from 1 to 10 such that
(i) D is 3 less than A (ii) B is the middle term
(iii) F is as much less than B as C is greater than D (iv) G is greater than F
What is the value (A–F)?
(a) 4 (b) 3 (c) 2 (d) Cannot be determined.

10. A snail is at the bottom of a 20-meter deep it. Everyday the snail climbs 5 meters upwards, but at night it slides 4 meters back downwards. How many days does it take before the small reaches the top of the pit?
(a) 20 (b) 19 (c) 17 (d) 16

11. Divide 50 in two parts such that the sum of reciprocals is $\frac{1}{12}$. What are the numbers?
(a) 20, 30 (b) 24, 26 (c) 28, 22 (d) 36, 14

12. What are the values of the digits A and B if the number 79A856776B is divisible by 8 and 9?
(a) 1, 0 (b) 9, 0 (c) 9, 8 (d) 8, 0

13. By reversing the digit of the number 13, the number is increased by 18. How many other two-digit numbers increase by 18, when their digits are reversed?
(a) 8 (b) 7 (c) 6 (d) 5

14. M men agree to purchase a gift for Rs. D. If three men drop out, how much more will each have to contribute towards the purchase of the gift?
(a) D/(M–3) (b) MD/3 (c) M/(D–3) (d) $3D/(M^2-3M)$

15. What is the next number in the series $\frac{2}{\sqrt{5}}, \frac{3}{5}, \frac{4}{5\sqrt{5}}, \frac{5}{25}, (...)$?

(a) $\frac{6}{5\sqrt{5}}$ (b) $\frac{6}{25\sqrt{5}}$ (c) $\frac{6}{125}$ (d) $\frac{7}{25}$

16. The number of 0's (zeros) in the value 1000! is:
(a) 100 (b) 200 (c) 249 (d) 250

17. The number of boys in a class is three times the number of girls. Which one of the following numbers cannot represent the total number of children in the class?
(a) 48 (b) 44 (c) 42 (d) 40

18. A father is three times as old as his son. Five years back, he was four times as old as his son. The age of the son is
(a) 12 (b) 15 (c) 18 (d) 20

19. If the operation, Λ is defined by the equation $x\Lambda y = 2x + y$, the value of a in $2\Lambda a = a\Lambda 3$ given by
(a) 1 (b) 0 (c) 1 (d) 2

20. Begin writing positive integers (in decimal notatio) sequentially. What is the 1, 000th digit written?
(a) 0 (b) 1 (c) 6 (d) None of the above.

21. Which letter follows 4 in the sequence:
1A 2D 3I 4_ 5Y
(a) A (b) Q (c) T (d) P

22. The digit in the unit's position of $(777)^{777}$ is:
(a) 9 (b) 3 (c) 7 (d) 1

23. A person while distributing some mangoes to his friends observes that if he gives 10 each, 9 mangoes are left with him; but if he gives 9 each, 8 are left. If he gives 8 each, 7 are left, and so on. Lastly if he gives 2 each, I is left. The number of mangoes the person has is
(a) 5220 (b) 5039 (c) 5219 (d) 5040

24. What is the last digit in 2^{54}?
(a) 2 (b) 4 (c) 6 (d) 8

25. The sum of two digits of a number is 15. If 9 is added to the number then the digits get reversed. Which of the following is FALSE about the number
(a) The number has the two digits separated by a difference of one.
(b) The number is divisible by 3.
(c) The number is divisible by 6
(d) The number is divisible by 9.

26. $100! = 1\times2\times3\times .....\times100$ ends exactly in how many zeroes?
(a) 24 (b) 10 (c) 11 (d) 21

27. Let 'a' and 'b' be two positive integers. The number of factors of 8^a7^b are
(a) $2^{(a+b)}$ (b) a + b + 2 (c) ab + 1 (d) (a + 1)(b+1)

28. If (12*22*35)/p is an integer, which of the following CANNOT be the value of p?
(a) 15 (b) 21 (c) 28 (d) 50

29. The age of a grandfather in years is the same as that of his grand-daughters in months. If their ages differ by 55 years, the age of the grand-daughter is

(a) $5\frac{1}{2}$ yrs (b) $5\frac{1}{2}$ months (c) 5 years (d) None of the above.

HINTS & SOLUTIONS

1. $(29)^{39} = (28+1)^{36}$
Answer is (b)

2. Let number is x
Multiply by 5, then 5x, added 25 then $5x + 25$
Result divided by 5 then $= x + 5$
Subtracted initial number then $x + 5 - x = 5$
Answer is (a)

3. Let number are 15x and 11x. HCF is 13
Thus only possible numbers are 13×15 and 13×11
Answer is (d)

4. $\text{LCM} = \dfrac{11.3.4}{7} = \dfrac{132}{7}$
Answer is (a)

5. $\sqrt{x} - \sqrt{y} = 1$ ... (i)
$\sqrt{x} + \sqrt{y} = 17$ ... (ii)
$\sqrt{x} = 9$ ... (i) + (ii)
$\sqrt{y} = 8$ ... (ii) - (i)
Answer is (d)

6. $1420 = 2 \times 2 \times 5 \times 7 \times 1$
Number of divisor $= (2+1)(1+1)(1+1) = 12$
Answer is (a)

7. Sumission of digits $= 3 + 7 + 5 + A = 15 + A$
Thus only possible value of A is 3.
Answer is (b)

8. Answer is (a)

9. (d) cannot be determined because integers are seven and value is 10.

10. In one day it climbs only 1 m. So, in 15 days it finally clombs 15 m and 16th day he climbs 5m and reach the top.
So, 16 day are required.
Answer is (d)

11. Let one part is x.
Then another part would be 50–x

$$\therefore \quad \frac{1}{x} + \frac{1}{50-x} = \frac{1}{12} \quad \Rightarrow \quad \frac{50}{x(50-x)} = \frac{1}{12}$$

$$x(50-x) = 12 \times 50 \quad \Rightarrow \quad 50x - x^2 = 600$$

$$x^2 - 50x + 600 = 0 \quad \Rightarrow \quad x^2 - 30x - 20x + 600 = 0$$

$$x(x-30) - 20(x-30) = 0 \quad \Rightarrow \quad (x-20)(x-30) = 0 \quad \Rightarrow \quad x = 20 \text{ or } 30$$

Answer is (a)

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12. Any number is divisible by 8 if last 3 digit of number is divisible by 8 also if sum of digit of given number is divisible by 9 then that number is divisible by 9. Hence, 9, 0 is our answer
Answer is (c and d)

13. Number is increased by 18 when difference of digit is 2
So, number is –24, 35, 46, 68, 79, 57
Answer is 6 i.e. (c)

14. Since, M men purchase a gift for = Rs. D
Therefore, 1 man share = D/M
If 3 men dropout then 1 man share = D/(m–3)

$$\text{Then more contribution} = \frac{D}{m-3} - \frac{D}{m} = \frac{Dm - Dm + 3D}{(m-3)m} = \frac{3D}{m^2 - 3m}$$

Answer is (d)

15. $\frac{2}{\sqrt{5}}, \frac{3}{5}, \frac{4}{5\sqrt{5}}, \frac{5}{25}, \frac{6}{25\sqrt{5}}$
Answer is (b)

16.

5	1000
5	200
5	40
5	8
	1

Hence number of zero = 200 + 40 + 8 + 1 = 249
Answer is (c)

17. The number of boys are 3 times number of girls.
So, Girl = x and Boy = 3x
Hence, total student = x + 3x = 4x
So, the number which is not divisible by 4 is our answer. Hence, 42
Answer is (c)

18. If Son = x year
Father = 3x year
5 hear back, Son = x–5
Father = 3x–5
Hence, 4(x–5) = 3x –5
4x – 20 = 3x – 5
4x – 3x = 20 – 5 $\Rightarrow x = 15$
Answer is (b)

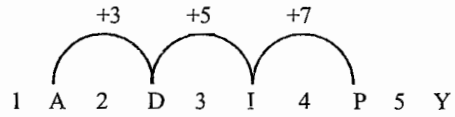
19. $\because \quad x \Lambda y = 2x + y \quad \Rightarrow 2 \Lambda a = 2 \times 2 + a = 4 + a$

$a \Lambda 3 = 2a + 3 \quad \Rightarrow 4 + a = 2a + 3 \quad \therefore \ a = 1$

Answer is (c)

20. Answer is (d)

21.



Answer is (d)

22. $777 \div 4$ given remainder 1
Hence, $(777)^1$'s unit is our result.
Hence, 7
Answer is (c)

23. If difference of divisor and remainder is same for each group. Then take LCM of divisor and substract difference of divisor-remainder from them.

2	10	9	8	7	6	5	4	3	2
2	5	9	4	7	3	5	2	3	1
3	5	9	2	7	3	5	1	3	1
5	5	3	2	7	1	5	1	1	1
	1	3	2	7	1	1	1	1	1

Therefore, LCM = $2\times2\times3\times5\times3\times2\times7 = 420\times6 = 2520$
But 2520 is not given hence another number = $2520 \times 2 = 5040$
Hence, Answer = $5040 - 1 = 5039$
Answer is (b)

24. $54 \div 4$ gives remainder 2
Hence, last digit $2^2 = 4$
Answer is (b)

25. Sum of digit is 15 and when 9 added number are reversed. Hence, number is 78
Check all option
Answer is (d)

26.

5	100
5	20
	4

Hence, number of zero = $20 + 4 = 24$
Answer is (a)

27. Answer is (d)

28. Because 50 cannot exactly divide the numerator, while other number can divide it.
Answer is (d)

29. Let grand father = x yrs.

Grand daughter = x month = $\frac{x}{12}$ yrs

Now, $x - \frac{x}{12} = 55 \quad \Rightarrow \frac{11x}{12} = 55$

$\therefore \quad x = 60 = 5$ yrs

Answer is (c)

TRY YOURSELF

1. If $a+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}=2$, then a = ?

Soln. $a+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}=2 \Rightarrow a+\cfrac{1}{1+\cfrac{1}{(13/4)}}=2 \Rightarrow a+\cfrac{1}{1+\cfrac{4}{13}}=2 \Rightarrow a+\cfrac{1}{17/13}=2$

$\Rightarrow a+\frac{13}{17}=2 \quad \Rightarrow a=2-\frac{13}{17} \quad \Rightarrow a=\frac{34-13}{17}=\frac{21}{17}=1\frac{4}{17}$

2. On children's day sweets were to be equallydistributed amongst 540 children. But on that particular day, 120 children were absent. Thus, each child got 4 sweets extra. How many sweets was each child originally supposed to get?

Soln. Suppose, each child was supported to get *x* sweets. Then we have,

$540\times x=(540-120)(x+4) \Rightarrow 120x=420\times 4 \quad \Rightarrow x=\frac{420\times 4}{120}=14$

Therefore, x = 14 sweets.

3. The difference between the squares of two consecutive odd integers is always divisible by which digits or numbers?

Soln. Let the two consecutive odd numbers be (2n + 1) and (2n + 3) respectively.

Then, $(2n+3)^2-(2n+1)^2=(2n+3+2n+1)(2n+3-2n-1)$

$=(4n+4)\times 2=8(n+1)$, which is divisible by 8.

4. "October 2, 2001" in MMDDYYYY format is a palindrome (a string that reads the same forwards as it does backwards example, 10/02/2001 ..... 10022001). When was the latest century before October 2, 2001 that is also a palindrome?

(a) 13th century (b) 14th century (c) 17th century (d) 20th century

5. Find the largest natural number which exactly divides the product of any 4 consecutive natural numbers.

Soln. Required number = $1\times2\times3\times4=24$

Therefore, required number = 24

[It is applicable for all 4 consecutive natural numbers].

6. Murari, Arun and Nitin start at the same time in the same direction to run around a circular stadium. Murari completes a round in 126 seconds, Arun in 154 seconds and Nitin in 99 seconds, all starting at the same point. After what time will they meet again at the starting point?

Soln. Required time = L.C.M. of 126 sec, 154 sec and 99 sec.

2 | 126, 154, 99

11 | 63, 77, 99

9 | 63, 7, 9

7 | 7, 7, 1

| 1, 1, 1

Therefore, required time = $2\times11\times9\times7=1386$ sec = 23 min. 06 sec.

7. Amit, Bijender and Chandu go walking round a circle 1 km in circumference at the rates of 10 m/min, 20 m/min and 40 m/min respectively. If they all start together and walk in the same direction, when will they be together at the same place?
(a) after 50 minute (b) after 100 minute (c) after 240 minute (d) after 800 minute

8. In an examination, a student average marks were 63 per paper. If he had obtained 20 more marks for his Biochemistry paper and 2 more marks for his Microbiology paper, his average per paper would have been 65. How many papers were there in the examinations?

Soln. Assume there be x papers.
Total marks of all papers = 62x
from question,
$65x - 63x = 20 + 2$
$\Rightarrow 2x = 22 \quad \therefore \quad x = 11$

CHAPTER

2

Average

Average : The average of a number is a measure of the central tendency of a set of numbers. In other words, it is an estimate of where the centre point of a set of numbers lies.

The basic formula for the average of n numbers $x_1, x_2, x_3, ...x_n$ is

$A_n = (x_1 + x_2 + x_3 + ...+ x_n)/n = (\text{Total sum of n numbers})/n$

This also means $A_n \times n =$ total sum of 'n' numbers.

The average is always calculated for a set of numbers.

$$\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observation}}\right)$$

Weighted average: When we have 2 or more groups whose individual average are known then to find the combined average of all the elements of all the groups we use weighted average. Thus if we have k groups with average $A_1, A_2, ..., A_k$ and having $n_1, n_2, ...n_k$ elements then the weighted average is given by the formula.

$$A_w = \frac{n_1A_1 + n_2A_2 + n_3A_3 + ...+ n_kA_k}{n_1 + n_2 + n_3 + ...+ n_k}$$

Example-1: There are two sections A and B of a class consisting of 36 and 44 students respectively. If the average weight of section A is 40 kg and that of section B is 35 kg. Calculate average weight of whole class.

$$A_w = \frac{36\times40 + 44\times35}{36+44} = 37.25$$

Important points about average:

(1) The average always lies above the lowest number of the set and the below highest number of the set.

(2) The net deficit due to the numbers below the average always equals the net surplus due to the numbers above the average.

(3) Ages and averages: If the average age of a group of persons is x years today then after 'n' years their average age will be (x+n).

Also, n years ago their average age would have been (x–n). This happens due to the fact that for a group of people, 1 years is added to each persons' age every year.

SOLVED EXAMPLES

1. There were 35 students in a hostel. Due to the admission of 7 new students the expenses of the mess were increased by Rs. 42 per day while the average expenditure per hand diminished by Rs. 1. The original expenditure of the mess was

(a) 450 rs (b) 320 rs (c) 550 rs (d) 420 rs

Soln. Let original average expenditure = x rs
then original expenditure for 35 students = $35x$ rs.
After admission of 7 new students
average expenditure = $(x-1)$ rs

hence total expenditure $= 42(x-1)$ rs

$$\therefore \quad 42(x-1)-35x = 42$$

$$42x-42-35x = 42$$

$$7x = 84 \Rightarrow 12 = x$$

then original expenditure = $35 \times 12 = 420$ rs

2. 9 persons went to a hotel for taking their meals. Eight of the spent 12 rs each on their meals and the ninth spent 8 rs. more than the average expenditure of all the time. What was the total money spent by them

(a) 115rs (b) 116rs (c) 117rs (d) 118rs

Soln. Let the average expenditure to all of them = x rs
then total expenditure = $9 \times x = 9x$ rs
Now, $12\times8 + (x+8) = 9x$

$$96 + x + 8 = 9x$$

$$8x = 104$$

$$x = 13$$

3. The average of 20 numbers is zero of them at most, how many may be greater than zero.

(a) 0 (b) 1 (c) 10 (d) 19

Soln. Average of 20 number = 0
Therefore, sum of 20 numbers = $0\times20 = 0$
Then it is possible that 19 of these number may be positive if their sum is equal to 20th negetive number.

4. The captain of a cricket team of 11 members is 26 years old and the wicket keeper is 3 yrs older. If the ages of these two are excluded, the average ages of remaining player is one year less than the average age of the whole team. What is the average age of the team?

(a) 23 yrs (b) 24 yrs (c) 25 yrs (d) none of these

Soln. Let the average age of whole team = x yrs
then total age = $11x$ yrs.
Now, after excluding 2 player the average age = $(x-1)$
hence, total age = $9(x-1)$ yrs

Now, $11x-(26+29) = 9(x-1)$

$$11x-55 = 9x-9$$

$$2x = 46 = 23$$

5. A car owner buys petrol at 7.50 rs, 8 rs and 8.50 rs per litre for three sucessive years. What approximately is the average cost per litre of the petrol if he spends Rs. 4000 each year

(a) 7.98 rs (b) 8 rs (c) 8.50 rs (d) 9 rs

Soln. Total quantity of petrol consumed in 3 yrs

$$\left(\frac{4000}{7.50}+\frac{4000}{8}+\frac{4000}{8.5}\right)\text{litre} = \frac{76700}{51}\text{ litres}$$

total amount spend = $3\times4000 = 12000$ rs

Hence, average cost $= \dfrac{\frac{12000}{76700}}{15} = \dfrac{12000 \times 51}{76700} = 7.98\ rs$

6. The average of 6 numbers is x and the average of three of these is y. If the average of remaining three is z. Then
(a) $x = y + z$ (b) $2x = y + z$ (c) $x = 2y + 2z$ (d) none of these

Soln. Let the numbers are $a_1, a_2, a_3, a_4, a_5, a_6$

hence, $\dfrac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} = x$

$\Rightarrow \quad a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 6x$

and $\dfrac{a_1 + a_2 + a_3}{3} = y, \qquad \dfrac{a_4 + a_5 + a_6}{3} = z$

$\Rightarrow \quad a_1 + a_2 + a_3 = 3y, \qquad a_4 + a_5 + a_6 = 3z$

hence, $3y + 3z = 6x$, $y + z = 2x$

7. Of the 3 numbers, the average of the first and second is greater than the average of second and third by 15. What is the difference between the first and the third of the three number
(a) 15 (b) 45 (c) 55 (d) none of these

Soln. Let the numbers are a_1, a_2 and a_3.

then $\dfrac{a_1 + a_2}{2} - \dfrac{(a_2 + a_3)}{2} = 15$

$\dfrac{a_1 + a_2 - a_2 - a_3}{2} = 15, \quad \dfrac{a_1 - a_3}{2} = 15$

$a_1 - a_3 = 2 \times 15 = 30$

8. If mean of a, b and c is M and $(ab + bc + ca) = 0$, then mean of a^2, b^2 and c^2 is
(a) $9M^2$ (b) $3M^2$ (c) M^2 (d) none of these

Soln. $\because \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$

$\Rightarrow \quad a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab+bc+ca)$

Now, mean of a, b, c is M

hence, $\dfrac{a+b+c}{3} = \mathrm{M} \quad \Rightarrow \quad a+b+c = 3\mathrm{M}$

then, $a^2 + b^2 + c^2 = (3\mathrm{M})^2 - 2 \times 0 = 9\mathrm{M}^2$

then mean of $a^2 + b^2 + c^2 = \dfrac{a^2 + b^2 + c^2}{3} = \dfrac{9\mathrm{M}^2}{3} = 3\mathrm{M}^2$

9. M, N, O, P and Q are 5 consecutive integers and the average of these 5 numbers is less than $\frac{1}{4}th$ of M. Then which of the following option is correct.
(a) M is positive (b) M is negative (c) M is even (d) Data inconsistent

Soln. Since, M, N, O, P and Q are positive integers. So, O is the average of these 5 numbers. And for O to be less than M, M must be greatest term in the A.P.
hence, N = (M–1) , O = (M–2), P = (M–3) and Q = (M–4)

Hence, average of M, N, O P and Q $= \frac{M+M-1+M-2+M-3+M-4}{5} = \frac{5M-10}{5} = M-2$

Now, $(M-2) < \frac{M}{4}$ $\Rightarrow 4M-8 < M \Rightarrow M < \frac{8}{3}$

But M is an integer

Hence, data inconsistent.

10. A two wheeler vehicle is fitted with new tyres (both front and rear) and it has one unused stepney. Each tyre can be used for 100 kms and after 100 kms, the tyres become useless. What is the maximum distance this two wheeler can travel.

(a) 100 km (b) 150 km (c) 200 km (d) 175 km

Soln. Use two tyres for first 50 km and change one with stepney. After next 50 kms first tyre which is still unchanged will become useless and the tyre which had been taken out can be refitted till next 50 km untill it becomes useless. So, total distance travelled = 50 + 50 + 50 = 150 km.

EXERCISE

1. The average of a batsman after 25 innings was 56 runs per innings. If after 26th innings his average increased by 2 runs then what was his score in 26th inning?

(a) 106 (b) 107 (c) 108 (d) 109

2. The average age of class of 30 students and a teacher is decreased by 0.5 years if we exclude the teacher. If the initial average is 14 years and find the age of class teacher.

(a) 27 (b) 28 (c) 29 (d) 30

3. The average marks of a group of 20 students on a test is reduces by 4 when the topper who scored 90 marks are replaced by a new student. How many marks did the new student have?

(a) 10 (b) 11 (c) 12 (d) 13

4. The average age of a family of five persons is 20 years. If the youngest member is 8 years old. What was the average age of the family at the birth time of the youngest member?

(a) 12 years (b) 15 years (c) 18 years (d) 16 years.

5. If a, b, c, d and e are five consecutive odd numbers, their average is:

(a) $5(a+4)$ (b) $\frac{abcde}{5}$ (c) $5(a+b+c+d+e)$ (d) $a+4$

6. The average weight of 35 students in a class is 35 kg. If the teacher is also included, the average weight increases to 36 kg. The weight of the teacher is:

(a) 36 kg (b) 35 kg (c) 70 kg (d) 71 kg

7. Of the three numbers, the first is four times the second and three times the third. If the average of all the three numbers is 95, then the third number is:

(a) 57 (b) 76 (c) 38 (d) 60

8. The average age of 10 members in a family is 21 years and due to death of one family member, the average age is reduced by 2 months. The age of the member who died is:

(a) 20 years 10 months (b) $20\frac{1}{3}$ years

(c) 22 years 6 months (d) $19\frac{1}{3}$ years

9. The average of three consecutive odd numbes is 14 more than one-third of the first of these numbers. What is the last of these numbers?

(a) 17 (b) 19 (c) 15 (d) Data inadequate.

10. The captain of a cricket team of 11 players is 25 years and the wicket keeper is 3 years older. If the age of the two are excluded, the average age of the remaining players is 1 year less than the average age of the whole team. The average age of the whole team is:
(a) 24 years (b) 21 years (c) 26 years (d) 22 years

11. In a coconut groove, (x + 2) trees yield 60 nuts per year, x trees yield 120 nuts per year and (x–2) trees yield 180 nuts per year. If the average yield per year is per tree be 100. Then the value of x is
(a) 8 (b) 4 (c) 12 (d) 10

12. A cricket player makes 200 runs in the 15 over. In doing so, his average at the end of the 14th over increases by 10. What was his average at the end of 15th over?
(a) 60 (b) 50 (c) 40 (d) 45

13. A body weighs 121 grams and 125.44 grams on two different pans of a faulty balance. Its true weight will be:
(a) 4.44 gm (b) 120 gms (c) 123.22 gm (d) 130 gms

14. The average of 9 numbers is 61. If the average of first 5 numbers is 58 and that of last 5 numbers is 63, what is the fifth number?
(a) 66 (b) 61 (c) 56 (d) 53

15. In a cricket match, five batsmen A, B, C, D and E scored an average of 36 runs. D scored 5 more than E; E scored 8 fewer than A; B scored as many as D and E combined; B and C scored 107 between them. How many runs did E score?
(a) 20 (b) 28 (c) 45 (d) 62

16. A cricketer's average in his first 18 innings was 16.5 runs. After a further 8 innings his average had increased to 32.5 runs. What was his average for the last 8 innings.
(a) 120 (b) 80.5 (c) 68.5 (d) 65.5

HINTS & SOLUTIONS

1. Runs in 26th innings = total runs after 26th inning – total runs after 25th innings.
$= 26\times58-25\times56 = 108$
Answer is (c)

2. Age of teacher = total age of student and teacher – total age of students.
$= 31 \times 14 - 30\times13.5 = 434 - 405$
$= 29$ years.
Answer is (c)

3. The replacement has the effect of reducing the average marks for each of 20 students by 4. Hence the replacement must be $20\times4 = 80$ marks below the original and hence answer is $90-80 = 10$
Answer is (a)

4. Now average of a family members = 20 years
Eight year ago average age = 12 years.
Total age eight years ago of all members = 12×5
But members are total 4 so average $= \frac{12\times5}{4} = 15$ years
Answer is (b)

5. Five consecutive odd number.
So, $b = a + 2, c = a+ 4, d = a + 6, e = a + 8$
$\text{Average} = \frac{a+a+2+a+4+a+6+a+8}{5} = \frac{5a+20}{5} = a+4$
Answer is (d)

6. Answer is (d)

7. Let numbers are a, b, c
According to problem $a = 4b = 3c$

$$\text{Average} = \frac{a+b+c}{3} = \frac{3c+\frac{3c}{4}+c}{3} = \frac{c\left(\frac{19}{4}\right)}{3} = 95$$

Thus $c = 60$
Answer is (d)

8. Answer is (c) 9. Answer is (d) 10. Answer is (d)

9. $$\text{Average yield per tree} = \frac{60(x+2)+120x+(x-2)180}{x+2+x+x-2} = 100$$
$$\Rightarrow 360x - 240 = 300x,\ 60x = 240 \Rightarrow 4 = x$$
Answer is (b)

10. Answer is (a)

11. $$\text{Should be average of two} = \frac{121+125.44}{2} = 123.22$$
Answer is (c)

12. Sum of 9 numbers = $61 \times 9 = 549$
Sum of first five number = $58 \times 5 = 290$
Sum of last five number = $63 \times 5 = 315$
Hence, 5th number = $315 + 290 - 549 = 605 - 549 = 56$
Answer is (c)

13. $A + B + C + D + E = 36 \times 5 = 180$
$D = E + 5$, $E = A - 8$, $B = D + E$, $B + C = 107$
$A + A - 8 + A - 3 + 10.7 = 180$
$3A = 180 - 96 = 84$
Therefore, $A = 28$
Hence, $E = 28 - 8 = 20$
Answer is (a)

16. Total runs after 18 innings = $16.5 \times 18 = 297.0$ runs
And after 26 innings total runs = $32.5 \times 26 = 845.0$ run
Total runs in 8 innings = $845 - 297 = 548$ run
Hence, Average = $548/8 = 68.5$
Answer is (c)

CHAPTER

3

Alligation or Mixture

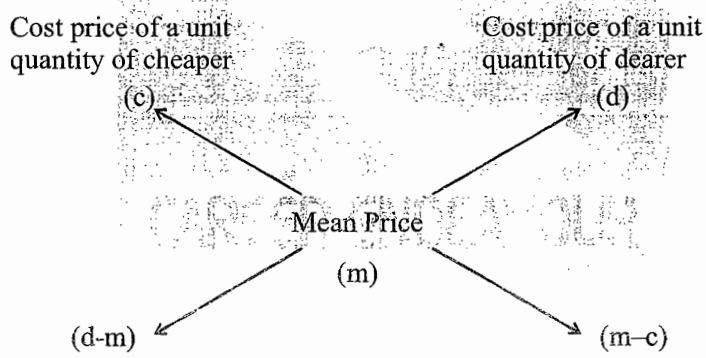
Alligation: It is rule that enables us to find the ratio in which two or more ingredient at the given price must be mixed to produced a mixture of desired price.

Mean price: The cost price of unit quantity of the mixture is called the mean price.

Rule of alligation: If two ingrediants are mixed, then

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} = \frac{\text{Cost price of dearer} - \text{mean price}}{\text{Mean price} - \text{cost price of cheaper}}$$

We present as under:



$$\therefore \quad \frac{\text{cheaper quantity}}{\text{dearer quantity}} = \frac{d-m}{m-c}$$

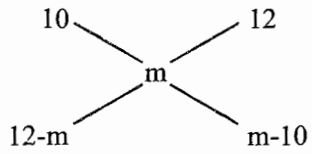
* When water is mixed with any liquid and then mixture is sold on cost price and earn a profit of r %, then the ratio of liquid and water in mixture must be always 100 : r %.

*. If any two container contains the mixture of any liquid and water (another liquid) in ratio of a : b and c : d respectively and if same quantity of mixture from both container are mixed in 3rd container, then new ratio of liquid and water in third container will be (2ac + ad + bc) : (2bd + ad + bc).

SOLVED EXAMPLES

1. Two varieties of rice at Rs. 10 per kg and Rs. 12 per kg are mixed together in the ratio 1:2. Find the average price of the resulting mixture.

Soln. If 'm' is average cost of two varieties of rice
Applying above format



Thus, $\frac{1}{2}=\frac{12-m}{m-10}$ $\boxed{m=11.33\ Rs/kg}$

2. In what ratio must water be mixed with milk to gain 20 % by selling the mixture at cost price.

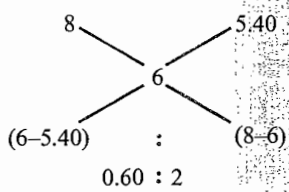
Soln. We know that the ratio will be 100 : r %
Hence, 20: 100 $\Rightarrow$ 1 : 5 (Ratio of water and milk respectively)

3. How many kgs. of wheat costing Rs. 8 per kg must be mixed with 36 kg of wheat costing Rs. 5:40 per kg so that 20% gain may be obtained by selling the mixture at Rs. 7.20 per kg?

Soln. Selling price of 1 kg mixture = Rs. 7.20, Gain = 20 %

We know that cost price = $S.P.\times\frac{100}{(100+\text{gain }\%)}=7.20\times\frac{100}{100+20}=\frac{7.20\times100}{120}=6$ Rs.

Hence,



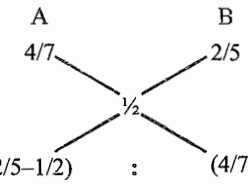
0.60 : 2
6 : 20
3 : 10

$\Rightarrow \frac{\text{wheat of first kind}}{\text{wheat of second kind}}=\frac{3}{10}\Rightarrow\frac{x}{36}=\frac{3}{10}\Rightarrow 10x=36\times3\Rightarrow x=\frac{36\times3}{10}=10.8$ kg

4. The poison and water in two vessels A and B are in the ratio 4 : 3 and 2 : 3 respectively. In what ratio the liquid in both the vessels be mixed to obtain a new mixture in vessel C containing half poison and half water.

Soln. Part of poison in vessel $A=\frac{4}{7}$, water $=\frac{3}{7}$; Part of poison in vessel $B=\frac{2}{5}$, water $=\frac{3}{5}$

Part of poison in new mixture $=\frac{1}{2}$, water $=\frac{1}{2}$



Hence, 1/10 : 1/14
7 : 5

Note: If two ingredients in any mixture are in ratio of a : b, then part of first $=\frac{a}{a+b}$ and part of second $=\frac{b}{a+b}$.

5. Any can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 litres of mixture are drawn off and the can is filled with B, the ratio of A and B becomes 7 : 9. How many litres of liquid A was contained by the can initially.

Soln. Let the quantity of A and B is 7x and 5x litres respectively when 9 litres are drawn off, then

$$\text{Quantity of A in mixture left} = \left(7x - \frac{7}{12}\times 9\right) = \left(7x - \frac{21}{4}\right) \text{litres}$$

$$\text{Quantity of B in mixture left} = \left(5x - \frac{5}{12}\times 9\right) = \left(5x - \frac{15}{4}\right) \text{litres}$$

then, $$\frac{\left(7x - \frac{21}{4}\right)}{\left(5x - \frac{15}{4}\right) + 9} = \frac{7}{9} \quad \Rightarrow \frac{28x - 21}{20x + 21} = \frac{7}{9} \quad \Rightarrow 252x - 189 = 140x + 147$$

$$112x = 336 \Rightarrow x = 3$$

So, the can contained liquid A = 7×3 = 21 litre

Container Problem: Suppose a container contains x units of liquid from which 'y' units are taken out and replaced by water. After 'n' operations, the quantity of pure liquid $\left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

6. A container contains 40 liter of milk from those container 4 liters of milk was taken out and replaced by water. This process was repeated two times more. How much milk is now contained by container.

Apply formula: $\left[x\left(1 - \frac{y}{x}\right)^n\right]$ where, $x = 40, y = 4, n = 3$

Thus amount of milk left after 3 operation $= 40\left(1 - \frac{4}{40}\right)^3 = 29.16$ liters

7. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask hold originally.

Soln. **Note:** when quantity of mixture is constant and their ingrediants are changed, then sum of ratio of elements is equal to total mixture.

Let initial quantity of wine is x liters.

then quantity of wine left after 4 operations $= x\left(1 - \frac{8}{x}\right)^4$ litre

$\because$ since now ratio of water and wine = 65 : 16

Hence, ratio of wine and mixture = 16 : (65 + 16) = 16: 81

Hence, $$\frac{x\left(1 - \frac{8}{x}\right)^4}{x} = \frac{16}{81} \quad \Rightarrow \left(1 - \frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4$$

$\therefore$ $$1 - \frac{8}{x} = \frac{2}{3} \quad \Rightarrow \frac{x - 8}{x} = \frac{2}{3} \quad \Rightarrow 3x - 24 = 2x \quad \Rightarrow x = 24$$

EXERCISE

1. How many kilograms of sugar costing Rs. 9 per kg must be mixed with 27 kg of sugar costing Rs 7 per kg so that there may be a gain of 10% by selling the mixture at Rs. 9.24 per kg?
(a) 36 kg (b) 42 kg (c) 54 kg (d) 63 kg

2. In what ratio must water be mixed with milk to gain $16\frac{2}{3}\%$ on selling the mixture at cost price?
(a) 1:6 (b) 6:1 (c) 2:3 (d) 4:3

3. In what ratio water be mixed with oil costing Rs. 12 per litre to obtain a mixture worth of Rs. 8 per liter
(a) 1 : 2 (b) 4 : 1 (c) 2 : 3 (d) 3 : 2 Ans. (a)

4. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. The percentage of water in the mixture is:
(a) 4% (b) $6\frac{1}{4}\%$ (c) 20% (d) 25%

5. Two vessels A and B contain spirit and water mixed in the ratio 5:2 and 7:6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8:5?
(a) 4:3 (b) 3:4 (c) 5:6 (d) 7:9

6. Two vessels A and B contain milk and water mixed in the ratio 8:5 and 5:2 respectively. The ratio in which these two mixtures be mixed to getr a new mixture containing $69\frac{3}{13}\%$ milk, is:
(a) 2:7 (b) 3:5 (c) 5:2 (d) 5:7

7. A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contains 50% water . How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3:5?
(a) 4 litres, 8 litres (b) 6 litres, 6 litres (c) 5 litres, 7 litres (d) 7 litres, 5 litres.

8. One quality of wheat at Rs.9.30 per kg is mixed with another quality at a certain rate in the ratio 8:7. If the mixture so formed be worth Rs. 10 per kg, what is the rate per kg of the second quality of wheat?
(a) Rs. 10.30 (b) Rs. 10.60 (c) Rs. 10.80 (d) Rs.11

9. The cost of type 1 rice is Rs. 15 per kg and type 2 rice is Rs. 20 per kg. If both are mixed in the ratio of 2 : 3, then the price per kg of the mixed variety of rice is
(a) Rs. 18 (b) Rs. 18.50 (c) Rs. 19 (d) Rs. 19.50

10. In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10%
(a) 3 : 2 (b) 3 : 4 (c) 3 : 5 (d) 4 : 5

11. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety in the ratio 1:1:2. If the mixture is worth Rs. 153 per kg, the price of the third variety per kg will be
(a) Rs. 169.50 (b) Rs. 170 (c) Rs. 175.50 (d) Rs. 180

12. A merchant has 1000 kg of sugar, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is:
(a) 400 kg (b) 560 kg (c) 600 kg (d) 640 kg

13.

14.

15 .

1.

2.

3.

4.

13. A jar full of whisky contains 40% alcohol. A part of this whisky is replaced by another containing 19% alcohol and now the percentage of alcohol was found to be 26%. The quantity of whisky replaced is:

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$

14. Three types of tea a, b, and c cost Rs 95/kg, Rs 100/kg and Rs 70/kg respectively. How many kg of each should be blended to produce 100 kg of mixture worth Rs 90/kg? Given that the quantities of b and c are equal.

(a) 70, 15, 15 (b) 50, 25, 25 (c) 60, 20, 20 (d) 40, 30, 30

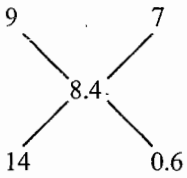
15. A clever servant is sent to a coffee shop where a standard blend of Plantation with Peaberry is sold in the ratio of 2:1. It is given that at that shop Plantation costs Rs. 0.33/gram, and Peaberry costs Rs. 0.24/gram. The servant finds that he does not have enough money to buy 100 grams of the standard blend. Being clever he interchanges the ratio and brings home 100 grams of the new blend for all money he had. How much money did the servant have?

(a) 84 Rs (b) 81 Rs (c) 30 Rs (d) 27

(e) None of the above.

HINTS & SOLUTIONS

1. Cost price of 1 kg of mixture $= 9.24 \times \frac{100}{110} = 8.40$

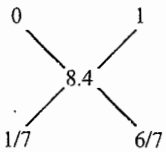


Ratio $\frac{1st}{2nd} = \frac{1.4}{0.6} = \frac{14}{6} = \frac{7}{3}$

Answer is (d)

2. Let cost price of milk be Re 1.

Cost of 1 liter of mixture $= \left(100 \times \frac{3}{50} \times 1\right) = \frac{6}{7}$



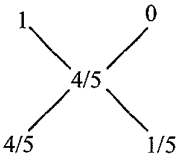
Ratio of water to milk $= \frac{1}{7} : \frac{6}{7} = \frac{1}{6}$

Answer is (a)

3. Answer is (a)

4. Cost price of 1 liter milk $= \frac{4}{5}$

Ratio of milk to water $\frac{4}{5}:\frac{1}{5}=4:1$

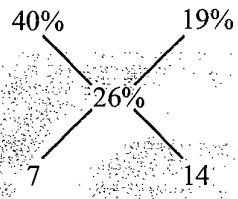


Hence, percentage of water in mixture $=\frac{1}{5}\times 100=20\%$

Answer is (c)

5. Answer is (d) 6. Answer is (a) 7. Answer is (b) 8. Answer is (c)
9. Answer is (a) 10. Answer is (a) 11. Answer is (c) 12. Answer is (c)

13. By the rule of alligation strength of 1st jar strength of 2nd jar



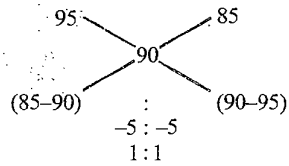
Ratio of 1st and 2nd jar $=\frac{7}{14}=\frac{1}{2}$

Therefore, the required quantity replaced $=\frac{2}{3}$

Answer is (b)

14. Answer is (b)

Since quantity of B and C are equal



Hence, average price of B and C $=\frac{100+70}{2}=85$ rs/kg

For 100kg of mixture quantity of A $=\frac{1}{2}\times 100=50$ kg

B and C = 50 kg both are equal.
Hence, (50, 25, 25) kg.
Answer is (b)

15. Average price of standard blend $=\frac{0.33\times 2+0.24\times 1}{2+1}=\frac{0.90}{3}=0.30$rs/gram

Hence, for 100 grams $=0.30\times 100=30$ rs

If he changed the ratio then Average price $=\frac{1\times 0.33+0.24\times 2}{3}=\frac{0.81}{3}=0.27$ rs/gram

For 100 gram $=0.27\times 100=27$ rs.
Answer is (d)

CHAPTER

Progressions

1. **Arithmatic Progression:** Quantities are said to be in arithemtic progression when they increase or decrease by a common difference. a, a +d, a + 2d, a + 3d, ... The common difference is found by subtracting any term of the series from the next term. Thus the n^{th} term of arithmetic progression is given by $T_r = a + (n-1)d$.

(i) To find the sum of the given number of terms in an Arithmetic progression:

Let 'a' and 'b' be two quantities and A be their arithmetic mean. Then since a, A, b are in A.P. We must have b–A = A–a. Each being equal to the common difference.

This gives us $A = \frac{(a+b)}{2}$

Between two given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in AP. The terms thus inserted are called the arithmetic means.

(ii) Process for finding the sum of an AP.

Once you can find a pair of corresponding terms for any AP, you can easily find the sum of the AP by using property of averages.

i.e. sum = number of term × average.

In fact, this is the best process for finding the sum of an AP it is much more superior to the process of finding the sum of an AP using the expression $\frac{n}{2}(2a + (n-1)d)$.

2. **Geometric Progression:** Quantities are said to be in Geometric Progression when they increase or decrease by a constant factor. The constant factor is also called the common ratio and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4, ...$

If 'n' be the number of terms and if l denote the last, or n^{th} term, we have $\ell = ar^{n-1}$

(i) To find the Geometric mean between two given quantities:

Let 'a' and 'b' be the given quantities and 'n' the required number of means to be inserted. In all there will be n +2 terms so that we have to find a series n + 2 terms in GP of which 'a' is the first and 'b' is the last.

Let 'r' be the common ratio:

Then $b = \text{term}(n+2)\text{ th term} = ar^{n+1}$;

$\therefore \quad r^{(n+1)} = \frac{b}{a} \qquad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

Hence, the required number of means are $ar, ar^2, ...ar^n$, where 'r' has the value found in above equation.

To find the sum of a number of terms in a geometric progression:

Let 'a' be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$\boxed{S_n = \frac{a(r^n - 1)}{(r-1)}} \qquad \ldots (1)$$

If $r < 1$, then

$$\boxed{S_n = \frac{a(1 - r^n)}{(1-r)}} \qquad \ldots (2)$$

Note: It will be convenient to remember both forms given above for S. Number (2) will be used in all cases execept when 'r' is positive and greater than one.

Sum of an infinite geometric progression when $r < 1$.

$$\boxed{S_\infty = \frac{a}{(1-r)}}$$

Obviously this formula is used only when the common ratio of the GP is less than one.

3. Some important results about series:

1. If the same quantity be added to or subtracted from all the terms of an AP the resulting terms will form at AP but with the same common difference as before.

2. If all the terms of an AP be multiplied or divided by the same quantity the resulting terms will form an AP but with a new common difference, which will be the multiplication/division of the old common difference(as the case may be)

3. If all the terms of a GP be multiplied or divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.

4. If a, b, c, d, ... are in GP, they are also in continued proportion, since, by definition.
   $a/b = b/c = c/d = ... = 1/r$
   Conversely, a series of quantities in continued proportion may be represented by x, xr, xr^2.....

5. If you have to assume 3 terms in AP, assume them as $a - d, a, a + d$ or as a, a+d and $a + 2d$.

   For assuming 5 terms of an AP we use: $a - 2d, a - d, a, a + d, a + 2d$. These are the most convenient in terms of problems solving.

   For assuming 4 terms: $a - 3d, a - d, a + d, a + 3d$

6. For assuming three terms of a GP assume them as a, ar and ar^2 or as $\frac{a}{r}$, a, ar

7. To find the sum of the first 'n' natural numbers.

   Let the sum be denoted by S then $S = 1 + 2 + 3 + ... + n$ is given by $S = \frac{n(n+1)}{2}$

8. To find the sum of square of the first n natural numbers.

Let the sum be denoted by S then $S = 1^2 + 2^2 + 3^2 + ... + n^2$

This is given by $S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

9. To find the sum of the cubes of the first 'n' natural numbers

Let the sum be denoted by S; then $S = 1^3 + 2^3 + 3^3 + ... + n^3$; $S = \left[\frac{n(n+1)}{2}\right]^2$

Thus, the sum of the cubes of the first 'n' natural numbers is equal to the square of the sum of these numbers.

10. To find the sum of the first 'n' odd natural numbers.

$S = 1 + 3 + 5 + ... + (2n-1) \rightarrow n^2$

11. How many numbers between 100 and 200 divisible by three.

Soln. The first number is 102 and the last number is 198. Hence, $\frac{(198-102)}{2} + 1 = 33$ (since both 102 and 198 are included).

12. A company models its annual profits using the function $P(x) = x^2 + 20x - 300$, where P represents profits and x gives the number of units sold. One year, their profits were Rs 167, 700. How many units of their product did they sell?

(a) 200 (b) 400 (c) 420 (d) 600

13. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which are in A.P. when 30 of the installments are paid, the dues leaving one third of the debt unpaid. Find the value of 8th installment.

Soln. Let the first instalment be a and common difference of A.P. be d.

Given, 3600 = sum of 40 terms $= \frac{40}{2}\{2a + (40-1)d\}$

$\Rightarrow \quad 3600 = 20\{2a + 39d\}$

$\Rightarrow \quad 2a + 39d = 180$ ... (i)

After 30 installments, one third of the debt is unpaid

Hence, $\frac{3600}{3} = 1200$ is unpaid and 2400 is paid.

Now, $2400 = \frac{30}{2}\{2a + (30-1)d\}$

$\Rightarrow \quad 2400 = 15\{2a + 29d\}$

$\Rightarrow \quad 2a + 29d = 160$ ... (ii)

Subtracting (ii) from (i), we get

$\Rightarrow \quad 10d = 20 \quad \therefore \; d = 2$

From (i), $2a = 180 - 39d = 180 - 39 \times 2 = 180 - 78 = 102$

Therefore, $a = 51$

Now value of the 8th instalment $= a + (8-1)d = 51 + 7 \times 2 = 65$

14. The population of bacteria culture doubles every 3 minutes. How many minutes will it take for the population to grow from 1000 to 512000 bacteria?

Soln. $a = 1000$, $r = 2$ and $t_n = 512000$

$t_n = ar^{n-1}$

$512000 = 1000(2)^{n-1} \Rightarrow 2^{n-1} = 2^9 \quad \Rightarrow n = 10$

Therefore, 9 times spaces.
Therefore, total time = 9×3 = 27 min

15. A lucky man finds 6 pots of gold coins. He counts the coins in the first 4 pots to be 60, 30, 20 and 15 respectively. If there is a definite progression, what would be the number of coins in the next two pots.
(a) 10 and 5 (b) 4 and 2 (c) 15 and 15 (d) 12 and 10
[NET LIFE SCIENCES 2013]

Soln. As we know,

$60 \times 1 = 60$

$60 \times \frac{1}{2} = 30$

$30 \times \frac{2}{3} = 20$

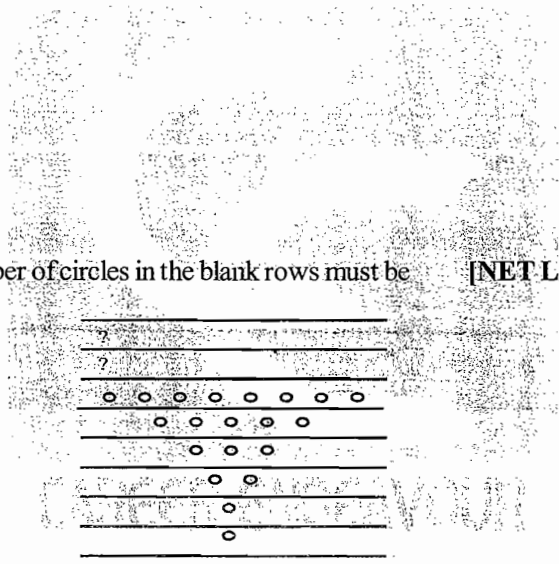
$20 \times \frac{3}{4} = 15$

$15 \times \frac{4}{5} = 12$

$12 \times \frac{5}{6} = 10$

Hence, 12 and 10

16. In the figure below the number of circles in the blank rows must be **[NET LIFE SCIENCES 2012]**



(a) 12 and 20 (b) 13 and 20 (c) 13 and 21 (d) 10 and 11

Soln. Number of circles are 1, 1, 2, 3, 5, 8, ?, ?
Here every next term is sum of previous two term.

1 + 1 = 2
2 + 1 = 3
3 + 2 = 5
5 + 3 = 8
8 + 5 = 13
13 + 8 = 21

Answer is 13 and 21.

17. The sum of n terms given by the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots..$ is given by

(a) $2^n - 1$ (b) $2^n - n - 1$ (c) $2^{-n} + n - 1$ (d) $1 - 2^{-n}$

Soln. We have $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ times

$$\Rightarrow \quad \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)+.....\text{n term}$$

$$\Rightarrow \quad (1+1+1+.....\text{n term})-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+.....\text{n terms}\right)$$

$$\Rightarrow \quad n-\frac{\frac{1}{2}\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} \quad \Rightarrow \quad n-\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{\frac{1}{2}} \quad \Rightarrow \quad n-\left(1-\frac{1}{2^n}\right)$$

$$\Rightarrow \quad 2^{-n}+n-1$$

18. If $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$ then a, b, c are in

(a) AP (b) GP (c) HP (d) none of these

Soln. We have

$$\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{a}+\frac{1}{c}$$

$$\Rightarrow \quad \frac{1}{b-a}-\frac{1}{c}=\frac{1}{a}-\frac{1}{b-c} \quad \Rightarrow \quad \frac{c-(b-a)}{(b-a)c}=\frac{b-c-a}{(b-c)a}$$

$$\Rightarrow \quad \frac{1}{(b-a)c}=\frac{-1}{(b-c)a} \quad \Rightarrow \quad (b-c)a=-c(b-a)$$

$$\Rightarrow \quad ab-ac=-bc+ac \quad \Rightarrow \quad 2ac=ab+bc$$

$$\Rightarrow \quad 2ac=b(a+c) \quad \Rightarrow \quad b=\frac{2ac}{a+c}$$

So, a, b, c are in HP

19. If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the AM between a and b, then the value of n is

(a) –1 (b) 0 (c) 1/2 (d) 1

Soln. Since $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}=\frac{a+b}{2}$

$$2\left(a^n+b^n\right)=(a+b)\left(a^{n-1}+b^{n-1}\right)$$

$$\Rightarrow \quad 2a^n+2b^n=a.a^{n-1}+ab^{n-1}+b.a^{n-1}+b.b^{n-1}$$

$$\Rightarrow \quad 2a^n+2b^n=a^n+ab^{n-1}+ba^{n-1}+b^n$$

$$\Rightarrow \quad a^n+b^n-ab^{n-1}-ba^{n-1}=0$$

$$\Rightarrow \quad a^n-ba^{n-1}+b^n-ab^{n-1}=0$$

$$\Rightarrow \quad a^{n-1}(a-b)+b^{n-1}(b-a)=0$$

$\Rightarrow \quad a^{n-1}(a-b)-b^{n-1}(a-b)=0$

$\Rightarrow \quad (a-b)\left(a^{n-1}-b^{n-1}\right)=0$

Since, $(a-b)\neq 0$

Hence, $a^{n-1}-b^{n-1}=0$

$$\left(\frac{a}{b}\right)^{n-1}=1=\left(\frac{a}{b}\right)^{0}$$

$\therefore \quad n-1=0 \Rightarrow \quad n=1$

20. A monkey while trying to reach the top of a pole of height 12 mtrs. take every time a jump of 2 mtr. but slips one meter while holding the pole. The number of required to reach the top of pole is

(a) 6 (b) 10 (c) 11 (d) 12

Soln. It is clear that in each jump monkey cover only 1 m finally. Hence 10 jump monkey cover only 10 m finally. Hence, 11 jump are required.

21. If a > 1, b > 1 then the minimum value of $\log_b^a+\log_a^b$ is

(a) 0 (b) 1 (c) 2 (d) none of these

Soln. As $AM \geq GM$

$$\frac{\log_b^a+\log_a^b}{2}\geq\left(\log_b^a.\log_a^b\right)^{1/2}$$

$$\frac{\log_b^a+\log_a^b}{2}\geq\left[\frac{\log a}{\log b}\times\frac{\log b}{\log a}\right]^{1/2}$$

$$\Rightarrow \quad \frac{\log_a^b+\log_b^a}{2}\geq 1 \Rightarrow \log_b^a+\log_a^b\geq 2$$

22. A person purchases one kg of tomatoes from each of the 4 places at the rate of 1kg, 2kg, 3kg and 4kg per rupee respectively. On the average he has purchased x kg of tomatoes per rupee, then the value of x is

(a) 2 (b) 2.5 (c) 1.92 (d) none of these

Soln.
$$HM=\frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}=\frac{4\times 12}{25}=\frac{48}{25}=1.92$$

HM = 1.92

23. Let $a_1, a_2 \ldots\ldots a_n$ be AP. If $\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\ldots\ldots+\frac{1}{a_na_1}=\frac{k}{a_1+a_n}\left\{\frac{1}{a_1}+\frac{1}{a_2}+\ldots\ldots+\frac{1}{a_n}\right\}$ then k is equal to

(a) 1 (b) 2 (c) 3 (d) none of these

Soln. Since, $a_1, a_2, a_3 \ldots\ldots\ldots a_n$ are in AP then

$a_1+a_n=a_2+a_{n-1}=a_3+a_{n-2}$

Now, $\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\ldots\ldots+\frac{1}{a_na_1}$

Now, multiply and divide by a_1+a_n, we get

$$\frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_1a_n}+\frac{a_1+a_n}{a_2\,a_{n-1}}+.....+\frac{a_1+a_n}{a_na_1}\right]$$

$$\Rightarrow \quad \frac{1}{a_1+a_n}\left[\frac{a_1+a_n}{a_na_1}+\frac{a_2+a_{n-1}}{a_{n-1}a_2}+\frac{a_3+a_{n-2}}{a_{n-2}a_3}+.....+\frac{a_n+a_1}{a_1a_n}\right]$$

$$\Rightarrow \quad \frac{1}{a_1+a_n}\left[\left(\frac{1}{a_n}+\frac{1}{a_1}\right)+\left(\frac{1}{a_{n-1}}+\frac{1}{a_2}\right)+.....+\left(\frac{1}{a_1}+\frac{1}{a_n}\right)\right]$$

$$\Rightarrow \quad \frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+.....+\frac{1}{a_n}\right]$$

Hence, k = 2.

24. If x, y, z are in AP, then $(x+2y-z)(2y+z-x)(z+x-y)$ is

(a) $4xyz$ (b) $2xyz$ (c) xyz (d) none of these

Soln. As x, y, z are in AP, hence

$$y=\frac{x+z}{2};\ 2y=x+z$$

Now, $(x+2y-z)(2y+z-x)(z+x-y)$

$$\Rightarrow \quad (x+x+z-z)(x+z+z-x)(2y-y)$$

$$\Rightarrow \quad (2x)(2z)(y)$$

$$\Rightarrow \quad 4xyz$$

25. If H is the harmonic mean between 'a' and 'b' then $\frac{H+a}{H-a}+\frac{H+b}{H-b}$ is

(a) 1 (b) 2 (c) –1 (d) –2

Soln. $\because \quad H=\frac{2ab}{a+b}$

$$\frac{H}{a}=\frac{2b}{a+b} \text{ and } \frac{H}{b}=\frac{2a}{a+b}$$

Using componendo& dividendo

$$\frac{H+a}{H-a}=\frac{2b+a+b}{2b-a-b} \qquad \frac{H+b}{H-b}=\frac{2a+a+b}{2a-a-b}$$

$$\frac{H+a}{H-a}=\frac{3b+a}{b-a} \qquad \frac{H+b}{H-b}=\frac{3a+b}{a-b}$$

Hence, $\frac{H+a}{H-a}+\frac{H+b}{H-b}=\frac{3b+a}{b-a}-\frac{3a+b}{b-a}=\frac{3b+a-3a-b}{b-a}=\frac{2b-2a}{b-a}=\frac{2(b-a)}{b-a}=2$

26. If a, b and c are positive. Then the minimum value of $a^{\log b-\log c}+b^{\log c-\log a}+c^{\log a-\log b}$ is

(a) 3 (b) 1 (c) 9 (d) 16

Soln. Using $AM \geq GM$

$$\Rightarrow \quad \frac{a^{\log b-\log c}+b^{\log c-\log a}+c^{\log a-\log b}}{3} \geq \sqrt[3]{a^{\log b-\log c}b^{\log c-\log a}c^{\log a-\log b}} \quad \ldots (i)$$

Now, let $x = a^{\log b-\log c}b^{\log c-\log a}c^{\log a-\log b}$

Hence, $\log x = (\log b-\log c)\log a+(\log c-\log a)\log b+(\log a-\log b)\log c$

$= \log ab-\log ac+\log bc-\log ab+\log ac-\log bc$

$\log x = 0$

$\log x = \log 1 \quad \Rightarrow x = 1$

from (i), we get

$$\frac{a^{\log b-\log c}+b^{\log c-\log a}+c^{\log a-\log b}}{3} \geq 1$$

27. If $S_n = 1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots..+\frac{1}{2^{n-1}}$ then the least value of n such that $2-S_n < \frac{1}{100}$ is

(a) 6 (b) 8 (c) 10 (d) none of these

Soln. $S_n = 1+\frac{1}{2}+\frac{1}{2^2}+\ldots.....+\frac{1}{2^{n-1}}$

$$\therefore \quad S_n = \frac{1\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} = 2-\frac{1}{2^{n-1}}$$

$$\because \quad 2-S_n < \frac{1}{100}$$

$$\Rightarrow \quad \frac{1}{2^{n-1}} < \frac{1}{100}$$

$$\Rightarrow \quad 2^{n-1} > 100 > 2^6 \quad \Rightarrow 2^{n-1} > 2^6 \quad \Rightarrow n-1 > 6 \quad \Rightarrow n > 7$$

EXERCISE

1. If sum of squares of two number is 3341 and the difference of their squares is 891 then numbers are
(a) 25, 36 (b) 25, 46 (c) 35, 46 (d) Insufficient data.

2. What is the sum of two consecutive even numbers the difference of whose squares is 84?
(a) 34 (b) 38 (c) 42 (d) 46

3. The sum of three consecutive odd numbesr is 20 more than the first of these number then what is the middle number?
(a) 7 (b) 9 (c) 12 (d) Insufficient data.

4. There are two numbers such that the sum of twice the first and thrice the second is 39 while the sum of thrice the first and twice the second is 36 then larger of the two number is:
(a) 6 (b) 9 (c) 12 (d) Insufficient data.

5. The sum of the digits of a two digit number is 15 and the difference between the digits is 3 what is this two digit number?
(a) 69 (b) 78 (c) 76 (d) None

6. A two digit number is such that the product of the digits is 8 when 18 is added to the number then digits are reversed the number is:
(a) 24 (b) 18 (c) 12 (d) 43

7. A number consists of two digits. If the digits interchange places and new number is added to the original number the resulting number is divisible by:
(a) 11 (b) 4 (c) 5 (d) 9

8. If 10 term of an AP is 84 and commond difference is 8, find the first term
(a) 10 (b) 12 (c) 14 (d) 16

9. Find the sum of $1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+.....$
(a) 1/2 (b) 3/2 (c) 2 (d) 3

10. Find the 14th term of $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}$
(a) $\frac{-1}{2^{12}}$ (b) $\frac{-1}{2^{13}}$ (c) $\frac{-1}{2^{14}}$ (d) $\frac{-1}{2^{15}}$

11. Manufacturer of a tennis ball announced that their new ball is the best in the market. If one were to drop it from any height, it would only rise to the extent of 10% of the height from which it was dropped. If the ball was dropped from 45 feet, how much would the ball travel before coming to rest?
(a) 45 ft (b) 50 ft (c) 55 ft (d) 49.5 ft

HINTS & SOLUTIONS

1. $a^2 + b^2 = 3341$ ... (i)

   $a^2 - b^2 = 891$ ... (ii)

   (i) + (ii) $2a^2 = 4232$, Thus $a = 46$

   Answer is (c)

2. $(a+2)^2 - a^2 = 84$; $a^2 + 4a + 4 - a^2 = 84$; $4a = 80$; $a = 20$

   Answer is (c)

3. Let number a, a+2, a + 4 Given: $a + a + 2 + a + 4 - a = 20$; $2a + 6 = 20$; $a = 7$

   Answer is (b)

4. Let first number is 'a' and second number is 'b'.

   $2a + 3b = 39$ ... (i)

   $3a + 2b = 36$ ... (ii)

   (i)×3 - (ii) ×2; $5b = 45$; $b = 9$

   Answer is (b)

5. Let number is a, b

   Thus, $a + b = 15$; $a - b = 3$

   Answer is (d)

6. Answer is (a)

7. Answer is (a)

8. $T_n = a + (n-1)d$ $84 = a + 9 \times 8$ $a = 12$

   Answer is (b)

9. $S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$

   Answer is (b)

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10. $a = 1$, $r = \frac{-1}{2}$, $T_{14} = a(r)^{13} = \frac{-1}{2^{13}}$ Answer is (b)

11. Distance covered by ball = 45 + (4.5 + 4.5) + (0.45 + 0.45) + (0.045 + 0.045) + ...+ ∞

    = 45 + 9 + 0.90 + 0.090 + ...

    = 54.99 ..... = 55 feet (Approx.)

CHAPTER

5

Surds and Logarithm

1. Important results for surds:

1. $\sqrt[n]{a} = a^{\frac{1}{n}}$
2. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$
3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
4. $\left(\sqrt[n]{a}\right)^n = a$
5. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
6. $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

2. Important results for indices:

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $\left(a^m\right)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. $a^0 = 1 \quad (a \neq 0)$

3. Logarithms: If "a" is a positive real number other than 1 and $a^m = x$ then we write $m = \log_a x$ and we say that the value of log x to the base "a" is m. Logarithm to a negative base is not defined.

Also logarithm of a negative number is not defined. Hence in the above logarithm equation, $\log_a m = x$ and we can say that m>0 and a>0.

Thus $a^x = m \Rightarrow x = \log_a m$ and $\log_a m = x \Rightarrow a^x = m$.

Laws of Logarithms:

(1) $\log_a (xy) = \log_a x + \log_a y$. That is log of product = Sum of logs.

(2) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$. That is log of quotient = Difference of logs.

(3) $\log(mnp) = \log m + \log n + \log p$.

(4) $\log_a (x^p) = p (\log_a x)$.

(5) $\log_a x = \frac{1}{\log_x a}$

(6) $\log_a x = \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a}$.

(7) $\log_b (a) = \frac{\log_c a}{\log_c b}$.

(8) $\log_a (b) = 1 / \log_b (a)$.

(9) log c to the base a^b is equal to $\frac{\log a^c}{b}$.

SOLVED EXAMPLES

1. If $\log 2 = 0.30103$, the number of digits in 4^{50} is what?

Soln. $\log 4^{50} = 50 \log 4 = 50 \log 2^2 = (50\times2) \log 2 = 100\times\log 2 = 100\times0.30103 = 30.103$

Therefore, characteristic = 30. Hence number of digit = 31

2. Find number of digits in 5^{20}?

Soln. $\log 5^{20} = 20\log 5 = 20\times\log\left(\frac{10}{2}\right) = 20[\log 10 - \log 2] = 20\,(1 - 0.30103\,) = 20 \times 0.6990 = 13.9800$

Therefore, characteristic = 13. Hence, number of digit = (13 + 1) =14

3. If $\log(0.57) = \bar{1}.756$ then the value of $\log 57 + \log(0.57)^3 + \log\sqrt{0.57}$ is what?

Soln. $\log(0.57) = \bar{1}.756 \quad \Rightarrow \log 57 = 1.756 \quad (\because$ Mantissa will remain the same$)$

$\therefore \quad \log 57 + \log(0.57)^3 + \log\sqrt{0.57}$

$\therefore \quad \log 57 + 3\log\left(\frac{57}{100}\right) + \log\left(\frac{57}{100}\right)^{1/2}$

$\therefore \quad \log 57 + 3\log 57 - 3\log 100 + \frac{1}{2}\log 57 - \frac{1}{2}\log 100$

$\therefore \quad \log 57 + 3\log 57 + \frac{1}{2}\log 57 - 3\log 100 - \frac{1}{2}\log 100$

$\therefore \quad \frac{9}{2}\log 57 - \frac{7}{2}\log 100$

$\therefore \quad \frac{9}{2}\times1.756 - \frac{7}{2}\log 10^2 \quad \therefore \quad 7.902 - \frac{7}{2}\times2\log 10$

$\Rightarrow \quad 7.902 - 7\times1 = 7.902 - 7 = 0.902$

4. The value of $\log_2 \log_2 \log_3 \log_3 27^3$ is what?

Soln. $\log_2 \log_2 \log_3\left[\log_3 (3^3)^3\right]$

$\log_2 \log_2 \log_3\left[\log_3 3^9\right]$

$\log_2 \log_2 \log_3\left[9\log_3 3\right]$

$\log_2 \log_2\left[\log_3 9\right]$

$\Rightarrow \quad \log_2 \log_2\left[\log_3 3^2\right]$

$\Rightarrow \quad \log_2 \log_2\left[\log_3 3^2\right]$

$\Rightarrow \quad \log_2 \log_2\left[2\log_3 3\right] \quad \because \log_3 3 = 1$

$\Rightarrow \quad \log_2(\log_2 2) = \log_2 1 = 0$

5. If $\log_a(ab) = x$, then $\log_b(ab)$ is what?

Soln. $\log_a(ab) = x \quad \Rightarrow \frac{\log ab}{\log a} = x \quad \Rightarrow \frac{\log a + \log b}{\log a} = x$

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$$\Rightarrow \quad \frac{\log a}{\log a}+\frac{\log b}{\log a}=x \qquad \Rightarrow 1+\frac{\log b}{\log a}=x \qquad \Rightarrow \frac{\log b}{\log a}=x-1$$

$$\Rightarrow \quad \frac{\log a}{\log b}=\frac{1}{x-1} \qquad \Rightarrow 1+\frac{\log a}{\log b}=1+\frac{1}{x-1} \qquad \Rightarrow \frac{\log b+\log a}{\log b}=\frac{x}{x-1}$$

$$\Rightarrow \quad \frac{\log(ab)}{\log b}=\frac{x}{x-1} \qquad \Rightarrow \log_b(ab)=\frac{x}{x-1}$$

6. If $\log_{12} 27 = a$, then $\log_6 16$ is what?

Soln. $\log_{12} 27 = a \quad \Rightarrow \frac{\log 27}{\log 12}=a \quad \Rightarrow \frac{\log 3^3}{\log\left(3\times 2^2\right)}=a \quad \Rightarrow \frac{3\log 3}{\log 3+2\log 2}=a$

$$\Rightarrow \quad \frac{\log 3+2\log 2}{3\log 3}=\frac{1}{a} \qquad \Rightarrow \frac{\log 3}{3\log 3}+\frac{2\log 2}{3\log 3}=\frac{1}{a} \qquad \Rightarrow \frac{1}{3}+\frac{2}{3}\frac{\log 2}{\log 3}=\frac{1}{a}$$

$$\Rightarrow \quad \frac{2}{3}\frac{\log 2}{\log 3}=\frac{1}{a}-\frac{1}{3} \qquad \Rightarrow \frac{2}{3}\frac{\log 2}{\log 3}=\frac{3-a}{3a} \qquad \Rightarrow \frac{\log 2}{\log 3}=\frac{3(3-a)}{6a} \qquad \Rightarrow \qquad \log 3=\frac{2a}{3-a}\log 2$$

Now, $\log_6 16=\frac{\log 16}{\log 6}=\frac{\log 2^4}{\log(2\times 3)}=\frac{4\log 2}{\log 2+\log 3}=\frac{4\log 2}{\log 2+\left(\frac{2a}{3-a}\right)\log 2}$

$$=\frac{4\log 2}{\log 2\left[1+\frac{2a}{3-a}\right]}=\frac{4}{\frac{3+a}{3-a}}=\frac{4(3-a)}{(3+a)}$$

7. Find the value of $16^{\log_4 5}$

Soln. We know that $a^{\log_a x}=x$

$$\Rightarrow \quad 16^{\log_4 5}=4^{2\log_4 5}=4^{\log_4(5^2)}=5^2=25$$

8. Find the value of $\left(2^{1/4}-1\right)\left(2^{3/4}+2^{1/2}+2^{1/4}+1\right)$

Soln. Putting $x=2^{1/4}$, we get $(x-1)\left(x^3+x^2+x+1\right) \Rightarrow (x-1)\left[x^2(x+1)+1(x+1)\right]$

$$\Rightarrow \quad (x-1)(x+1)\left(x^2+1\right) \Rightarrow \left(x^2-1\right)\left(x^2+1\right) \quad \Rightarrow x^4-1$$

$$\Rightarrow \quad \left(2^{1/4}\right)^4-1=2-1=1$$

9. If $abc=1$, then find the value of $\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}$

Soln. Multiply second term by b^{-1} and third term by a

$$\frac{1}{1+a+b^{-1}}+\frac{b^{-1}}{b^{-1}+bb^{-1}+b^{-1}c^{-1}}+\frac{a}{a+ac+aa^{-1}}$$

$$\Rightarrow \quad \frac{1}{1+a+b^{-1}}+\frac{b^{-1}}{b^{-1}+1+a}+\frac{a}{a+b^{-1}+1} \qquad \left(\because\ abc=1 \Rightarrow a=\frac{1}{bc}=b^{-1}c^{-1}\right)$$

$$\left(\because\ abc=1 \Rightarrow ac=\frac{1}{b}=b^{-1}\right)$$

$$\Rightarrow \quad \frac{1+a+b^{-1}}{1+a+b^{-1}}=1$$

10. If $x=5+2\sqrt{6}$, then find the value of $\frac{x-1}{\sqrt{x}}$

Soln. $x=5+2\sqrt{6}=3+2+2\sqrt{6}=\left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2+2.\sqrt{3}.\sqrt{2}=\left(\sqrt{3}+\sqrt{2}\right)^2$

Also, $x-1=4+2\sqrt{6}=2\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)$

$$\therefore \quad \frac{x-1}{\sqrt{x}}=\frac{2\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)}{\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}}=\frac{2\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)}=2\sqrt{2}$$

11. If $\log_{\sqrt{8}} x=3\frac{1}{3}$, find x

Soln. $\log_{\sqrt{8}} x=3\frac{1}{3}=\frac{10}{3}$

$$\Rightarrow \quad x=\left(\sqrt{8}\right)^{\frac{10}{3}}=\left(2^{3/2}\right)^{\frac{10}{3}}=2^{\left(\frac{3}{2}\times\frac{10}{3}\right)}=2^5=32 \qquad \therefore\ x=32$$

12. If $\log_{10} 2=0.3010$ and $\log_{10} 3=0.4717$, find the values of $\log_{10} 25$ and $\log_{10} 4.5$.

Soln. $\log_{10} 25=\log_{10}\left(\frac{100}{4}\right)=\log_{10} 100-\log_{10} 4=\log_{10} 10^2-\log_{10} 2^2$

$$=2-2\log_{10} 2=2-2\times 0.3010=1.3980$$

$$\log_{10} 4.5=\log_{10}\left(\frac{9}{2}\right)=\log_{10} 9-\log_{10} 2=\log_{10} 3^2-\log_{10} 2$$

$$=2\log_{10} 3-\log_{10} 2=2\times 0.4771-0.3010=0.6532$$

EXERCISE

1. If $2^a = 4^b = 8^c$ and $\left(\frac{1}{2a}+\frac{1}{4b}+\frac{1}{6c}\right)=\frac{24}{7}$, then value of c is

   (a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

2. If $2^m = 3^n = 6^{-p}$, then $\left(\frac{1}{m}+\frac{1}{n}+\frac{1}{p}\right)$ is equal to

   (a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

3. If $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, then the value of 'x' is

   (a) 1 (b) 2 (c) 3 (d) 4

4. If $x = 3+2\sqrt{2}$, then the value of $\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)$ is:

   (a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $3\sqrt{2}$

5. Given that $10^{0.48} = x, 10^{0.70} = y$ and $x^z = y^2$ then the value of z is close to:

   (a) 1.45 (b) 1.88 (c) 2.9 (d) 3.7

6. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^?$

   (a) 3.5 (b) 4.5 (c) 6 (d) 7

7. The value of $\log_2(\log_5 625)$ is:

   (a) 2 (b) 5 (c) 10 (d) 15

8. If $\log_2\left[\log_3(\log_2 x)\right]=1$, then 'x' is equals to

   (a) 0 (b) 12 (c) 128 (d) 512

9. If $\log_4 x + \log_2 x = 6$, then 'x' is equals to

   (a) 2 (b) 4 (c) 8 (d) 16

10. If $\log_5(x^2+x)-\log_5(x+1)=2$, then the value of 'x' is:

    (a) 5 (b) 10 (c) 25 (d) 32

11. If $\log x + \log y = \log(x+y)$, then

    (a) x = y (b) xy = 1 (c) $y=\frac{x-1}{x}$ (d) $y=\frac{x}{x-1}$

12. $\left[\frac{1}{(\log_a bc)+1}+\frac{1}{\log_b ca+1}+\frac{1}{\log_c ab+1}\right]$ is equal to

    (a) 1 (b) $\frac{3}{2}$ (c) 2 (d) 3

13. If $\log 3 = 0.477$ and $(1000)^x = 3$ then 'x' equal to

    (a) 0.0159 (b) 0.0477 (c) 0.159 (d) 10

14. If $\log 2 = 0.30103$, the number of digits in 2^{64} is:
(a) 18 (b) 19 (c) 20 (d) 21

15. If $\log P = \frac{1}{2}\log Q = \frac{1}{3}\log R$, then which of the following option is true
(a) $P^2 = Q^3 R^2$ (b) $Q^2 = PR$ (c) $Q^2 = R^3P$ (d) $R = P^2Q^2$

16. The value of $\left(\frac{32}{243}\right)^{-4/5}$ is:
(a) $\frac{4}{3}$ (b) $\frac{9}{4}$ (c) $\frac{16}{81}$ (d) $\frac{81}{16}$

17. The value of $5^{1/4} \times (125)^{0.25}$ is:
(a) $\sqrt{5}$ (b) 5 (c) $5\sqrt{5}$ (d) 25

18. If $2^{2n-1} = \frac{1}{8^{n-3}}$ then the value of n is:
(a) 3 (b) 2 (c) 0 (d) –2

19. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals to
(a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$ (c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

20. $\frac{1}{1+a^{n-m}} + \frac{1}{1+a^{m-n}} = ?$
(a) 0 (b) 1/2 (c) 1 (d) a^{m+n}

21. The value of $\log_{\sqrt{2}} 32$ is:
(a) $\frac{5}{2}$ (b) 5 (c) 10 (d) $\frac{1}{10}$

22. The value of $\log_{10}(0.0001)$ is:
(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) –4 (d) 4

23. If $\log_x^4 = 0.4$ value of x is:
(a) 1 (b) 4 (c) 16 (d) 32

24. $\left(\log_5{}^3\right) \times \left(\log_3{}^{625}\right)$ equals
(a) 1 (b) 2 (c) 3 (d) 4

25. $a = b^x$, $b = c^y$ and $c = a^z$ then the value of xyz is equal to
(a) –1 (b) 0 (c) 1 (d) abc

26. If $a, b \neq 0$, $\left(\frac{a}{b}\right)^{(x-1)} = \left(\frac{b}{a}\right)^{(x-3)}$. Then x =
(a) 7/2 (b) 2 (c) 1 (d) 1/2

ANSWER KEY

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	c	a	d	b	c	d	a	d	d	c
Q.	11	12	13	14	15					
Ans.	d	a	c	c	b					

HINTS & SOLUTIONS

16. $\left(\frac{32}{243}\right)^{-4/5}=\left(\left(\frac{2}{3}\right)^5\right)^{-4/5}=\left(\frac{2}{3}\right)^{5\times\left(\frac{-4}{5}\right)}=\left(\frac{2}{3}\right)^{-4}=\left(\frac{3}{2}\right)^4=\frac{81}{16}$

Answer is (d)

17. $5^{1/4}\times(125)^{0.25}=5^{0.25}\times\left(5^3\right)^{0.25}=5^{0.25}\,5^{0.75}=5^{(0.25+0.75)}=5^1=5$

Answer is (b)

18. $2^{2n-1}=\frac{1}{8^{n-3}}=\frac{1}{2^{3n-9}}=2^{-3n+9}$

Thus, $2n-1=-3n+9;\ \Rightarrow\ n=2$

Answer is (b)

19. Let $a^x=b^y=c^z=k$

Then $a=k^{1/x}, b=k^{1/y}$ and $c=k^{1/z}$

$b^2=ac;\ k^{2/y}=(k)^{1/x}(k)^{1/z}$

Thus, $\frac{2}{y}=\frac{1}{x}+\frac{1}{z};\qquad y=\frac{2xz}{x+z}$

Answer is (d)

20. $\frac{1}{1+a^{n-m}}+\frac{1}{1+a^{m-n}}=\frac{1}{1+\frac{a^n}{a^m}}+\frac{1}{1+\frac{a^m}{a^n}}=\frac{a^m}{a^m+a^n}+\frac{a^n}{a^m+a^n}=1$

Answer is (c)

21. Let $\log_{\sqrt{2}}32=n$; then $\left(\sqrt{2}\right)^n=32;\ 2^{n/2}=2^5;\frac{n}{2}=5\ \Rightarrow n=10$

Answer is (c)

22. Let $\log_{10}(0.0001)=n$, Then $10^n=0.0001=10^{-4}$

$n=-4$

Answer is (c)

23. $\log_x^4 = 0.4 \Rightarrow \frac{4}{10} = \frac{2}{5}$

Thus, $x^{2/5} = 4;\ x = 4^{5/2} = 2^5 = 32$

Answer is (d)

24. Given equation, $\frac{\log 3}{\log 5} \times \frac{\log 625}{\log 3} = \frac{\log 625}{\log 5} = \frac{\log 5^4}{\log 5} = 4$

Answer is (d)

25. $a = b^x$ takes log both side

$\log a = x \log b;\ x = \frac{\log a}{\log b}$

Similarly, $y = \frac{\log b}{\log c}, z = \frac{\log c}{\log a};\ xyz = \frac{\log a}{\log b} \cdot \frac{\log b}{\log c} \cdot \frac{\log c}{\log a}$

Answer is (c)

26. Answer is (b)

CAREER ENDEAVOUR

1.

Soln.

2.

Soln.

3.

Soln.

CHAPTER

6

Percentage and Discount

Rule-1: If a number is increased by x% and then further increased by y% then net % change in the number is given by:

$$\left(x+y+\frac{xy}{100}\right)\%.$$

Rule-2: If a number is decreased by x% and then further decreased by y% then net % change in the number . is given by:

$$\left(-x-y+\frac{xy}{100}\right)\%.$$

SOLVED EXAMPLES

1. If length and breadth of a rectangle are changed by +20% and -10% respectively then what is the % change in area of rectangle

Soln. Net % change in area will be $\left(20-10+\frac{20\times-10}{100}\right)\%=+8\%$ so area will be increased by 8%.

2. If person A's salary increases by 20% and then decreases by 20%. what is the net percentage change in A's salary?

Soln. Net % change will be $\left(20-20+\frac{20\times-20}{100}\right)\%=-4\%$ so salary will be decreased by 4%

3. A trader marks up the price of his good by 20% but to a particularly handling customer he ends up giving a discount of 10% on the marked price. What is the percentage profit he makes?

Soln. Net % change will be $\left(20-10+\frac{20\times-10}{100}\right)\%=8\%$ Hence the percentage profit is 8%.

Concept of discount: If two successive discounts of an article are x% and y% respectively then a single discount is equivalent to $\left(x+y-\frac{xy}{100}\right)\%$

4. Find the single discount which is equivalent to successive discount of 50% and 40%.

Soln. $\left(50+40-\frac{50\times 40}{100}\right)\%=70\%$

5. A trader given successive discounts of 10%, 20% and 10% respectively. The percentage of the original cost price he will recover is:

Soln. $X\left(1-\frac{10}{100}\right)\left(1-\frac{20}{100}\right)\left(1-\frac{10}{100}\right)$

$=X\left(\frac{90}{100}\right)\left(\frac{80}{100}\right)\left(\frac{90}{100}\right)=0.648X$

Hence the overall discount is 35.2% and the answer is 64.8%.

Rule-3: If the price of a commodity increases by R% then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{(100+R)}\times 100\right]\%$$

If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{(100-R)}\times 100\right]\%$$

6. If B's salary is 25% more than A's salary. By what percent is A's salary less than B's salary?

Soln. $\frac{R}{(100+R)}\times 100\%=\frac{25}{(100+25)}\times 100\%=20\%$

Alternative: $100(\text{A})\xrightarrow[+25]{25\%\uparrow}125(\text{B})\xrightarrow{\quad\quad-25\quad\quad}100(\text{A})$ A drop of 25 on 125 gives a 20% drop.

Hence A's salary is 20% less than B's.

Rule-4: Percentage change in Population of a town be P now and suppose it increases at the rate of R% per annum then:

1. Population after n years $=P\left(1+\frac{R}{100}\right)^n$. 2. Population n years ago $=\frac{P}{\left(1+\frac{R}{100}\right)^n}$.

7. During one year the population of a town is increased by 5% and during the next year population was decreased by 5% if the total population is 9975 at the end of the second year then what the population size is in beginning of first year.

Soln. Population 2 years ago $=\frac{9975}{\left(1+\frac{5}{100}\right)\left(1-\frac{5}{100}\right)}=9975\times\frac{20}{21}\times\frac{20}{19}=10000$

8. Monika's math test has 75 problems i.e., 10 arithmetic, 30 algebra and 35 Geometry problems. Although she answered 70% of the arithmetic 40% of the algebra and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more question she would have needed to answer correctly to earn a 60% passing grade.

Soln. Number of questions attempted correctly = (70% of 10 + 40% of 30 + 60% of 35)
= (7 + 12 + 21) = 40
questions to be answered correctly for 60% grade = 60% of 75 = 45 required number of questions = 45–40 = 5

9. A salesman commission is 5% on all sales upto Rs. 10000 and 4% on all sales exceeding this. He remits Rs. 31100 to his parent company after deducting his commission. Find the total sales.

Soln. Suppose total sales = Rs. x
then total sales – commission = 31100 Rs
Therefore, [x – (5% of 10000 + 4% of (x–10000)] = 31100

$$\Rightarrow \quad x-\left[\frac{5}{100}\times 10000+\frac{4}{100}\times(x-10000)\right]=31100$$

$$\Rightarrow \quad x-500-\frac{(x-10000)}{25}=31100 \Leftrightarrow x-\frac{x}{25}=31200 \Leftrightarrow \frac{24x}{25}=31200$$

$$\Rightarrow \quad x=\frac{31200\times 25}{24}=32500$$

10. Santosh spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%. Find the percentage increase in savings.

Soln. Let original income = Rs. 100, then expenditure = Rs 75, saving = Rs. 25.

New income = Rs.120, New expenditure = $75\times\frac{110}{100}=\frac{165}{2}$ Rs

New savings $=\frac{75}{2}-25=\frac{25}{2}$ Rs

Therefore, increase % $=\frac{25/2}{25}\times 100=50\%$

11. In an examination, 35% of total students failed in Physics, 45% failed in Chemistry and 20% in both. Find the percentage of students
(a) Who passed in both subjects. (b) Failed in Physics only
(c) Failed in Chemistry only. (d) Passed in any one subject.
(e) Failed in any one subject.

Soln. Let A = failed in Physics = 35%, B = failed in Chemistry = 45%,
failed in both = 20%

(a) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$ (Therefore, failed in anyone subject)

$$n(A\cup B)=35+45-20=60$$

Henced passed in both subject = 100 – failed in anyone subject = 100–60 = 40%
(b) Failed in Physics only = failed in physics – failed in both subjects = 35 - 20 = 15%
(c) Failed in Chemistry only = failed in chemistry – failed in both = 45 – 20 = 25%
(d) Passed in anyone subject = 100 – failed in both subject = 100 – 20 = 80%
(e) Failed in any one subject = failed in physics + failed in chemistry – failed in both
= 35 + 45 – 20 = 60%

12. Due to a reduction of $6\frac{1}{4}\%$ in the price of wheat, Arun is able to buy 1 kg more for Rs. 120. Find the original and reduced rate of wheat.

Soln. Suppose original rate = Rs. x

$$\text{Reduced rate} = \left[\left(100-\frac{25}{4}\right)\times\frac{1}{100}\right] = \frac{15x}{16}\ \text{Rs}$$

$$\therefore \quad \frac{120}{15x/16}-\frac{120}{x}=1 \Leftrightarrow \frac{128}{x}-\frac{120}{x}=1 \Leftrightarrow x=8$$

Hence, original price = Rs. 8/kg.

$$\text{Reduced rate} = \frac{15x}{16} = \frac{15\times 8}{16} = 7.50\ \text{Rs/kg}$$

13. A student multiplied a number by $\frac{3}{5}$ instant of $\frac{5}{3}$. What is the percentage error in calculation

(a) 34% (b) 44% (c) 54% (d) 64%

Soln. Let the number be x

$$\text{Then error } \frac{5}{3}x-\frac{3}{5}x=\frac{16x}{15}$$

$$\text{Hence, error \%} = \frac{16x/15}{5x/3}\times 100 = \frac{16x}{15}\times\frac{3}{5x}\times 100 = 64\%$$

14. In an election between two candidates, one got 55% of the total valid votes and 20% votes were invalid. If the total number of votes was 7500, the number of vallid votes that the other candidate got,

(a) 2700 (b) 2900 (c) 3000 (d) 3100

Soln. If 20% of the votes are invallid, hence 80% of votes are valid. Then

$$\text{Number of valid votes} = 7500\times\frac{80}{100} = 6000\ \text{votes}$$

If first candidate got 55% of valid votes, then other will get only 45%.

$$\text{Hence, number of votes of 2nd candidate} = 6000\times\frac{45}{100} = 2700$$

15. Gauri went to the stationers and bought things worth Rs. 25, out of which 30 paise went on salestax on taxable purchases. If the tax rate, was 6%, then what was the cost of the tax free item.

(a) 15 rs (b) 15.70 rs (c) 19.70 rs (d) 20 rs

Soln. Let the amount taxable purchases be x rs.
then 6% of x = 30 paise
$x \times 6 = 30$

$$\text{Hence, cost of tax free items} = \left[25-(5+0.30)\right] rs = 19.70\ rs$$

16. Neeraj buys goods worth Rs 6650. He gets a rebate of 6% on it after getting the rebate, he pays sales tax @10%. Find the amount he will have to pay for the goods

(a) 6876.10 rs (b) 6999.20 re (c) 6654 rs (d) 7000 rs

Soln. $$\text{Amount of rebate} = 6\%\ \text{of}\ 6650\ \text{rs} = \frac{6}{100}\times 6650 = 399\ rs$$

$$\text{Amount of sale tax} = 10\%\ \text{of}\ (6650\text{–}399)\ \text{rs} = \frac{10}{100}\times 6251 = 625.10\ rs$$

Hence, final amount pay for goods = (6251 + 625.10) = 6876.10 rs

17. A reduction of 10% in price of sugar enables a housewife to buy 5 kg more for Rs. 300. Find the reduced price per kg of sugar

(a) 5 (b) 4.5 (c) 6 (d) none of these

Soln. Let initial price of sugar = x rs/kg

Amount of sugar for 300 rs = $\frac{300}{x}$ kg.

Now reduce price of sugar = $x \times \frac{(100-10)}{100} = \frac{9x}{10} rs$

The quantity of sugar for 300 rs = $\frac{300}{9x/10} = \frac{3000}{9x} kg$

then $\frac{3000}{9x} - \frac{300}{x} = 5$

$\frac{3000-2700}{9x} = 5 \quad \Rightarrow \frac{300}{9x} = 5 \quad \Rightarrow x = \frac{300}{9\times 5} = 6.66\, rs$

hene, reduced price = $6.66 - 6.66 \times \frac{10}{100} = 6\, rs$

18. In a list of weights of candiates appearing for police selections, the weight of A is marked as 58 kg instead of 46.4 kg. Find the percentage of correction required.

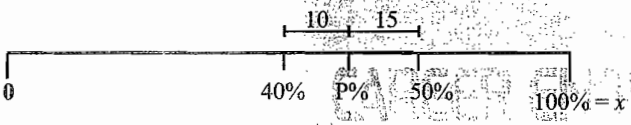
(a) 30 (b) 20 (c) 24 (d) none of these

Soln. Required percentage correction = $\frac{58-46.4}{58} \times 100 = 20\%$

19. In an examination, a student scores 40% and fails by 10 marks. If he scored 50% of marks, he would pass by 15 marks. Find the maximum marks required to pass the examination.

(a) 250 (b) 100 (c) 110 (d) 125

Soln.



Let P is the pass marks % and x is the maximum marks.

Then $x \times 50\% - x \times \frac{40}{100} = 25$

$\Rightarrow \quad x = \frac{25\times 100}{(50-40)} = 250$

Note: In an exam, if a student scores y% of marks and fails by a mark. If he scored z% of marks, he would pass by b marks. Then

$$\text{Max marks} = \frac{(a+b)100}{z-y}$$

and pass marks = $\left(\text{Max marks} \times \frac{y}{100} + a\right)$ or $\left(\text{Max marks} \times \frac{z}{100} - b\right)$

20. A shop is offering discounts on shirts costing 20 rs each. If Rajeev buys 2 shirts, he will be offered a discount of 15% on the first-shirt and another 10% discount on the reduced price for the second shirt. How much would Rajeev pay for two shirts at this shop

(a) 30.9 rs (b) 32.3 rs (c) 37.45 rs (d) 33.3 rs

Soln. The reduced price for first shirt $= 20 \times \frac{(100-15)}{100} = 17\ rs$. The reduced price for second shirt = after 10% discount of reduced price for first shirt $= 17 \times \frac{(100-10)}{100} = 15.3\ rs$.

Hence, the total cost for 2 shirts = 17 + 15.3 = 32.3 rs.

21. The monthly salary of a shop assistant is the sum at a fixed salary of Rs. 500 plus 5% of all monthly sales. What should the montly sales be so that his monthly salary reaches 1500 rs.

(a)30000 rs (b) 1000 rs (c) 20000 rs (d) 40000 rs

Soln. Let x be the monthly sales.

Hence, $500 + x \times 5\% = 1500$

$$x \times \frac{5}{100} = 1000$$

$$x = \frac{1000 \times 100}{5} = 20000\ rs$$

22. The profit percentage on the three articles A, B and C is 10%, 20% and 25% and the ratio of the cost price is 1 : 2 : 4. Also the ratio of number of articles sold of A, B and C is 2 : 5 : 2, then the over all profit percentage is

(a) 18.5% (b) 21 (c) 75 (d) none of these

Soln. Let cost price of A, B and C = x, $2x$ and $4x$ rs and number of articles sold = 2y, 5y and 2y

Hence, total cost price of A, B and C $= x \times 2y + 2x \times 5y + 4x \times 2y = 2xy + 10xy + 8xy = 20xy$

Profit on A $= 2xy \times \frac{10}{100} = 0.2xy$

Profit on B $= 10xy \times \frac{20}{100} = 2xy$

and profit on C $= 8xy \times \frac{25}{100} = 2xy$

Hence, total profit $= 0.2xy + 2xy + 2xy = 4.2xy$

Then profit % $= \frac{4.2xy}{20xy} \times 100 = 21\%$

23. Even after a discount of q% on marked price, a trader gains by P%. What is the markup percentage over the cost price

(a) $\frac{p+q}{q-p} \times 100$ (b) $\frac{p+q}{100-p} \times 100$ (c) $\frac{p+q}{100-q} \times 100$ (d) not possible

Soln. Then $S = M - \frac{q}{100} \times M = \frac{100M - qM}{100}$

Now, there is a gain at p%. Hence,

$$C = S \times \frac{100}{(100+p)};\quad C = \frac{M(100-q)}{100} \times \frac{100}{(100+p)} = \frac{M(100-q)}{100+p}$$

Now, marked up price = M – C = $M - M\dfrac{(100-q)}{(100+p)} = \dfrac{M\left[(100+p)-(100-q)\right]}{(100+p)}$

Hence, mark up % = $\dfrac{\dfrac{M[100+p-100+q]}{(100+p)}}{\dfrac{M(100-q)}{(100+p)}} \times 100 = \dfrac{M(p+q)}{(100+p)} \times \dfrac{(100+p)}{M(100-q)} \times 100 = \dfrac{p+q}{(100-q)} \times 100$

24. A person sold an electronic watch at 96 rs in such a way that his percentage profit is same as the cost price of the watch. If he sells it at twice the percentage profit at its previous percentage profit, then the new selling price will be

(a) 132 rs (b) 150 rs (c) 192 rs (d) 180 rs

Soln. Let CP of the watch = x rs

$x + x \times \dfrac{x}{100} = 96\ rs$

$x = 60\ rs$

Hence, SP of twice profit % = $60 + 60 \times \dfrac{120}{100} = 132\ rs$

25. A man sells two wrist watches one at a profit of 10% and another at a loss of 10% but the SP of each watch is 200. Find the profit or loss%

(a) Profit of 1% (b) Loss of 1% (c) Profit of 2% (d) Loss of 2%

Soln. If SP of 2 articles are same and one is sold on x% profit and other is sold on x% loss, then there is alway

loss = $\left(\dfrac{x^2}{100}\right)\%$

Therefore, loss% = $\dfrac{10^2}{100} = 1\%$

26. The ratio of cost price and marked price of an article is 2 : 3 and the ratio of percentage profit and percentage discount is 3 : 2. What is the discount percentage.

(a) 16.66% (b) 20% (c) 25% (d) 33.33%

Soln. Let CP = $2x$ and MP = $3x$

Profit % : discount % = 3 : 2

Let us suppose CP = 200 and MP = 300

then MP – CP = 300–200 = 100

but $\dfrac{3y}{100} \times 200 + \dfrac{2y}{100} \times 300 = 100$

$6y + 6y = 100$

$y = \dfrac{100}{12} = 8.33\%$

Hence, discount percentage = 2×8.33 = 16.66%

27. A trader sells 20 kg of sugar at 400 rs. A customer asks 20% discount and he agrees to it but instead of 1kg he gives 4% less sugar. What is the effective discount that the customer gets

(a) 16% (b) 16.66% (c) 15.5% (d) 19.6%

Soln. Let the MP = 1 rs per kg then

Weight	MP	Rate
100	100	1
96	80	80/96

Effective discount $= 1 - \frac{80}{96} = \frac{16}{96}$

Hence, effective discount % $= \frac{16}{96} \times 100 = 16.66\%$

28. A retailer cheats both to his whole-seller and his customer by 10% by his faulty balance i.e., he actually weighs 10% more while purchasing from whole seller and weighs 10% less while selling to his customer, what is his net profit percentage, when he sells at CP

(a) $22\frac{2}{11}\%$ (b) $22\frac{2}{9}\%$ (c) 20% (d) 21%

Soln. Amount purchased = 1100 gm
Amount sold = 900 gm

hence, profit % $= \frac{1100-900}{900} \times 100 = \frac{200}{900} \times 100 = 22\frac{2}{9}\%$

29. The profit percentage of A and B are same on selling the article at 1800 rs each but A calculates his profit on SP while B calculates it correctly on the CP which is equals to 20%. What is the difference in their profit.

(a) 360 rs (b) 60 rs (c) 540 rs (d) 450 rs

Soln. Profit of A $= 1800 \times \frac{20}{100} = 360\ rs$

CP of B $= SP \times \frac{100}{(100+\text{gain})} = 1800 \times \frac{100}{120} = 1500\ rs$

hence, profit of B = 1800–1500 = 300 rs
Difference in profit of A and B = 360–300 = 60 rs

30. A trader procures his goods from a whole seller whose balance needs 1200 gm for 1000 gm. The trader sells all the procured goods to a customer after marking up the goods at 20% above the cost price. What is the his overall percentage profit or loss in the whole transaction.

(a) 38% profit (b) 50% profit (c) no profit no loss (d) none of the above

Soln. Let CP of goods = 1 rs per gram
then he pays Rs 1200 for 1000 gm
Again he obtains Rs. 1200 for 1000 gm (selling at 20% profit). Thus no profit no loss.

31. A larger tank filled with water is to be emptied by removing half of the water present in it everyday. After how many days will there be closest to 10% water left in the tank

(a) 1 (b) 2 (c) 3 (d) 4

Soln. Let initial quantity of water = 100%

After first day amount of water left $= 100 \times \frac{1}{2} = 50\%$

After second day amount of water left $= 50 \times \frac{1}{2} = 25\%$

After third day amount of water left $= 25 \times \frac{1}{2} = 12.5\%$

Which is closest to 10%.

32. The rabbit population in community A increase at 25% per year while that in B increase at 50% per year. If the present population of A and B are equal, the ratio of the number of the rabbits in B to that in A after 3 yrs. will be

(a) 1.44 (b) 1.728 (c) 1.90 (d) 1.25

Soln. Let present population of A and B = 100

Population of B at the end of 3 yrs. $= 100 \times \frac{150}{100} \times \frac{150}{100} \times \frac{150}{100}$

Population of A at the end of 3 yrs $= 100 \times \frac{125}{100} \times \frac{125}{100} \times \frac{125}{100}$

Hence, ratio of B to A $= \frac{100 \times \frac{150}{100} \times \frac{150}{100} \times \frac{150}{100}}{100 \times \frac{125}{100} \times \frac{125}{100} \times \frac{125}{100}} = \frac{216}{125} = 1.728$

EXERCISE

[Level-I]

1. 40% of the students of a college are from West Bengal and out of this, 40% are from Kolkata. What percentage of the students is not from Kolkata?

(a) 60 (b) 16 (c) 40 (d) 84

2. A school has a student population of 560. The number of girls is $14\frac{2}{7}\%$ of the number of boys. How many girls are in the school?

(a) 100 (b) 70 (c) 80 (d) 140

3. Two numbers are 20% and 25% less than the third number. By how much per cent is the second number to be enhanced to make it equal to the first number?

(a) $6\frac{2}{3}$ (b) $6\frac{1}{4}$ (c) 25 (d) $33\frac{1}{3}$

4. There is an increase of 30% in the production of milk chocolates in Amul Dairy in one month. If now it is 9100 milk chocolates per month, what was it one month ago?

(a) 13000 (b) 10300 (c) 8400 (d) 7000

5. When 30 per cent of a number is added to another number the second number increases to its 140 per cent. The second number = x% of the first number. The value of x is

(a) 130 (b) 75 (c) $133\frac{1}{3}$ (d) $33\frac{1}{3}$

6. If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area?

(a) No change (b) 16% increase (c) 8% decrease (d) 16% decrease

7. If two numbers are respectively 20% and 50% of the third number, then what % is the first number of the second?

(a) 30 (b) 70 (c) 40 (d) 30

8. If there is an error of x% in measuring the edge of a cube, then the percent error in estimating its volume is

(a) x (b) 3x (c) x/3 (d) x^3

9. A watermelon weighs 500 gm. It turns out that 99% of the weight is due to water in the watermelon. After the watermelon was put in a drying room for sometime, it turned out that it is only 98% water by weight. What is the weight of the watermelon now?
(a) 495 gm (b) 250 gm (c) 5 gm (d) 450 gm

10. In a country with three major scooter manufacturers, Brand C sells three times as many as Brand A while Brand C sells three times as many as Brand A while Brand A sells half as many as Brand B. It implies that Brand C holds a market share of about
(a) 50% (b) 33% (c) 66% (d) None of the above.

11. In an examination, the percentage of students qualified to the number of student appeared from school A is 70%. In school B, the number of students appeared in 20% more than the students appeared from school A and the number of students qualified from school B is 50% more than the students qualified from school A. What is the percentage of students qualified to the number of students appeared from school B.
(a) 30% (b) 70% (c) 75.5% (d) 87.5%

12. Fresh grapes contain 68% water and dry grapes contain 20% water. How much dry grapes can be obtained from 100 kg of fresh grapes.
(a) 32 kg (b) 40 kg (c) 52 kg (d) 30 kg

13. A large watermelon weighs 20 kg with 96% of its weight being water. It is allowed to stand in the Sun and same of the water evaporates so that only 95% of its weight is water. Its reduced weight will be
(a) 16 kg (b) 16.5 kg (c) 17 kg (d) 18 kg

14. How much pure alcohol has to be added to 400 ml of a solution containing 15% alcohol to change the concentration of alcohol in the mixture of 32%
(a) 60 ml (b) 68 ml (c) 100 ml (d) 128 ml

15. Wine contains 5% water. What quantity of pure wine should be added to 10 litres of wine to reduced this to 2%?
(a) 5 lit (b) 7 lit
(c) can not be determined (d) 15 lit

16. In an examination, 80% of the students passed in english, 85% of maths and 75% in both. If 40 students failed in both the subjects, find the total number of students
(a) 400 (b) 360 (d) 420 (d) 380

HINTS & SOLUTIONS

1. Let the total student are x

From the bengal $= \frac{40}{100}x$

From kolkata $= \left(\frac{40}{100}\right)x\left(\frac{40}{100}\right) = \frac{16}{100}x$

Answer is (d)

2. Answer is (b)

3. Let the number are a, b and c

a is 20% less than c. Thus $a = \frac{80}{100}c$

b is 25% less than c. Thus, $b = \frac{75}{100}c$

Suppose b is x% enhance to equal value of a.

$\left(\frac{x}{100}\right)b = a$ $\left(\frac{x}{100}\right)\left(\frac{75c}{100}\right) = \left(\frac{80c}{100}\right)$ $x = \frac{85}{75}100 = \left(100 + \frac{20}{3}\right)$

Answer is (a)

4. Suppose one month earlier production of chocolate is x.

Thus, $x \times \frac{130}{100} = 9100$

$x = 700$

Answer is (d)

5. Let number are a, b

Given $b + \frac{30a}{100} = \frac{140b}{100}$; $\frac{b}{a} = \frac{3}{4}$

Second number = $\frac{x}{100}$ of first number

$b = \frac{x}{100}a$; $x = 100\frac{b}{a} = 100 \times \frac{3}{4} = 75$

Answer is (b)

6. Answer is (d) 7. Answer is (c) 8. Answer is (d)

9. Material in 500 gram = $500 \times \frac{1}{100} = 5$ gram

Water = 495 gram

Now, weight of material = 2%

$x \times 2\% = 5$; $x \times \frac{2}{100} = 5$

$x = 250$ gram.

Answer is (b)

10. $C = 3A$ $A : C = 1 : 3$

$B = 2A$ $A : B = 1 : 2$

$B : A = 2 : 1$

B A

A C

2 1 1 3

B : A : C

2 : 1 : 3

$\frac{3}{6} \times 100 = 50\%$

Answer is (a)

11. Answer is (d) 12. Answer is (b) 13. Answer is (a) 14. Answer is (c)

15. Answer is (d) 16. Answer is (a)

[LEVEL-II]

1. In fresh grapes, 80% of weight is because of water and in dried grapes 20% of weight is beacause of water. In a basket 72% of the weight is fresh grapes and the rest is because of dried grapes. What is the percentage reduction in weight if all the fresh grapes turn into dried grapes?

Soln. From given information we can conclude that when fresh grapes turn into dry grapes, the weight reduces to $\frac{1}{4}$th of the original weight. Because pulp that constitutes 20% is now 80% of the weight in dried grapes.

So, the final weight as a % of the original weight $= \frac{1}{4}\times(72\%)+28\% = 18\%+28\% = 46\%$

2. A chemical engineer added 10% impurities to 50 ml of pure alcohol. Later he took 25 ml of the above alcohol and added 10 ml of pure alcohol to it. What percentage of impurity was there in the final 35 ml of alochol?

Soln. Total alcohol $= 50\left(1+\frac{10}{100}\right) = 55$ ml

In 55 ml of alcohol there is 5 ml impurity

In 25 ml of alcohol there is $\frac{5\times 25}{55} = 2.3$ml impurity

Therefore, in 35 ml of alochol there is 2.3 ml impurity.

Therefore, impurity $= \frac{2.3\times 100}{35} = 6.6\%$

3. 16 litres of a mixture contains milk and water in the ratio 5 : 3. If 4 litres of this mixture is replaced by 4 litres of milk, then what is the ratio of the milk and water in the new mixture?

Soln. According to question,

In 16 litres of mixture, milk $= \left(16\times\frac{5}{8}\right)$ litres $= 10$ litres

Then water = 6 litres

New mixture contains 14 litres of milk and 6 litres of water

Therefore, Milk : water = 14 : 6 = 7 : 3

4. The pressure of a gas is directly proportional to its temperature and inversely proportional to its volume. The intial pressure, volume and temperature are 150 Pascals, 20 m^3 and 100°K respectively. Find the final temperature if the final volume and pressure of the same gas are 8 m^3 and 150 Pascals respectively under the same conditions.

Soln. According to question,

$P \propto T$ and $P \propto \frac{1}{V}$

After combining, $P \propto \frac{T}{V}$ or $P = K\frac{T}{V}$. Hence, $\frac{PV}{T} = K$, or, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = K$

Or, $\frac{150\times 20}{100} = \frac{150\times 8}{T_2} \Rightarrow 20T_2 = 8\times 100 \quad \therefore T_2 = \frac{8\times 100}{20} = 40°\text{Kelvin}$

Alternate Method:

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ [because pressure is constant]

$\Rightarrow \frac{20}{100} = \frac{8}{T_2} \quad \therefore T_2 = \frac{8\times 100}{20} = 40°\text{K}$

TRY YOURSELF

1. Seats for maths, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats 40%, 50% and 75% respectively. The new ratio is
(a) 2 : 3 : 4 (b) 6 : 7 : 8 (c) 6 : 8 : 9 (d) None of these

2. 15L of mixture contains 20% alcohol and the rest water. If 3 litres of water be mixed with it, the percentage of alcohol in the new mixture would be
(a) 15% (b) $16\frac{2}{3}\%$ (c) 17% (d) $18\frac{1}{2}\%$

3. 20L of a mixture contains milk and water in the ratio 5 : 3. If 4L of this mixture be replaced by 4 litres of milk. The ratio of milk to water in the new mixture would be
(a) 2 : 1 (b) 7 : 3 (c) 8 : 3 (d) 4 : 3

4. The ratio of the number of boys and girls in a school is 3 : 2. If 20% of the boys and 25% of the girls are scholarship holders, percentage of the student who does not get schlorship
(a) 56 (b) 70 (c) 78 (d) 80

5. The electricity bill of a certain establishment is partly fixed and partly varies as well as the number of units of electricity consumed. When in a certain month 540 units are consumed, the bill is Rs. 1800. In another month 620 units are consumed and the bill is Rs. 2040. In yet another month 500 units are consumed, the bill for that month would be
(a) Rs. 1560 (b) Rs. 1680 (c) Rs. 1840 (d) Rs. 1950

6. The ratio of incomes of A and B is 5 : 4 and the ratio of thier expenditures is 3 : 2. If at the end of the year, each saves 1600 rs. then the income of A is
(a) Rs. 3400 (b) Rs. 3600 (c) Rs. 4000 (d) Rs. 4400

7. How many kg of pure salt must be added to 30 kg of 2% solution of salt and water to increase it to a 10% of solution.
(a) $2\frac{2}{3}$ (b) $2\frac{2}{3}$ (c) $3\frac{2}{3}$ (d) none of these

8. 8% of the people eligible to vote are between 18 and 21 yrs of age. In an election, 85% of those eligible to vote, who were between 18 to 21, actually voted. In that election, the number of persons between 18 and 21, who actually voted was what percent of those eligible to vote
(a) 4.2 (b) 6.4 (c) 6. 8 (d) 8

9. Two tailors X and Y are paid a total of 550 rs per week by their employer. If X is paid 120% of the sum paid to Y. How much is Y paid per week.
(a) 200 (b) 250 (c) 300 (d) none of these

10. In two alloys the ratios of gold to silver are 4 : 5 and 7 : 2 (be weight). How many kilograms of the first alloy and of the the second alloy should be mixed together to obtain 54 kilogram of a new alloy with equal contents of gold and silver?
(a) 45 kg, 9 kg (b) 36 kg, 18 kg (c) 27 kg, 27 kg (d) 40 kg, 14 kg

Answer Key

Q	1	2	3	4	5	6	7	8	9	10
Ans.	a	b	b	c	b	c	b	c	b	a

CHAPTER

7

Distance, Trains and Boats

(1) $\text{Speed} = \left(\frac{\text{Distance}}{\text{Time}}\right), \text{Time} = \left(\frac{\text{Distance}}{\text{Speed}}\right)$ (2) $x \text{ km/hr} = \left(x \times \frac{5}{18}\right) \text{m/sec}$

(3) $x\,m/\sec = \left(x \times \frac{18}{5}\right) km/hr$

There is a direct proportionality between time and distance when the speed is constant &(time $\propto$ distance)

BOAT & STREAM

Short cut Methods:

(i). If a man capable of rowing at a speed of x km/h in still water, rows the same distance up and down a stream which flows at a rate of y km/h, then this average speed through out journey is

$$= \frac{\text{up stream} \times \text{down stream}}{\text{Man's rate in still water}} = \frac{(x-y)(x+y)}{x} km/h$$

(ii). A man can row a boat in still water at x km/h and stream flowing at y km/h. If he takes 't' hours more in upstream than to go downstream for the same distances, then distance is given by $\frac{(x^2-y^2)t}{2y} km$

(iii). A man rows a certain distance in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream by y km/h then the speed of the man in still water is given by

$$y\left(\frac{t_2+t_1}{t_2-t_1}\right) km/hr$$

(iv). A man can row a boat in still water at x km/h in a stream flowing at y km/hr. If it takes him 't' hours to row a place and come back. Then the distance between the two places is $\frac{t(x^2-y^2)}{2x} km$

(v). A boat (or a swimmer) takes 'n times' as long to upstream as to row downstream the river. If the speed of boat is x km/hr and the speed of stream by y km/hr. Then

$$x = y\left(\frac{n+1}{n-1}\right)$$

SOLVED EXAMPLES

1. A train meets with an accident and moves at ¾ its original speed. Due to this it is 20 minutes late. Find the original time for the journey beyond the point of accident.

Soln. Speed becomes ¾ (Time becomes 4/3). Extra time = 1/3 of normal time = 20 minutes
Normal time = 60 minutes.

Acceleration: Acceleration is defined as the rate of change of speed. Acceleration can be positive (speed increases) or negative (speed decreases) which is also known as deceleration.
The unit of acceleration is speed per unit time square e.g. m/s^2.

$$\textbf{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

Rule 1: suppose a man covers a certain distance at x km / hr and an equal distance at y km / hr. Then the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right) km/hr$.

Rule 2: If a moving body travels $d_1, d_2, d_3, d_4 \ldots\ldots d_n$ meters with different speeds $v_1, v_2, v_3, v_4 \ldots\ldots v_n$ m/sec in time $t_1, t_2, t_3, t_4 \ldots\ldots t_n$ respectively then average speed $= \frac{\text{Total distance}}{\text{Total time taken}}$

1. If $d_1, d_2, d_3, d_4 \ldots\ldots d_n$ and $t_1, t_2, t_3, t_4 \ldots\ldots t_n$ are known then:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{d_1 + d_2 + d_3 + d_4 \ldots\ldots + d_n}{t_1 + t_2 + t_3 + t_4 \ldots\ldots + t_n}$$

2. If $d_1, d_2, d_3, d_4 \ldots\ldots d_n$ and $v_1, v_2, v_3, v_4 \ldots\ldots v_n$ are known then:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{d_1 + d_2 + d_3 + d_4 \ldots\ldots + d_n}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \frac{d_3}{v_3} + \frac{d_4}{v_4} \ldots\ldots + \frac{d_n}{v_n}}$$

3. If $v_1, v_2, v_3, v_4 \ldots\ldots v_n$ and $t_1, t_2, t_3, t_4 \ldots\ldots t_n$ are known then:

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time taken}} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + v_4 t_4 \ldots\ldots + v_n t_n}{t_1 + t_2 + t_3 + t_4 \ldots\ldots + t_n}$$

2. A car travels at 60 km/h from Agra to Delhi and at 120 km/h from Delhi to Agra. what is the average speed of the car for the entire journey?

Soln. $\text{Average speed} = \frac{2 \times 60 \times 120}{60 + 120} = 80 \text{Km / h}$

3. An aeroplane files along four sides of a square at the speed of 200,400,600 and 800 km/hr. Find the average speed of plane around the field.

Soln. Let each side of square is a km and let the average speed of plane around the field be b km/hr then:

$$\frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800} = \frac{4x}{y} \Rightarrow \frac{25x}{2400} = \frac{4x}{y} \Rightarrow y = 384$$ So average speed is 384 km/hr.

Motion in a straight line: Two or more bodies starting from the same point and moving in the same direction then the relative speed is $(v_1 - v_2)$. Moving in the opposite direction then the relative speed given by $v_1 + v_2$.

(i) General formula for train:

Time to cross an object by a train moving in direction/opposite of train is:

$$T = \frac{\text{Length of train} + \text{Length of object}}{\text{speed of train} - \text{speed of object}} = \frac{L_t + L_o}{v_t \mp v_o}$$

(ii) Modification in general formula:

1. If object is stationary then put speed of object = 0
2. If object is of negligible length then put length of object = 0

4. A train crosses a pole in 8 seconds. If the length of the train is 200 meters, find the speed of the train.

Soln. In this case it is evident that the situation is one of the train crossing a stationary object without length. Hence case 1 is applicable here.

Thus $v_t = 200/8 = 25$ m/s $\rightarrow 25 \times \frac{18}{5} = 90$ kmph.

5. Train crosses a man traveling in another train in the opposite direction in 8seconds. However the train requires 25 seconds to cross the same man if the trains are traveling in the same direction. If the length of the first train is 200 meters and that of the train in which the man is sitting is 160 meters find the speed of the first train.

Soln. Here it is clear that one of the train crossing a moving object without length. Thus the length of the man's train is useless or redundant data.

For opposite direction: $T = \frac{L_t}{v_t + v_o} \Rightarrow 8 = \frac{200}{v_t + v_o}$. ... (1)

For same direction: $T = \frac{L_t}{v_t - v_o} \Rightarrow 25 = \frac{200}{v_t - v_o}$ ... (2)

By solving equation (1) and (2) one get = 59.4km/h.

Boats and streams: .

Important points:

1. Still water means speed of water in river is zero while stream water means water in river is moving.
2. In water the direction along the stream is called downstream. And the direction against the stream is called upstream.
3. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/ hr then speed of downstream = (u + v) km/hr and speed of upstream = (u – v) km/hr.

So the speed of the movement of the boat is also dependents on current speed.

6. A man can row 4 Km/h in still water and he finds that it takes him twice as long to row up as to row down the river. Find the stream of river.

Soln. If the speed of a man in still water is u km / hr and the speed of the stream of river is v km/hr then speed of down-stream = (u + v) km/hr and speed of upstream = (u – v) km/hr.

If distance covered is d then according to given condition:

$$u = 4km/h$$

Time to travel in up stream$(T_1) = \frac{d}{u - v}$ *and* Time to travel in down stream $(T_2) = \frac{d}{u + v}$

$$But\ T_1 = 2T_2 \Rightarrow 2(u - v) = u + v \Rightarrow u = 3v \Rightarrow v = \frac{4}{3} km/h.$$

7. A man takes 6 hours 35 minutes in walking to a certain place and the riding back. He would have taken two hours less by riding both ways. What would be the time he would take to walk both ways?

Soln. Given, walking + riding = 6 hrs 35 minutes ... (i)
Riding + riding = 4 hrs 35 minutes ... (ii)
2 riding = 4 hrs 35 mintes, mutually equation (1) by two we get
2 walking + 2 riding = 13 hrs 10 minutes ... (iii)
Put the value of equation (ii) in equaton (iii)
Therefore, 2 walking = 8 hrs 35 minute.

8. A train 135 meters long is running with a speed of 49 km/hr. In what time will it pass a man who is walking at 5 km/hr in the direction opposite to that of the train?

Soln. Given, length of first train (x) = 135 metres.
Length of man (y) = 0
Speed of first train (u) = 49 km/hr
Speed of man (v) = 5 km/hr

So, relative speed = u + v = 49 + 5 = 54 km/hr = $54 \times \frac{5}{18} = 15$ m / sec

Hence, time taken $= \frac{x+y}{u+v} = \frac{135}{15} = 9$ second

9. Excluding stoppages, the speed of a bus is 54 km/h and including stoppage it is 45 km/h. For how many minutes does the bus stop per hour?
(a) 4 (b) 6 (c) 8 (d) none of these

Soln. Due to stoppages, it covers 9 km less (54-45) = 9km.

hence, bus stop per hour is $\left(\frac{9}{54} \times 60\right)$ min = 10 min

10. A train can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 kms away from A at the same time. On the way, however, the train lost about 12.5 min while stopping at the stations. The speed of car is
(a) 100 km/h (b) 110 km/h (c) 120 km/h (d) 130 km/h

Soln. Let speed of car = x km/h

then speed of train = $x \times \frac{150}{100} = \frac{3x}{2} km/h$

time taken by car = $\frac{75}{x}$ hours.

time taken by train = $\frac{75}{3x/2} = \frac{150}{3x} = \frac{50}{x} hour$

Now, according to question

$$\frac{50}{x} + 12.5 = \frac{75}{x}$$

$$\frac{75}{x} - \frac{50}{x} = \frac{125}{600}$$

$$\frac{25}{x} = \frac{5}{24}$$

$$x = \frac{25 \times 24}{5} = 120\ km/h$$

11. A flight of 600 km, an aircraft was slowed down due to bad wheather. Its average speed for the trip was reduced by 200 km/h and the time of flight increased by 30 minutes. The duration of the flight is
(a) 1 hour (b) 2 hour (c) 3 hour (d) 4 hour

Soln. Let initial speed = x km/h

then time of flight = $\frac{600}{x}$ *hour*

New speed is $(x-200)$ km/h

then time of flight = $\frac{600}{x-200}$ *hour*

then $\frac{600}{(x-200)} - \frac{600}{x} = \frac{30}{60}$

$\Rightarrow \quad \frac{600x-600x+120000}{x^2-200x} = \frac{1}{2} \Rightarrow \quad x^2-200x-240000=0$

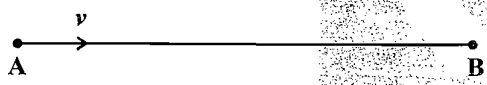
$\Rightarrow \quad (x-600)(x+400)=0 \quad \Rightarrow x = 600 \text{ or } -400$

Now, $x \neq -400$ then $x = 600$ *km / h*

Hence, duration of flight = $\frac{600}{600} = 1$ *hour*

12. Two guns were fired from the same place at an interval of 12 minutes, but a person in the train approaching the place hears the second of 10 min after first. The speed of the train, if speed of sound 330 m/s, is
(a) 65 m/s (b) 65 km/hr (c) 56 m/s (d) none of these

Soln.



Let train be located at point A and firing place is point B. Now, speed of train is v m/sec and speed of sound is 330 m/sec. Then distance between A and B = speed × time = 330×12×60 m
Distance by the help of second fire = $(v+330)10\times60$ m
Hence, $(v+330)\ 10\times60 = 330\times12\times60$
$v + 330 = 396$
Hence, $v = 396 - 330 = 66$ m/esc.

13. Sumeer started the journey with his car from the bad lands with tank full of fuel 12 gallons exactly. However the moment he started out the fuel tank sprung a leak he travelled at 50 mp/h untill. He rank out of fuel exactly 4h later knowing that the car runs 25 miles for each gallon. How much fuel did he lose through the leak-miles
(a) 4 gallons (b) 5 gallons (c) 6 gallons (d) 3 gallons

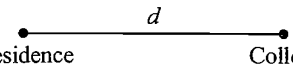
Soln. Since he travelled at 50 mph for 4 hours
then distance covered by them = 50×4 = 200 miles
Now distance covered for each gallon = 25 miles

Hence, number of gallon used = $\frac{200}{25} = 8$ gallons

Hence, fuel lost by him (12–8) = 4 gallons

14. Walking at 10 km/h umesh reaches his college 2 hrs early and if he walks at 15 km/h, he reaches 2 hours and 40 min early. The distance between his college and residence is
(a) 27.8 km (b) 22.5 km (c) 20 km (d) none of these

Soln.



Time taken by the first speed = d/10 hrs
time taken by the second speed = d/15 hrs

Hence, $\frac{d}{10}-\frac{d}{15}=\frac{40}{60}=\frac{3d-2d}{30}=\frac{2}{3}$

$\therefore \quad d = 20\ km$

15. A man travels 35 km partly at 4 km/h and partly at 5 km/h. If he covers the former distance at 5 km/h, he could cover 2 km more in the same time. Then the time taken to cover the whole distance at the original rate is
(a) 4.5 hrs (b) 7.5 hrs (c) 7.4 hrs (d) 9 hrs

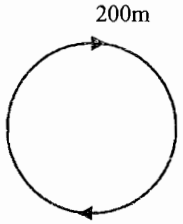
Soln. If he coveres former distance at 5 km/h also, the entire journey will covered by 5 km/h
then total distance of journey = 35 + 2 = 37 km

Now time of travel = $\frac{37}{5}=7.4\ hrs$

which is same as the time taken at the original rate.

16. On a circular track of 200m, shiva runs twice as fast as Pawan in a race. If shiva gives a lead of 100m to Pawan, when will they meet for the first time (after what meters)
(a) 150 m (b) 200 m (c) 250 m (d) 50 m

Soln.



Since, Shiva gives a start of 100m to Pawan
Now, ratio of speeds of Shiva and Pawan = 2 : 1
Then in same time Shiva covered twice distance than Pawan.
Hence, if Pawan covers 100m then Shiva will go = 200m
Total distance for Pawan = 100m + 100m = 200m

17. In a 100m race sujeet beats Rishi by 5 meter and Rishi beats Praveen by 5m. By what distance does Sujeet beat Praveen
(a) 10m (b) 9m (c) 11m (d) 9.75m

Soln. Ratio of speeds of Sujeet and Rishi = 100 : 95 = 20 : 19
and ratio of speeds of Rishi and Praveen = 100 : 95 = 20 : 19
Hence, ratio of speeds of Sujeet, Rishi and Praveen = 20 19

$$= (20\times20) : (19\times20) : (19\times19)$$

Ratio of speeds of Sujeet and Praveen = $20^2 : 19^2 = 400 : 361$
Now when Sujeet goes 400m, Praveen goes = 361 m

hence, Sujeet goes 100m, Praveen goes = $\frac{361}{400}\times100=90.25m$

Hence, lead given by Sujeet = 100–90.25 = 9.75m

18. Deepak and Vinay start travelling in the same direction at 8 km/h and 13 km/h respectively. After 4 hrs. Deepak doubled his speed and Vinay reduced his speed by 1 km/h and reached the destination at same time. How long did the entire journey last
(a) 9 hrs (b) 10 hrs (c) 11 hrs (d) 8 hrs

Soln. Let the entire journey last in x hour.

then distance covered by Deepak = $8\times4 + (x-4)16$
and distance covered by Vinay = $13\times4 + (x-4)12$
then $32 + 16x - 64 = 52 + 12x - 48$

$\Rightarrow \quad 16x - 12x = 52 - 48 + 64 - 32$

$\Rightarrow \quad 4x = 36$

$x = 9\ hrs$

19. After travelling 30 min, a train_met with an accident and was stopped there for 45 min. Due to the accident, its speed reduced by 2/3 of its former speed and train reached its destination 1 hrs 30 min late. If the accident occured 60 km after the point it occurred earlier the train would have reached 30 min earlier. The length of journey is

(a) 90 km (b) 120 km (c) 150 km (d) 180 km

Soln. Let the length of journey = x km
and speed of train = v km/h

after accident the speed = $v\times\frac{2}{3} = \frac{2v}{3}\ km/h$

then normal time of entire journey = $\frac{x}{v}h$

Now according to first oscillation

$\frac{1}{2} + \frac{3}{4} + \frac{x - v/2}{2v/3} - \frac{x}{v} = \frac{3}{2}$ (converting minuts to hours)

and $\frac{v/2+60}{v} + \frac{3}{4} + \frac{x - v/2 - 60}{2v/3} - \frac{x}{v} = 1$

Solving both equation we will get

$v = 60\ km/h$

$x = 120\ km$

20. Assume that the distance that a car runs on 1L petrol is inversely proportional to the square of the speed at which it is driven it gives a run of 25 km/L at a speed of 36 km/hr. At what speed would it be driven to get a run of 36 km/L.

(a) 36 km/h (b) 30 km/h (c) 42 km/h (d) 35 km/h

Soln. Let x be the distance per litre of petrol and v = speed at which it is driven

$\Rightarrow \quad x \propto \frac{1}{v^2}$

$\Rightarrow \quad x = k.\frac{1}{v^2}$

$\Rightarrow \quad xv^2 = k\ (\text{constant})$

Now, $25(36)^2 = 36(v_1)^2$

$\Rightarrow \quad v_1^2 = \frac{25\times36\times36}{36} = 30\ km/h$

21. There is a train of length 500m, in which a man is standing at the rear end. At the instant the rear end crosses a stationary observer on a plateform, the man starts walking from the rear to the front and the front to the rear of the train at a constant speed of 3 km/h. The speed of the train is 80 km/h. The distance of the man from the observer at the end of 30 min is

(a) 41.5 km (b) 40.5 km (c) 40 km (d) 41 km

Soln. In 30 min, distance covered by train $= 80 \times \frac{1}{2} = 40\ km$

In 30 min, distance covered by man $= 3 \times \frac{1}{2} = \frac{3}{2} km = 1500\ m$

Now length of train is = 500 m
then man will go rear end to front, come front to rear and again rear to front. So, distance of observer from man = 40 km + 500 m = 40.5 km

22. A can beat B by 20 m in a 200 m race. B can beat C by 10 m in a 250 m race. By how many metres can A beat C in a 100 m race

Soln. When A runs 200m, B runs 180 m
When B runs 250 m then C runs = 240 m

When B runs 180 m then C runs $= \frac{172.8}{200} \times 100 = 86.4\ m$

Hence, A beats C by (100–86.4) = 13.6 m

23. Sanjay and Samit can cover a distance of 660 m in 81 sec and 88 sec respectively. By how many seconds will Samit win if he was a head sart. If 60 m
(a) 1 sec (b) 2 sec (c) 7 sec (d) 11 sec

Soln. Samit can cover 660 m = 88 sec

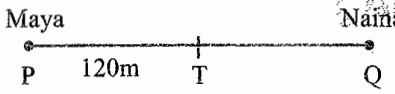
Samit can cover 1m $= \frac{88}{660}$ sec

Samit can cover 600 m $= \frac{88}{660} \times 600 = 80$ sec

Hence, Samit will win by (81–80) = 1 sec

24. Maya and Naina start simultaneously from P and Q towards Q and P respectively. They meet on the way at T which is at a distance of 120m from P. If Maya and Naina take 16 sec and 25 sec to reach their respective destinations from T. Then distance between P and Q is
(a) 214 m (b) 200 m (c) 240 m (d) 216 m

Soln.



It is clear that 120 m covered by Naina = 25 sec
Hence, speed of Naina = 120/25
Ratio of time from point P to reach their destinations by
Maya : Naina = 16 : 25

Hence, $\frac{\text{speed of maya}}{\text{speed of Naina}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

So, speed of Maya $= \frac{5}{4} \times$ speed of Naina $= \frac{5}{4} \times 4.8 = 6\ m/\text{sec}$

Now, from point T, Maya reach her destination in 16 sec.
So, distance between T and Q = 6×16 = 96 m
Hence, distance between P and Q = PT + TQ = 120 + 96 = 216 m.

EXERCISE

1. A car travels a certain distance at 60 km/h and comes back at 50 km/h. Find the average speed for total journey?
(a) 50 km/h (b) 45 km/h (c) 48 km/h (d) None of these

2. A monkey ascends a greased pole 12 meters high. He ascends 2 meters in first minute and slips down 1 meter in the alternate minute. In which minute, he reaches the top?
(a) 10^{th} (b) 21^{st} (c) 12^{th} (d) 12^{th}

3. If a man walks at 4 km/h, he misses the bus by 10 min. If he walks at 5 km/h, he reaches 5 min. before the arrival of the bus. How far is the bus stand?
(a) 10 km (b) 12 km (c) 15 km (d) 5 km

4. A car driver completes a 180 km trip in 4 hours. If he averages 50 km/h during the first 3 hours of trip, what was his speed in the final hour?
(a) 44 km/h (b) 45 km/h (c) 35 km/h (d) 30 km/h

5. A train of unknown length crosses a platform L_1 meters in t_1 seconds and also crosses a telegraph post in t_0 seconds. What is speed of the train?
(a) $L_1(t_1 - t_0)$ (b) $\frac{L_1}{t_1 - t_0}$ (c) $\frac{L_1 t_1}{t_0^2}$ (d) $\frac{L_1 t_1}{t_1 + t_0}$

6. A train consists of 12 bogies, each boggy 15 meters long. The train crosses a telegraph post in 18 seconds. Due to some problem, two bogies were detached. The train now crosses a telegraph post in
(a) 18 s (b) 12 s (c) 15 s (d) 20 s

7. A car travels from point A to point B (a distance of one mile) at 30 miles per hour. How fast would the car have to travel from point B to point C (also a distance of one mile) to average 60 miles per hour for the entire trip?
(a) 90 miles per hour (b) 120 miles per hour
(c) the car has reached point C already (d) Not possible to average 60 miles per hour.

8. 2 hours after a freight train leaves Delhi, a passanger train leaves the same station travelling in the same direction at an average speed of 16 km/hr. After travelling 4 hours, the passenger train overtakes the freight train. The average speed of the freight train was
(a) 10 km/hr (b) 14 km/hr (c) 18 km/hr (d) None of the above.

9. Suppose an ant is placed on one corner of a sugar cube, which has equal sides of 1.5 cm each. If the ant may walk only along the edges of the cube, what is the maximum distance the ant may walk on the cube without retracting its path?
(a) 9 cm (b) 18 cm (c) 10.5 cm (d) 13.5 cm

10. On a straight road XY, 100 meters long, five heavy stones are placed 2 meters apart beginning at the end X. A worker, starting at X, has to transport all the stones to Y, by carrying only one stone at a time. The minimum distance he has to travel (in meters is):
(a) 422 (b) 480 (c) 744 (d) 860

HINTS & SOLUTIONS

1. Let the distance is x

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} = \frac{x+x}{\frac{x}{60}+\frac{x}{50}} = 54\frac{6}{11}\text{ km/h}$$

Answer is (d)

2. Answer is (b)

3. Let distance is d

$$\text{At speed 4 km/h time taken} = \frac{d}{4}$$

$$\text{At speed 3 km/h time taken} = \frac{d}{5}$$

$$\text{According to question, } \frac{d}{4}-\frac{10}{60}=\frac{d}{5}+\frac{5}{60}; \qquad d = 5$$

Answer is (d)

4. Answer is (d)

5. Let length of train is l.

$$\text{velocity} = \frac{\ell + L_1}{t_1} = \frac{\ell}{t_0}; \qquad \ell = \frac{L_1}{f_1 - f_0}$$

Answer is (b)

6. Length of train = 12×15 = 180 m

$$\text{Velocity of train} = \frac{180\text{ m}}{18} = 10\text{ m/s}$$

If two boggies detacted then length of train = 150 m

$$\text{After that time to iron pole} = \frac{150}{10} = 15\text{ sec}$$

Answer is (c)

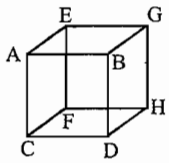
7. Answer is (c)

8. In 4 hours distance covered by passenger train = 16×4 = 64 km
It is distance equal to covered by frieght train in 6 hours.

$$\text{Hence, speed of frieght train} = \frac{64}{6} = 10.66\text{ km/h}$$

Answer is (d)

9.



Ant may walk along AB, BD, DC, CA and AE then EG, GH, HF and FE without retracing its path.
Hence, 9 edges × 1.5 = 13.5
Answer is (d)

10. X ● ● ● ● ● Y
A B C D

Five stones are put at a point X, A, B, C and D which are 2m from each other.
Firstly, we put all stones at a point D.
Then, for point X we have to walk 8m.
For point A, we have to come D to A and then A to D.
It means 6 + 6 = 12 m
For point B, 4 + 4 = 8 m
For point C, 2 + 2 = 4 m
Now, all stones are at point D.
Total distance = 8 + 12 + 8 + 4 = 32 m.
Now, from D to Y the distance is 92.
For 4 stones we have to make round trip and for 5 stones we have to covered only D to Y.
Hence, total distance covered = 92× 9 = 828
Hence, total covered distance = 828 + 32 = 860.
Answer is (d)

CAREER ENDEAVOUR

CHAPTER

8

Work, Pipe and Cistern

1. **Important formula:**

$$\frac{\text{Man}_1 \times \text{days}_1 \times \text{work rate}_1}{\text{Amount of work done}_1} = \frac{\text{Man}_2 \times \text{days}_2 \times \text{work rate}_2}{\text{Amount of work done}_2}$$

Man days required per unit work is always same.

Generalize above formula:

$$\frac{N_1 \times D_1 \times R_1 \times E_1}{W_1} = \frac{N_2 \times D_2 \times R_2 \times E_2}{W_2}$$

$N_1, N_2 \rightarrow$ No. of workers $D_1, D_2 \rightarrow$ Time of work

$R_1, R_2 \rightarrow$ work rate of workers or meachine

$E_1, E_2 \rightarrow$ Efficiency of workers or machine

$W_1, W_2 \rightarrow$ Amount of work done

2. **Time and work:**

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If A does a work in a days then in one day A does $\rightarrow \frac{1}{a}$ of the work.

If B does a work in b days then in one day B does $\rightarrow \frac{1}{b}$ of the work.

Then in one day If A and B work together then their combined work is $\frac{1}{a}+\frac{1}{b}$ or $\frac{a+b}{ab}$ of the work.

So, to finish whole work it require $\frac{ab}{a+b}$ days

Rule: Efficiency $\propto \frac{1}{\text{day}}$

SOLVED EXAMPLES

1. A contractor estimates that he will finish the road construction project in 100 days by employing 50 men. However, at the end of the 50th day when as per his estimation half the work should have been completed He finds that only 40% of his work is done.
(a) How many more days will be required to complete the work?
(b) How many more men should he employ in order to complete the work in time?

Soln. The contactor has completed 40% of the work in 50 days
If the number of men working on the project remains constant the rate of work also remains constant. Hence to complete 100% work he will have to complete the remaining 60% of the work. For this he would require 75 more days.
In order to complete the work on time it is obvious that he will have to increase the number of men working on the project. This can be solved as: 50 men working for 50 days $50 \times 50 = 2500$ man-days.
2500 man-days has resulted in 40% work completion. Hence the total work to be done in terms of the number of man-days is got by using unitary method: work left = 60% = $2500 \times 1.5 = 3750$ man-days.
This has to be completed in 50 days. Hence the number of men required per day is $3750/50 = 75$ men. Since 50 men are already working on the project the contractor needs to hire 25 more men.

Equating men, women and children: This is directly derived from the concept of efficiencies.

2. 8 men can do a work in 12 days while 20 women can do it in 10 days. In how many days can 12 men and 15 women complete the same work?

Soln. Total work to be done = $8 \times 12 = 96$ man-days or total work to be done = $20 \times 10 = 200$ woman-days. Since the work is the same we can equate 96 man-days = 200 woman-days.
Hence 1 man-day = 2.08333 woman-days.
Now if 12 men and 15 women are working on the work we get
12 men are equal to $12 \times 2.08333 = 25$ women
Hence the work done per day is equivalent to 25 +15 women working per day.
That is 40 women working per day.
Hence 40×no. of days = 200 women days so number of days = 5 days.

3. A is twice as efficient as B. If they complete a work in 30 days find the time required by each to complete work individually.

(A + B) in one day do = 1 / 30 of total work
But efficiency of A is equal to efficient of 2×efficiency of B.

Soln. So 3B work in one day = 1 / 30 of total work $\Rightarrow$ B work in one day = 1 / 90 of total work
$\Rightarrow$ B can finish whole work in 90 days & A can finish whole work in 45 days

4. A is thrice as efficient as B. If they complete a work in 30 days then find the time required by each to complete work individually.

Soln. Time required by A to complete work is 40 days while that by B to complete work is 120 days.

5. A and B can do a piece of work in 10 days and 15 days respectively. They work together for 3 days and then B leaves while A finishes the remaining work alone. In how many days total work finished.

Soln. 8 days.

6. If the output of 3 men is equals to that of 4 women who can complete a job in 12 days working together. How much time would 5 women and 3 men together take to complete the same job

(a) 27 days (b) 24 days (c) $5\frac{1}{3}$ days (d) none of these

Soln. Since, 3 men = 4 women
Hence, 3 men + 5 women = 4W + 5W = 9W
$M_1 D_1 = M_2 D_2$

$4\times12 = 9\times D_2$

$$D_2 = \frac{4\times12}{9} = \frac{16}{3} = 5\frac{1}{3}$$

7. 3 men earn as much as 4 women, 4 women earn as much as 6 boys and 8 boys earn as 10 girls. If a girl earn Rs. 50 a day then the earning of a man would be

(a) Rs 115 (b) Rs 135 (c) Rs 125 (d) Rs 150

Soln. Earning of a girl = Rs 50

Earning of 10 girl = 50×10 = Rs 500

Therefore, earning of 8 boys = Rs 500

$$\text{Therefore, earning of 6 boys} = \frac{6}{8}\times500 = \text{Rs.375}$$

Therefore, earning of 4 women = Rs. 375

Therefore, earning of 3 men = Rs. 375

$$\text{Therefore, earning of 1 man} = \frac{375}{3} = \text{Rs 125}$$

8. The work done by a man, a woman and a child are in the ratio 3 : 2 : 1. If daily wages of 20 men, 30 women and 36 children amount to Rs. 78. What will be the wages of 15 men, 21 women and 30 children for 18 week

(a) Rs. 7371 (b) Rs. 8645 (c) Rs. 9000 (d) Rs. 7173

Soln. Let a man, a woman and a child put in 3, 2, 1 units per day. Then they are paid Rs. 78 for putting in ($20\times3 + 30\times2 + 36\times1$) units = 156 units.

Hence, $15M + 21W + 30C = (15\times3 + 21\times2 + 30\times1) = 117$ units

Since, 156 units get per day = Rs. 78

$$\text{Therefore, 117 unit get per day} = \frac{117}{156}\times18\times7 = \text{Rs.7371}$$

9. A, B and C working together complete a job in 18 days. A and B together work twice as much as C, A and C together work thrice as much B. A alone can finish the work in

(a) 18 days (b) 43.2 days (c) 54 days (d) 72 days

Soln. Let A, B and C can finish the job alone in a, b and c days respectively.

$$\text{Hence, } \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{18} \quad \ldots (i)$$

$$\text{and } \frac{1}{a}+\frac{1}{b}=\frac{2}{c} \quad \ldots (ii)$$

$$\text{and } \frac{1}{a}+\frac{1}{c}=\frac{3}{b} \quad \ldots (iii)$$

Subtracting (ii) from (i), we get

$$\frac{1}{c}=\frac{1}{18}-\frac{2}{c};\ \frac{3}{c}=\frac{1}{18};\ c = 54 \text{ days}$$

subtracting (iii) from (i), we get

$$\frac{1}{b}=\frac{1}{18}-\frac{3}{b};\ \frac{4}{b}=\frac{1}{18}$$

$b = 72$ days

$$\text{Hence, } \frac{1}{a}+\frac{1}{72}=\frac{2}{54}$$

$$\frac{1}{a}=\frac{1}{27}-\frac{1}{72} \Rightarrow \frac{1}{a}=\frac{5}{216} \Rightarrow a=\frac{216}{5}=43.2 \text{ days}$$

10. If 15 men or 24 women or 36 boys can do a work in 12 days working 8 hrs a day. How many men must be associated with 12 women and 6 boys to do another work, $2\frac{1}{4}$ times as great in 30 working days hrs. 6 per day.

(a) 10 (b) 15 (c) 8 (d) 112

Soln. $15M = 24W = 36B$

$\Rightarrow \quad 1W = \frac{15}{24}M \quad \Rightarrow 12W = \frac{15}{24}\times 12M = \frac{15}{2}M = \frac{15}{2}M$

$\Rightarrow \quad 1B = \frac{15}{36}M \quad \Rightarrow 6B = \frac{15}{36}\times 6M = \frac{15}{6}M$

Let x men and 12W and 6 boy do the work

$$Mx + \frac{15}{2}M + \frac{15}{6}M = \frac{6xM + 45M + 15M}{6} = \frac{6xM + 60M}{6} = xM + 10M$$

$m_1 = 15, d_1 = 12, h_1 = 8, w_1 = 1$

$m_2 = (x+10), d_2 = 30, h_2 = 6, w_2 = \frac{9}{4}$

$M_1 d_1 h_1 W_2 = M_2 d_2 h_2 W_1$

$15\times 12\times 8\times \frac{9}{4} = (x+10)\times 30\times 6\times 1$

$x + 10 = 18$

$x = 8$ *men*

3. Pipe and Cisterns:

Inlet: A pipe connected with a tank or a cistern or a reservoir that fills it is known as an inlet.

Outlet: A pipe connected with a tank or a cistern or a reservoir emptying it is known as an outlet.

(1) Two pipes A and B can fill a cistern in x and y hours respectively alone. If both the pipes are open together, then the time taken to fill the cistern $= \frac{xy}{(x+y)}$ *hrs*.

(2) Three pipes A, B and C can fill a cistern in x, y and z hours alone respectively. If all pipes are opened together, the time taken to fill the cistern is $\frac{xyz}{xy + yz + xz}$ *hrs*

(3) Two pipes A and B can fill a cistern in x and y hours respectively, there is also an outlet C. If all pipes are opened together, the tank is filled in z hours. Then the time taken by C to empty the full tank is given by $\frac{xyz}{xz + yz - xy}$ *hrs*

(4) A tank takes x hours to be filled by a pipe out but due to a leak it is filled in y hrs. The amount of time in which the leak can empty the tank $\frac{xy}{y-x}$ *hrs*.

(5) Pipe A is 'k' times faster than B and B can fill a cistern in x hours. If both the pipes are opened together, then the cistern will be filled $\frac{x}{k+1}$ hours. If A can fill a cistern in y hrs, then after opening both pipe, cistern will be filled in $\left(\frac{k}{k+1}\right) y$ *hrs*

(6) If one pipe A is k times faster than B and A takes x min less than B to fill a tank. If both the pipes are opened together. The cistern will filled in $\left(\frac{kx}{(k-1)^2}\right)$ min and A will fill the cistern in $\left(\frac{x}{k-1}\right)$ min and B will fill the cistern in $\left(\frac{kx}{k-1}\right)$ min

Rule 1: If a pipe can fill a tank in x hours then part filled in 1 hour $= \frac{1}{x}$

If a pipe can empty a full tank in y hours then part emptied in 1 hour $= \frac{1}{y}$

11. There is an empty reservoir whose capacity is 30L. There is an inlet pipe A which fill at 5L/minute and B an outlet pipe which empties at 4L/min. Both the pipes function alternatively for one-one minute assuming that A is the first one to function then B. How much time will it take of the reservoir to the filled up to its capacity

(a) 60 (b) 51 (c) 50 (d) 40

Soln. In first minute pipe A will fill = 5L and in second minute pipe B will empty = 4 L

Hence, in two minutes, reservoir will filled = 5L – 4L = 1L

then in 50 minutes, it will filled $= \frac{50}{2} = 25L$ and in next minute, the turn of pipe a will come and will fill gain 5L.

So, 25L + 5L = 30 L

Required time = 50 + 1 = 51 min.

12. A larger tanker can be filled by two pipes A and B respectively in 60 min and 40 min. How many minutes will it take to fill the tanker from empty if A and B both are used for half of time and B alone is used for half of time

(a) 15 min (b) 30 min (c) 45 min (d) 1 hour

Soln. Let tanker be filled in x min then $\frac{x}{2}\left[\frac{1}{60}+\frac{1}{40}\right]+\frac{x}{2}\times\frac{1}{40}=1$

$$\Rightarrow \quad \frac{x}{120}+\frac{x}{80}+\frac{x}{80}=1 \Rightarrow \frac{2x+3x+3x}{240}=1 \Rightarrow \frac{8x}{240}=1 \Rightarrow x=\frac{240}{8}=30\,\text{min}$$

13. Bucket P was thrice the capacity as bucket Q. So, it takes 60 turns for bucket P to fill the empty drum. How many turns it will take for both the bucket P and Q having each turn together to fill the empty drum

(a) 30 (b) 40 (c) 45 (d) 90

Soln. Let capacitiy of bucket Q is 1L

then capacity of bucket P = 3L

and capacity of drum = 60×3L = 180L

Now each turn of P and Q both the drum is filled = 3L + 1L = 4L.

Hence, required turns $= \frac{180}{4} = 45$

14. Pipe A can fill the tank in 4 hrs while pipe B can fill it in 6 hours working alone. Pipe C can empty whole the tank in 4 hours. He opened the pipe simultaneously to fill the empty tank. He wanted to adjust his alarm so that he could open the pipe C when it was half filled but he mistakely adjust his alarm at a time when the tank would $\frac{3}{4}th$ filled. What is the time difference between both the cases to fill the tank fully.

(a) 48 min (b) 54 min (c) 30 min (d) none of these

Soln. If both the pipe A and B are opened the tank will filled in $\frac{4\times6}{4+6}=\frac{24}{10}=2.4$ hrs

Then half of it is filled in $=\frac{2.4}{2}=1.2$ hrs and $\frac{3}{4}$th of it $=2.4\times\frac{3}{4}=1.8$ hours

When all three pipes are opened the tank will filled $=\frac{1}{4}+\frac{1}{6}-\frac{1}{4}$

in one hour $=\frac{1}{6}=6$ hrs and $\frac{1}{4}$ of it $=6\times\frac{1}{4}=3$hrs

In first case time required for filling the tank = 1.2 + 3 = 4.2 hrs = 4 hours and 12 min
In second case for filling the tank = 1.8 + 1.5 = 1 hour 48 min + 1 hour 30 min = 3 hour & 18 min.
Hence, time difference = 4 hour : 12 min – 3 hour : 18 min = 54 min.

15. Two pipes A and B can fill a cistern in 15 hours and 10 hours respectively. While tap C can empty the full cistern in 30 hours. All the three taps were open for 1 hours when it was remebered that the emptying tap had been open left, then it was closed. How many hours would it take for the cistern to be filled
(a) 30 min (b) 5.2 hour (c) 24 min (d) 35 min

Soln. In 1 hours work of all taps $=\frac{1}{15}+\frac{1}{10}-\frac{1}{30}=\frac{2+3-1}{30}=\frac{4}{30}=\frac{2}{15}$

Now, $\frac{2}{15}$ part of cistern has filled. Then remaining part $=1-\frac{2}{15}=\frac{13}{15}$

Now, work of A and B tab for one hour $=\frac{1}{15}+\frac{1}{10}=\frac{2+3}{30}=\frac{5}{30}=\frac{1}{6}$

Now, required more times $=\frac{13}{15}\div\frac{1}{6}=\frac{13}{15}\times\frac{6}{1}=\frac{26}{5}=5.2$hrs

16. Two pipes A and B can fill a tank in 36 min and 45 min respectively. A water pipe C can empty tank in 30 minutes .First A and B are opened after 7 minutes C is also opened then in how much time the tank is full.

Soln. Part filled in 7 minute $=7\left(\frac{1}{36}+\frac{1}{45}\right)=\frac{7}{20}$ so remaining part $=1-\frac{7}{20}=\frac{13}{20}$

Now part filled in 1 minute when A B and C are opened $=\left(\frac{1}{36}+\frac{1}{45}-\frac{1}{30}\right)=\frac{1}{60}$

$\frac{1}{60}$ part is filled in 1 min; $\therefore$ $\frac{13}{20}$ part is filled in $\left(\frac{13}{20}\times60\right)=39$ min

Total time taken to fill tank=39+7=46 minute.

17. A cistern has two tapes which fill it in 12 min and 15 min respectively. There is also a waste pipe in the Cistern .when all the three are opened the empty cistern is full in 20 minutes. How long the waste pipe take to empty the full cistern?

Soln.
$$\left(\frac{1}{12}+\frac{1}{15}-\frac{1}{x}\right)=\frac{1}{20}$$
$$\Rightarrow \quad \frac{1}{x}=\frac{1}{12}+\frac{1}{15}-\frac{1}{20}\Rightarrow x=\frac{60}{6}=10$$

So, waste pipe will empty the full cistern in 10 minutes.

Rule 4: Calculation of capacity of cistern:

18. Two pipes A and B can separately fill a cistern in $7\frac{1}{2}$ and 5 minutes respectively and a waste pipe can carry off 14 liters per minute .If all the pipes are opened when the cistern is full it is emptied in 1 hour. How many litres does it hold?

Soln. Net work done by all three pipes in 1 minutes $= -\frac{1}{60}$

Work done by inlets in one minutes-work done by outlets in one minute $= -\frac{1}{60}$

$$\left(\frac{2}{15}+\frac{1}{5}-\frac{1}{x}\right)=-\frac{1}{60}$$

So, work done by waste pipe in one min $=\frac{1}{5}+\frac{2}{15}+\frac{1}{60}=\frac{7}{20}$

$$\text{Capacity of cistern} = \frac{\text{Flow rate in 1 min by waste pipe}}{\text{work done by watse pipe in 1 min}} = \frac{14}{\frac{7}{20}} = 40 \text{ litre}$$

19. The work done by a child is one third that by a man and half that by a woman. If one man, one woman and one child together can complete a work in 2 days, in how many days can 4 children together complete the same work?

Soln. We have, 1M = 3C and 1W = 2C (where, M = male work, C = child work, W = woman work)
Therefore, 1M + 1W + 1C = 6C

Hence, the required number of days $= \frac{6\times 2}{4} = 3$ days

25. In a central government office, there are 5 working days and for each day, the working hours are 8. A employee gets Rs. 2.40 per hour for regular work and Rs. 3.20 per hour for overtime. If he earns Rs. 432 in 4 weeks, how many hours he work for?
(a) 195 (b) 160 (c) 175 (d) 180

EXERCISE

1. A man and a boy together can do a certain amount of digging in 40 days. Their skills in digging are in the ratio of 8 : 5. How many days will the boy take, if engaged alone.
(a) 52 days (b) 104 days (c) 68 days (d) 80 days

2. If 10 men wave 10 mats in 10 days, how many men will be required to weave 100 mats in 100 days?
(a) 100 (b) 1000 (c) 10 (d) 1

3. One man can paint a house in 'r' days and another man can do it in 't' days. If they can together do it in 'd' days, then 'd' is given by :

(a) $\frac{1}{r}+\frac{1}{t}$ (b) $\frac{rt}{r+t}$ (c) $\frac{24rt}{r+t}$ (d) $\frac{rt}{t+24r}$

4. If a factory "A" turns out "X" cars an hour and factory "B" turns out "Y" cars every 2 hours, the number of cars that both factories turn out in 8 hours is:

(a) 8(X + Y) (b) $8X+\frac{Y}{2}$ (c) $8\left(X+\frac{Y}{2}\right)$ (d) X + Y

5. From a leaking tap 'a' drop come out in 'b' min. If there are 'c' drops in a liter, then in how many hours one liter of water will be wasted?

(a) $\frac{60a}{bc}$ (b) $\frac{a}{bc}$ (c) $\frac{abc}{60}$ (d) $\frac{bc}{60a}$

6. Two fill pipes A and B can fill a tank in 15 and 12 hours respectively. Pipe B alone is kept open for of 7.5 hours time and both pipes are kept open for remaining time. In how many hours, the tank will be full?

(a) 18 h (b) 20 h (c) 10 h (d) 13.5 h

7. Nine men and seven women pick as much corn in five days as seven men and eleven women pick in four days. Who are the better corn pickers and by how much?

(a) Women are better workers by a ratio of 17:9
(b) Women are better workers by a ratio of 2:1
(c) Men are better workers by a ratio of 13:1
(d) Men are better workers by a ratio of 2 : 1

8. Ram can complete a piece of work in 12 days and Shyam is 60% more efficient than Ram. In how many days Shyam will complete the same work?

(a) 9 days (b) 5 days (c) 7.5 days (d) 5.5 days

9. A tank is drained by two taps, T1 and T2. If both tapes are open the tank empties in 20 minutes. If T2 is open and T1 is closed the tank empties in 30 minutes. If T2 is closed and T1 is open how long will it take for the tank to empty?

(a) 45 min (b) 50 min (c) 60 min (d) 50 min
(e) None of the above.

10. A and B can do a job together in 10 days. A and C together can do the same job in 15 days. B and C together require 30 days to do the same job. The number of days required to do the job individually by A, B and C respectively is

(a) 15, 30, never (b) 30, 15, never (c) never, 15, 30 (d) 15, 30, 100
(e) None of the above.

11. It takes 6 technicians a total of 10 hours to install a new equipment from scratch, with each working at the same rate. If six technicians start to install the same equipment at 11:00 am, and one technician per hour is added beginning at 5:00 pm, at what time will the equipment installation be complete?

(a) 6:40 pm (b) 7:00 pm (c) 7:20 pm (d) 8:00 pm

12. 3 small pumps and a large pump are filling a tank. Each of the three small pumps works at $\frac{2}{3}$rd the rate of the large pump. In what fraction of the time that all 4 pumps working together will fill the tank in comparison to the time taken by the large pump alone?

(a) $\frac{4}{7}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

13. A wealthy woman had three servants S_1, S_2 and S_3 was estimating that the amount of grain in her storeroom would suffice for S_1 and S_2 for 45 days. S_1 and S_3 would eat all the grain in 60 days. It would take S_2 and S_3 90 days to eat it all. Now if she were to feed S_1, S_2 and S_3 together, how long would it have taken them to eat all the grain?

(a) 40 days (b) 65 days (c) 180 days (d) None of the above.

HINTS & SOLUTIONS

1. Let efficiency of man and boy are respectively 8k and 5k

   Work completed in 40 days, then $40(8k+5k)=1$; $\quad k=\frac{1}{40\times13}$; $\quad 5k=\frac{1}{104}$

   Answer is (b)

2. Answer is (c)

3. Answer is (b)

4. Each hour A turn out x cars
   Each hour B turn out y/2 cars.

   In 8 hours cars turn out $=\left(x+\frac{y}{2}\right)8$

   Answer is (c)

5. Drops out per minute $=\frac{a}{b}$ drop/minute. $\quad$ Drops out per hour $=\frac{60\,a}{b}$ drop/hour

   Liter out per hour $=\frac{c}{60\left(\frac{a}{b}\right)}=\frac{bc}{60\,a}$. $\quad$ Thus time taken for one liter $=\frac{60\,a}{bc}$.

   Answer is (a)

6. Cistern filled per hour $=\frac{1}{10}+\frac{1}{8}-\frac{1}{15}=\frac{19}{20}$

   Time taken to complete filled $=\frac{120}{19}=6\frac{6}{19}$ hour

   Answer is (c)

7. Answer is (c)

8. (9M + 7W)5 = (7M + 11W) 4
   45M + 35W = 28M + 44W
   45M–28 M = 44W – 35 W
   17M = 9W

   $\frac{M}{W}=\frac{9}{17}$ $\quad$ M : W = 9 : 17

   Answer is (a)

9. Efficiency of Ram : Shyam = 100 : 160 = 5 : 8
   Hence, number of days = 8 : 5
   8 part = 12 day
   Hence, 1 part = 12/8 day

   Hence, 5 part $=\frac{12}{8}\times5=\frac{15}{2}=7.5$ days

   Answer is (c)

10. $(T_1 + T_2)$'s 1 min work $= \frac{1}{20}$

T_2's 1 min work $= \frac{1}{30}$

Hence, T_1's 1 min work $= \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$

Hence, T_1 can empty it in = 60 min
Answer is (c)

11. (A + B)'s 1 day work $= \frac{1}{10}$

+ (A + C)'s 1 day work $= \frac{1}{15}$

(B + C)'s 1 day work $= \frac{1}{30}$

2(A + B + C)'s 1 day work $= \frac{1}{10} + \frac{1}{15} + \frac{1}{30} = \frac{6+4+2}{60} = \frac{12}{60} = \frac{1}{5}$

Therefore, (A + B + C)'s 1 day work $= \frac{1}{5\times 2} = \frac{1}{10}$

{(A + B + C)'s – (A + B)'s} 1 day work $= \frac{1}{10} - \frac{1}{10} = 0 \Rightarrow$ C = never

(A + B + C)'s – (A + C)'s 1 day work $= \frac{1}{10} - \frac{1}{10} = \frac{1}{30} \Rightarrow$ B = 30 days

(A + B + C)'s – (B + C)'s 1 day work $= \frac{1}{10} - \frac{1}{30} = \frac{2}{30} = \frac{1}{15} \Rightarrow$ A = 15 days

Answer is (a)

12. 6 tech. take = 10 hrs
hence 1 tech. take = 60 hrs
hence, work of 1 tech. in 1 hrs = 1/60 part
6 tech. 1 hrs work = 1/60×6 = 1/10
In 6 hrs they complete = 6×1/10 = 3/5 part
Remaining part = 1–3/5 = 2/5

Now, 5 + 02 there are 7 tech. work = 7/60 $= \frac{2}{5} - \frac{7}{60} = \frac{24-7}{60} = \frac{17}{60}$

Now, 6 to 7 there are 8 tech. work $= \frac{8}{60} = \frac{2}{15} = \frac{17}{60} - \frac{2}{15} = \frac{9}{60} = \frac{3}{20}$

Remaining work = 3/20
Hence, 8 : 00
Answer is (d)

13. Let us suppose if large pump fill the tank in 1 hrs then small pump will take 3/2 hrs
Large pump in = x hrs. then in 1 hrs = 1/x part
Small pump in = 3/2x hrs. then in 1 hrs = 2/3x part

For 3 small pumps = $\frac{2}{3x} \times 3$ part = $\frac{2}{x}$ part

All four pump can fill in 1 hrs = $\frac{1}{x} + \frac{2}{x} = \frac{3}{x}$ part

Hence, whole tank = $\frac{x}{3}$ hrs. Therefore, $\frac{x}{3}/x = \frac{1}{3}$

Answer is (b)

14. One day required grain for

$$S_1 + S_2 = \frac{1}{45}$$

$$+ \quad S_1 + S_3 = \frac{1}{60}$$

$$S_2 + S_3 = \frac{1}{90}$$

$$2(S_1 + S_2 + S_3) = \frac{1}{45} + \frac{1}{60} + \frac{1}{90} = \frac{9}{180} = \frac{1}{20}$$

Therefore, for $(S_1 + S_2 + S_3)$, one day required grain = $\frac{1}{40}$

$S_0 = 40$ days.
Answer is (a)

CAREER ENDEAVOUR

TRY YOURSELF

1. In a camp, 95 men had provisions for 200 days. After 5 days, 30 men left the camp. For how many days will the remaining food last now?

   (a) 180 (b) 285 (c) $139\frac{16}{19}$ (d) None of these

2. A garrison of 500 men had provisions for 27 days. After 3 days a reinforcement of 300 men arrived. For how many more days will the remaining food last now?

   (a) 15 (b) 16 (c) $17\frac{1}{2}$ (d) 18

3. A garrisons had provisions for a certain number of days. After 10 days, 1/5 of the men desert and it is found that the provisions will now last just as long as before. How long was that?
   (a) 15 days (b) 25 days (c) 35 days (d) 50 days

4. A contractor undertook to do a certain piece of work in 9 days. He employed a certain number of men, but 6 of them being absent from the very first day, the rest could finish the work in 15 days. The number of men originally employed were
   (a) 12 (b) 15 (c) 18 (d) 24

5. A contractor undertakes to do a piece of work in 40 days. He engages 100 men at the begining and 100 more after 35 days and complete the work at stipulate time. If he had not engaged the additional men, how many days behind schedule would it be finished.
   (a) 3 (b) 5 (c) 6 (d) 9

6. A contractor employed 30 men to do a piece of work in 38 days. After 25 days, he employed 5 men more and the work was finished one day earlier. How many days he would have been behind, if he had not employed additional men

   (a) 1 (b) $1\frac{1}{4}$ (c) $1\frac{3}{4}$ (d) $1\frac{1}{2}$

7. A booster pump can be used for filling as well as for emptying a tank. The capacity of tank is 2400 m^3. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs 8 minute lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump?
   (a) 50 m^3/min (b) 60 m^3/min (c) 72 m^3/min (d) none of these

8. Three pipes A, B and C can fill a tank from empty to full in 30 min, 20 min and 10 min respectively. When the tank is empty, all the three pipes are opened. A, B and C discharge chemical solutions P, Q and R respectively. What is the proprtion of solution R in the liquid in the tank after 3 minutes.
   (a) 5/11 (b) 6/11 (c) 7/11 (d) 8/11

9. 24 men can complete a work in 16 days. 32 women can complete the same work in 24 days. 16 men and 16 women started working and worked for 12 days. How many more men are to be added to complete the remaining work in 2 days?
   (a) 16 (b) 24 (c) 36 (d) 48

10. Twenty women can do a work in 16 days. Sixteen men can complete the same work in 15 days. What is the ratio between the capacity of man and a woman
    (a) 3 : 4 (b) 4 : 3 (c) 5 : 3 (d) data inadequate

Answer Key

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	b	a	d	b	b	a	a	b	b	b

1.

2.

CHAPTER

9

Mensuration & area of plane figures

1. Plane Figure:

Any figure bounded by three or more straight lines or bounded by a closed line is a **Plane Figure**. The space closed within that figure is called its **Area.** The measurement of the length of the lines enclosing the space called its **Perimeter.**

Parallel lines: Two straight lines are parallel if they lie on the same plane and do not intersect.
Transversal: It is a straight line that intersects two parallel lines.

2. Polygons:

Polygons are figures formed by a closed series of rectilinear (straight) segments.
Example: Triangle, Rectangle, Pentagonal (5 sides), Hexagonal (6 sides) .....
Polygons can broadly be divided into two types:
(i) **Regular polygons:** Polygons with all the sides and angles equal.
(ii) **Irregular polygons:** Polygons in which all the sides or angles are not of the same measure.

Properties:
(i) Sum of the angles of a polygon with n sides = (2n -4)p/2 Radians = (n–2) 180° degrees
(ii) Sum of the exterior angles = 360° i.e. in the figure below: $\theta_1 + \theta_2 + ... + \theta_6 = 360°$

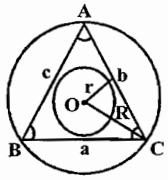
In general, $\theta_1 + \theta_2 + ... + \theta_n = 360°$
(iii) Number of sides = 360°/exterior angle.
(iv) Perimeter = n×s.

(a) Triangles (Δ):
A triangle is a polygon having three sides. Sum of all the angles of a triangle = 180°.
Types of Triangle:
(i) Acute angle triangle: Triangles with all angles acute (less than 90°)
(ii) Obtuse angle triangle: Triangles with one of the angles obtuse (more than 90°).
(iii) Right angle triangle: Triangle with one of the angles equal to 90°.

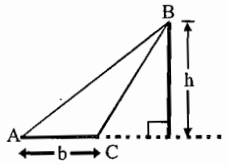
- **Important points about triangles (Δ):**

(i) Sum of the length of any two sides of a triangle has to be always greater than the third side.
(ii) Difference between the lengths of any two sides of a triangle has to be always lesser than the third side.
(iii) Side opposite to the greatest angle will be the greatest and the side opposite to the smallest angle the smallest.
(iv) The sine rule: a/sin A = b/sin B = c/sin C = 2R (where R = circum radius).
(v) The cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

This is true for all sides and respective angles.



• Formulas for area of a triangle:



(i) Area $= \frac{1}{2}$ base × height . Height = Perpendicular distance between the base and vertex opposite to it.

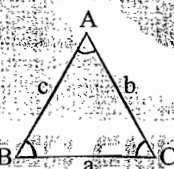
(ii) Area $= \sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$ where, s is half perimeter. (a, b and c being the length of the sides of the triangle).

(iii) Area = rs (where r is inradius).

(iv) Area $= \frac{1}{2}$ × product of two sides × sine of the included angle.

$= \frac{1}{2}ac \sin B = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A.$

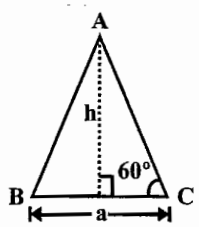


CAREER ENDEAVOUR

(v) Δ Area $= \frac{abc}{4R}$, where R = circumradius.

• Equilateral triangles (of side a): Triangle having all the angles equal to 60°.

(i) $h = \frac{a\sqrt{3}}{2}$ $\left(\because \sin 60 = \sqrt{3}/2 = h/side\right)$ (ii) Area $= \frac{1}{2}$ (base) ×(height) $= \frac{1}{2} \times a \times \frac{a\sqrt{3}}{2} = \frac{\sqrt{3}}{4}a^2$



(iii) R (circum radius) $= \frac{2h}{3} = \frac{a}{\sqrt{3}}$ (iv) r (in radius) $= \frac{h}{3} = \frac{a}{2\sqrt{3}}$

• **Properties of equilateral triangle:**

(i) The incenter and circumcentre lies at a point that divides the height in the ratio 2:1.

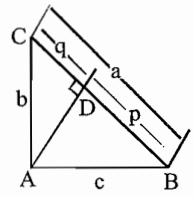
(ii) The circum radius is always twice the in radius [R=2r].

(iii) Among all the triangles that can be formed with a given perimeter, the equilateral triangle will have the maximum area.

(iv) An equilateral triangle in a circle will have the maximum area compared to other triangles inside the same circle.

• **Right angled triangle:** Triangle with one of the angles equal to 90°.

Pythagoras Theorem: In the case of a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



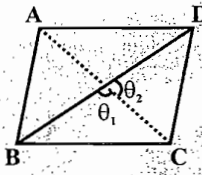
In the given figure , for triangle ABC, $a^2 = b^2 + c^2$

Area = $\frac{1}{2}$ (product of perpendicular sides) = $\frac{1}{2}$ (bc)

3. Quadrilateral:

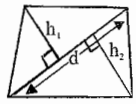
Formulas for area of quadrilaterals:

(i) Area = $\frac{1}{2}$ (product of diagonals) × (sine of the angle between, them)



If θ_1 and θ_2 are the two angles made between themselves by the two diagonals, we have by the property of intersecting lines $\rightarrow \theta_1 + \theta_2 = 180°$. Then the area of quadrilateral $= \frac{1}{2} d_1 d_2 \sin\theta_1 = \frac{1}{2} d_1 d_2 \sin\theta_2$.

(ii) Area = $\frac{1}{2}$ × diagonal × sum of the perpendiculars to it from opposite vertices = $\frac{d(h_1 + h_2)}{2}$



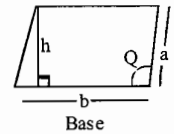
(iii) Area of a circumscribed quadrilateral $A = \sqrt{(S-a)(S-b)(S-c)(S-d)}$ Where, $S = \frac{a+b+c+d}{2}$

(Where a, b, c and d are the lengths of the sides.).

Types of Quadrilaterals:

(i) Parallelogram:

A parallelogram is a quadrilateral with opposite sides parallel



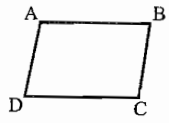
Formulas for area of parallelogram:

(i) Area = Product of any two adjacent sides × sine of the included angle.= ab sin Q

$$\text{Area of parallelogram} = \frac{1}{2} h \cdot (a+b)$$

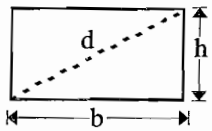
(ii) Perimeter = 2(a + b) where a and b are any two adjacent sides.

Properties of parallelogram:



(i) Diagonals of a parallelogram bisect each other.
(ii) Bisectors of the angles of a parallelogram form a rectangle.
(iii) A parallelogram inscribed in a circle is a rectangle.
(iv) A parallelogram circumscribed about a circle is a rhombus.
(v) The opposite angles in a parallelogram are equal.
(vi) The sum of the squares of the diagonals is equal to the sum of the squares of the four sides in the figure.

(ii) Rectangles: A rectangle is a parallelogram with all angles 90°.



(i) Area = Base × Height = b× h.

(ii) Diagonal $(d) = \sqrt{b^2 + h^2}$ → by Pythagoras theorem.

Properties of a rectangle:
(i) Diagonals are equal and bisect each other.
(ii) Bisectors of the angels of a rectangle form another rectangle.
(iii) All rectangles are parallelogram but the reverse is not true.

(c) Rhombus: A parallelogram having all the sides equal is a rhombus.

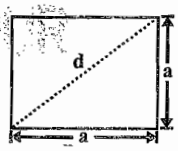
(i) Area = $\frac{1}{2}$ × product of diagonals × sine of the angle between them.

$= 1/2 \times d_1 \times d_2 \sin 90°$ (Diagonals in rhombus intersect at right angles) $= 1/2 \times d_1 d_2$

(ii) Area = product of adjacent sides × sine of the angle between them.

Properties of Rhombus:
(i) Diagonals bisect each other at right angles.
(ii) All rhombuses are parallelogram but the reverse is not true.
(iii) A rhombus may or may not be square but all squares are rhombus.

(d) Squares:
A square is a rectangle with sides are equal or a rhombus with each angle 90°.
(i) Area = base × height = a^2.

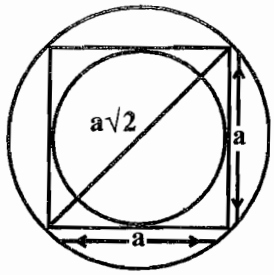


(ii) Area = $\frac{1}{2}$ (diagonal)^2 = $\frac{1}{2}$ d^2 (square is a rhombus too).

(iii) Perimeter = 4a(a = side of the square)

(iv) Diagonal = $a\sqrt{2}$

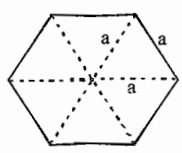
(v) In radius = $\frac{a}{2}$



Properties of Square:
(i) Diagonals are equal and bisect each other at right angles.
(ii) Side is the diameter of the inscribed circle.
(iii) Diagonal is the diameter of the circumscribing circle.

$\Rightarrow$ Diameter = $a\sqrt{2}$ Circumradius = $\frac{a}{\sqrt{2}}$

(e) Regular Hexagon:



(i) Area = $\left[\left(3\sqrt{3}\right)/2\right](\text{side})^2 = \frac{3\sqrt{3} \times a^2}{2}$

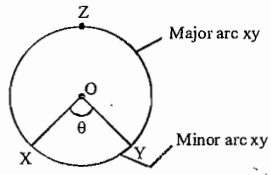
(ii) A regular hexagon is actually a combination of 6 equilateral triangles all of side 'a'.
(iii) If you look at the figure closely it will not be difficult to realize that circumradius (R) = a; i.e. the side of the hexagon is equal to the circumradius of the same.

4. Circle:

(i) Area = πr^2. (ii) Circumference = $2\pi r$ (r = radius). (iii) Area = ½ × circumference × r.

Arc of circle: It is a part of the circumference of the circle. The bigger one is called the major arc and the smaller one the minor arc.

(iv) Length $(\text{Arc XY}) = \frac{\theta}{360} \times 2\pi r$



(v) Sector of a circle is a part of the area of a circle between two radii.

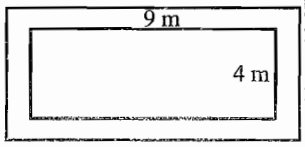
(vi) Area of a sector = $\frac{\theta}{360} \times \pi r^2$ (where θ is the angle between two radii)

$$= \left(\frac{1}{2}\right) r \times \text{length (arc xy)} \quad (\because \quad \pi r\theta / 180 = \text{length arc xy})$$

$$= \frac{1}{2} \times r \times \frac{2\pi r\theta}{360}$$

SOLVED EXAMPLES

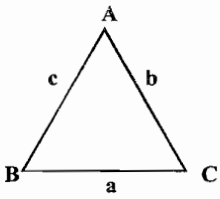
1. Pradeep wishes to make a gravel path around his rectangular pond. The path must be the same width all the way round, as shown in the diagram. The pond measures 4m by 9m and he has enough gravel to cover an area of 48m². How wide around the path be?



(a) 2 metre (b) 3 metre (c) 1.5 metre (d) 8 metre

2. What is the minimum value of the perimeter of a triangle, if two of its sides are 5cm and 7cm respectively? (the sides have integer volumes)

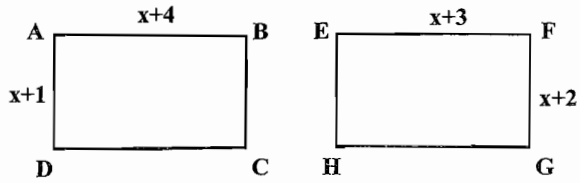
Soln. In a triangle, c > b–a



Therefore, c > 7–5 or c > 2

The minimum length of the third side is 3 cm
Hence, the minimum value of the perimeter = 5 + 7 + 3 = 15 cm

3. What is the area of rectangle EFGH of given figure, if the area of rectangle ABCD is 100?



Soln. Area of rectangle ABCD = $(x + 1) \times (x + 4) = x^2 + 5x + 4$
Area of rectangle EFGH = $(x + 2) \times (x + 3) = x^2 + 5x + 6$
If we compare the area of rectangle ABCD and EFGH, the area of rectangle EFGH is 2 more than the area of rectangle ABCD.
Therefore, Area of rectangle EFGH = 100 + 2 = 102

4. Euclid has a triangle in mind. Its longest side 20 units and another of its sides has length 10 units. Its area is 80 square units. What is the exact length of its third side?

(a) $\sqrt{270}$ units (b) $\sqrt{240}$ units (c) $\sqrt{250}$ units (d) $\sqrt{260}$ units

5. If a square and an equilateral triangle have equal perimeters, what is the ratio of the area of the triangle to the area of the square?

(a) $\frac{4\sqrt{3}}{9}$ (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) None of these

EXERCISE

1. In two triangles, ratio of the areas in 4:3 and the ratio of their heights is 3:4. Find the ratio of their bases.

(a) $\frac{16}{9}$ (b) $\frac{9}{16}$ (c) $\frac{12}{9}$ (d) $\frac{9}{12}$

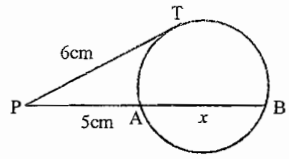
2. A rectangular gram plot 100m by 65m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per square metre.

(a) Rs 620 (b) Rs 660 (c) Rs 680 (d) Rs 640

3. AB is a chord of a circle of radius 14 cm. The chord subtends a angle of 90° at the centre of the circle. Find the area of minor segment.

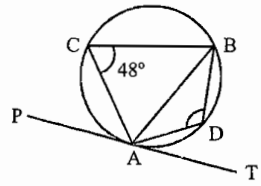
(a) 98 sqcm (b) 56 sqcm (c) 112 sqcm (d) 100 sqcm.

4. Find the value of 'x' in the given figure



(a) 2.2 cm (b) 1.6 cm (c) 3 cm (d) 2.5 cm

5. In the given figure, find angle ADB



(a) 130° (b) 132° (c) 122° (d) 120°

6. In a triangle ABC, point D is an side AB and point E is on side AC, such that BCED is a trapezium. DE:BC = 3:5. Calculate ratio of area of triangle ADE and the trapezium BCED.
(a) 3:4 (b) 9:16 (c) 3:5 (d) 9:25

7. A motocycle stuntman, belonging to a fair, rides over the vertical walls of a circular well at an average speed of 54 kmph for 5 minutes. If the radius of the well is 5 meters, then the distance travelled is:
(a) 2.5 km (b) 3.5 km (c) 4.5 km (d) 5.5 km

8. The least perimeter of an equilateral triangle in which a circle of radius 'r' can be inscribed is:
(a) $3r\sqrt{3}$ (b) $\sqrt{3}\,r$ (c) $\frac{6r}{\sqrt{3}}$ (d) $6r\sqrt{3}$

9. Consider a square of side 6 cm. A circle is inscribed inside the square. Another circle circumscribes the square. The ratio of the areas of the inscribed circle to the circumscribed circle is
(a) $1:\frac{\pi}{4}$ (b) $1:\pi$ (c) 1:1.5 (d) 1 : 2

10. A carpenter is designing a box. The floor of the box is a rectangular whose length is two feet more than its width. How long should the box be (in feet) if the area of the floor of the box were to be 15 square feet?
(a) 3 (b) 5 (c) 10 (d) 15

11. A wire of some length is bent in circular form and has an area of 308 sq. cm. If the same length of wire is straightended out and bent in the form of a square, the approximate area of the square in sq. cm may be
(a) 242 (b) 121 (c) 308 (d) 69.29

12. Consider a sqaqure circumscribed by a circle with a radius of 4 units. The area of the square in square units is
(a) $16\sqrt{2}$ (b) 16π (c) 32 (d) 64

13. Four cows are tethered at four corners of a square plot of side 14 meters so that the adjacent cows can just reach one another. There is a small circular pond of area 20 m^2 at the centre. The area left ungrazed is:
(a) 22 m^2 (b) 42 m^2 (c) 84 m^2 (d) 168 m^2.

14. Instead of walking along two adjacent sides of a rectangluar field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is
(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

15. There are two cubes on a table in which the volume of the second is half that of the first. If the first cube occupies a certain area (Y) on the table, how much area (approxiimately) does the second occupy?
(a) $\frac{Y}{\sqrt[3]{2}}$ (b) $\frac{Y}{2}$ (c) $\frac{Y}{\sqrt[3]{4}}$ (d) $\frac{Y}{\sqrt{2}}$

16. A paper of zie $\ell \times w\,(\ell > w)$ is folded in half along the longer dimension. It is then folded in half along the other dimension and a third time, along the direction of the first fold. What are the dimension of the folded paper?
(a) $\frac{\ell}{4}\times\frac{w}{4}$ (b) $\frac{\ell}{8}\times\frac{w}{4}$ (c) $\frac{\ell}{8}\times\frac{w}{8}$ (d) $\frac{\ell}{4}\times\frac{w}{2}$

17. Paper sizes are given by A_0, A_1, A_2 etc. such that A_0 is two times larger (in area) than A_1, A_1 is two times larger than A_2 and so on. The longer dimension of each smaller size is equal to the shorter dimension of the larger size. For example, the longer dimension of A_2 is the same as the shorter dimension of A_1. In this scheme if A_4 is 210 × 297 mm in size, what are the dimensions of A_0 in mm?
(a) 840×594 (b) 420×594 (c) 840×1188 (d) None of the above.

HINTS & SOLUTIONS

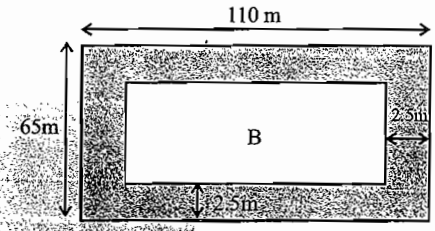
1. Let the bases of the two triangles be x and y and their heights be 3h and 4h respectively.

Then, $$\frac{\frac{1}{2}(x)(3h)}{\frac{1}{2}(y)(4h)}=\frac{4}{3} \quad \Rightarrow \frac{x}{y}=\left(\frac{4}{3}\right)\left(\frac{4}{3}\right)=\frac{16}{9}$$

Answer is (a).

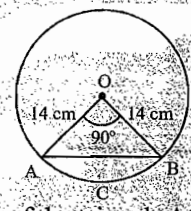
2. Area of plot = 110 × 65 = 7150 m².
Area of excluding the path = (100–5) (65–5) = 6300 m².
Area of path = (7150 – 6300) m² = 850 m².

Cost of gravelling the path = $850\left(\frac{80}{100}\right)$ Rs = Rs 680



Answer is (c)

3. Area of sector ACBO = $\frac{90^\circ \pi r^2 / 4^2}{360^\circ}$ = 154 sqcm



Area of triangle AOB = $\frac{14\times14}{2}$ = 98 sqcm

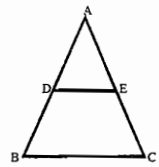
Area of segment ACB = Area of sector ACBO – Area of the triangle AOB = 154 – 98 = 56 sqcm.

Answer is (b)

4. In the figure, $PT^2 = PA \times PB$, $6^2 = 5(5 + x)$, x = 2.2 cm
Answer is (a)

5. ADBC is a cyclic qudrilateral as all its fair vertices are on the circumference of the circle. Also, the opposite angles of the cyclic qudrilateral a supplementary.
Therefore, $\angle ADB = 180° - 48° = 132°$
Answer is (b)

6. ΔADE is a similar to ΔABC
Thus, DE : BC = 3:5
Area of ΔADE : Area of ΔABC = 9:25
Area of trapezium = area of ABC – area of APE = 25–9 = 16
Area of ΔADE : Area of trapezium EDBC = 9/16
Answer is (b)



7. Speed = 54 km/h

Time = 5 min = $\frac{5}{60}$ hrs

Hence, D = Speed × time = $54\times\frac{5}{60}$ = 4.5 km

Answer is (c)

8. The radius of incircle of any equilateral triangle $r = \frac{a}{2\sqrt{3}}$

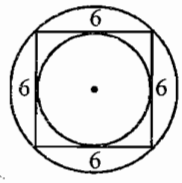
Hence, side of triangle is $2\sqrt{3}.r$

Hence, perimeter $= 3\times a = 3\times 2\sqrt{3}.r = 6\sqrt{3}\ r$

Answer is (d)

9. Diameter of inscribed circle = side of square = 6

$r = 3$



Hence, area $= \pi 3^2 = 9\pi$

Diameter of circumcircle = diagonal of square $= 6\sqrt{2}$

$$R = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

Hence, Area $= \pi\left(3\sqrt{2}\right)^2 = 18\pi$

Hence, ratio $= 9\pi : 18\pi \Rightarrow 1:2$

Answer is (d)

10. Let width = x, hence length = x + 2

Then area, $x(x+2) = 15$

$\Rightarrow$ $x = 3$. Then length $= 3 + 2 = 5$

Answer is (b)

11. Area of circle = 308 cm

Then, $\pi r^2 = 308 \Rightarrow r^2 = \frac{308}{22}\times 7 \Rightarrow r = 7\sqrt{2}$

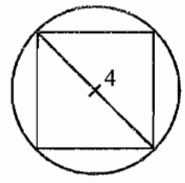
Hence, circumference $= 2\pi r = 2\times\frac{22}{7}\times 7\sqrt{2} = 44\sqrt{2}$ cm

This is equals to perimeter of square $= 4 \times$ side $= 44\sqrt{2} \Rightarrow$ side $= 11\sqrt{2}$

Hence, area $= A^2 = \left(11\sqrt{2}\right)^2 = 242$

Answer is (a)

12. Diagonal of square $= a\sqrt{2}$

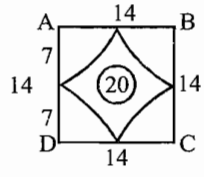


$\Rightarrow$ $a\sqrt{2} = 8 \Rightarrow a = 4\sqrt{2}$

Hence, area $= \left(4\sqrt{2}\right)^2 = 32$

Answer is (c)

13. Area of grazed by cow $= 4\times\frac{1}{4}\times\frac{22}{7}\times 7^2 = 154\ m^2$



There is a pond $= 20m^2$

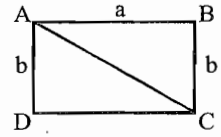
Then area of square is $= 14^2 = 196$

Hence, ungrazed area $= 196 - 154 - 20 = 22m^2$

Answer is (a)

14. Let longer side is 'a' and smaller side is 'b'. If a person walk along a two adjacent side.

Then total distance = a + b and go through diagonal = $\sqrt{a^2+b^2}$



$\therefore \quad (a+b)-\sqrt{a^2+b^2}=\frac{a}{2}$

$\therefore \quad a+b-\frac{a}{2}=\sqrt{a^2+b^2}$

Squaring both sides, we get, $\frac{a^2}{4}+b^2+2.\frac{a}{2}.b=a^2+b^2$

$\Rightarrow \quad a^2-\frac{a^2}{4}=ab \qquad \Rightarrow \frac{3a^2}{4}=ab \qquad \Rightarrow \frac{b}{a}=\frac{3}{4}$

Answer is (d)

15. Let side of first cube is A and second cube is 'a'

Hence, $V_I = A^3$ and $V_{II} = a^3$

$\frac{V_I}{V_{II}}=\frac{2}{1}=\frac{A^3}{a^3} \Rightarrow a^3=\frac{A^3}{2} \Rightarrow a=\frac{A}{\sqrt[3]{2}}$

Now, $A^2 = Y \quad \therefore a^2=\left(\frac{A}{\sqrt[3]{2}}\right)^2=\frac{A^2}{\sqrt[3]{4}}=\frac{Y}{\sqrt[3]{4}}$

Answer is (c)

16. Answer is (d)

17. A_4 = ☐ 297 $\Rightarrow$ A_3 = ☐ 210×2 =420

210 297

A_2 = ☐ 297×2 =594 $\Rightarrow$ A_1 = ☐ 420×2=840

210×2=420 594

A_0 = ☐ 594×2 =1188 hence, 840×1188

840

Answer is (c)

CAREER ENDEAVOUR

CHAPTER

10

Volume & surface area of solid figures

Solid Figure: Any figure bounded by one or more surfaces is called a solid figure. When plane surfaces bind by solid, they are called its "faces" and solid is called a Polyhedron.

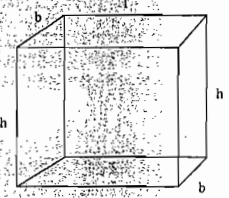
(i) Cuboid:

Let length = l, breadth = b and height = h units. Then,

(i) Volume = ($l \times b \times h$) cubic units.

(ii) Surface area = $2(lb + bh + lh)$ sq. units.

(iii) Diagonal = $\sqrt{l^2 + b^2 + h^2}$ units.



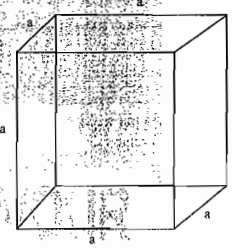
(ii) Cube:

Let each edge of a cube be of length a. Then,

(i) Volume = a^3 cubic units.

(ii) Surface area = $6a^2$ sq. units.

(iii) Diagonal = $\sqrt{3}\,a$ units.



(iii) Cylinder:

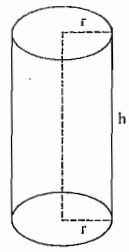
Let radius of base = r and Height (or length) = h. Then,

(i) Volume = ($\pi r^2 h$) cubic units.

(ii) Curved surface area = ($2\pi rh$) sq. units.

(iii) Total surface area = ($2\pi rh + 2\pi r^2$) sq. units

= $2\pi r\,(h + r)$ sq. units.



(iv) Cone:

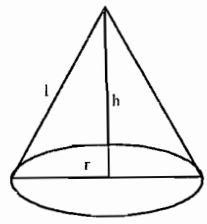
Let radius of base = r and Height = h. Then,

(i) Slant height, $l = \sqrt{h^2 + r^2}$ *units.*

(ii) Volume = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.

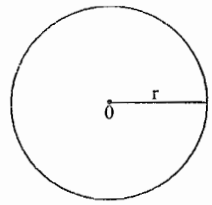
(iii) Curved surface area = (πrl) sq. units.

(iv) Total surface area = ($\pi rl + \pi r^2$) sq. units.



(v) Sphere:

Let the radius of the sphere be r. Then,

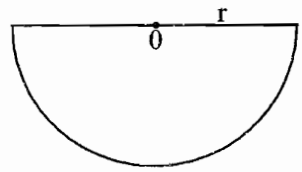


(i) Volume $= \left(\frac{4}{3}\pi r^3\right)$ cubic units.

(ii) Surface area $= (4\pi r^2)$ sq. units.

(f) Hemisphere:

Let the radius of a hemisphere be r. Then,



(i) Volume $= \left(\frac{2}{3}\pi r^3\right)$ cubic units.

(ii) Curved surface area $= (2\pi r^2)$ sq. units.

(iii) Total surface area $= (3\pi r^2)$ sq. units.

EXERCISE

1. The surface area of cube is 726 cm^2. Find the volume of the cube.
(a) 1314 m^3 (b) 1331 m^3 (c) 1741 m^3 (d) 726 m^3

2. A cube has a diagonal 34.64 m long. Find the volume.
(a) 6000 m^3 (b) 8000 m^3 (c) 7000 m^3 (d) 9000 m^3

3. Find the number of lead balls, each 1 cm in diameter that can be made from a sphere of diameter 12 cm.
(a) 1700 (b) 1688 (c) 1278 (d) 1728

4. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. Find the ratio of their volumes.
(a) 1:2:5 (b) 1:3:2 (c) 1:2:3 (c) 3:2:1

5. Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recost into a sphere. Find the diameter of the sphere.
(a) 4.1 cm (b) 4.2 cm (c) 4.6 cm (d) 5 cm

6. A solid wooden toy is in the shape of a right circular cone mainted an a hemisphere. It the radius of the hemisphere is 4.2 cm and total height of the toy is 10.2 cm, find the volume of the wooden toy.
(a) 244.030 (b) 266.112 (c) 148.125 (d) 278.112

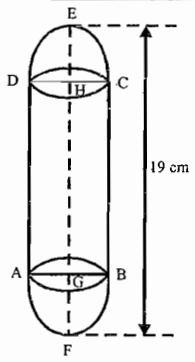
7. Find the number of bricks, each measuring, required construct a wall 12 m long, 5 m high and 0.25 m thick, while the sand and the cement mixture occupies 5% of the total volume of wall. If the dimensions of bricks are 25×12.5×7.5
(a) 6080 (b) 1520 (c) 3040 (d) 1216

8. A spherical common of ball, 28 cm in diameter is melted and cost into a right circular conical mailed the base of cone is 35 cm in diameter. Find the height of the cone correct up to two places of decimals.
(a) 8.69 cm (b) 35.84 cm (c) 5.97 cm (d) 17.92 cm

9 If V be the volume of a cuboid of dimention x, y, z and A is surface, then A / V will be equal to

(a) $x^2y^2z^2$ (b) $\frac{1}{2}\left(\frac{1}{xy}\times\frac{1}{yz}\times\frac{1}{zx}\right)$ (c) $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ (d) $\frac{1}{xyz}$

10. A solid is in the form of a cylinder with hemispherical ands. The total height of the solid is 19 cm and the diameter of the cylinder is 7 cm. Find the total surface area of the solid. (Use = 22/7)



(a) 418 cm^2 (b) 318 cm^2 (c) 468 cm^2 (d) 368 cm^2

HINTS & SOLUTIONS

1. In a cube volume $V = a^3$
surface area $s = 6a^2$

$$V^2 = a^6;\ V^2 = \left(\frac{5}{6}\right)^3;\ V = \left(\frac{5}{6}\right)^{3/2} = \left(\frac{726}{6}\right)^{3/2} = 1331\ m^3$$

Answer is (b)

2. In a cube volume $= a^3$; Diagonal $= a\sqrt{3}$

$$\text{Volume} = \left(\frac{d}{\sqrt{3}}\right)^3 = \left(\frac{34.64}{\sqrt{3}}\right)^3 = 8000\ m^3$$

Answer is (b)

3. Volume of larger sphere $= \frac{4}{3}\pi \times 6 \times 6 \times 6 = 288\pi\ cm^3$

Volume of a small lead ball $= \frac{4}{3}\pi \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{\pi}{6} cm^3$

Number of lead ball $= \frac{288\pi}{\frac{\pi}{6}} = 288\pi \times \frac{6}{\pi} = 1728$

Answer is (d)

4. Let R be the radius of each. Height of hemisphere = Its radius = R, Height of each = R

Ratio of volume = Cone: Hemisphere : Cylinder $= \frac{1}{3}\pi R^3 : \frac{2}{3}\pi R^3 : \pi R^2 \times R = 1:2:3$

Answer is (c)

5. Volume of sphere = Volume of 2 cones $= \frac{1}{3}\pi(2.1)^2 \times 4.1 + \frac{1}{3}\pi(2.1)^2 \times 4.3 = \frac{1}{3}\pi(2.1)^2(8.4) cm^3$

Let the radius of the sphere be R.

Thus $\frac{4}{3}\pi R^3 = \frac{1}{3}\pi(2.1)^3 \times 4$

$2R = 4.2$ cm

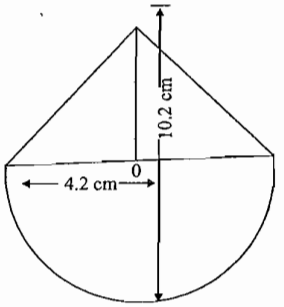
Answer is (b)

6. Volume of the cone $= \frac{1}{3}\pi r^2 h$

here, r = 4.2 cm, h = 10.2, h = 6 cm

Volume of the cone $= \frac{1}{3}\pi(4.2)^2 \times 6$

$= 110.88 \text{ cm}^3$



volume of the hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = 155.23$

Total volume = 110.88 + 155.232 = 266.112 cm^3

Answer is (b)

7. Volume of wall = $1200 \times 500 \times 25 = 15000000 \text{ cm}^3$

Volume of cement = 5% of 15000000 = 750000 cm^3

Remaining volume = 15000000 – 750000 = 14250000 cm^3

Volume of brick $= 25 \times 12.5 \times 7.5 = 2343.75 \text{ cm}^3$

Number of bricks used $= \frac{14250000}{2343.75} = 6080$

Answer is (a)

8. The volume in both cases will be equal, let the height cone is h.

$$4 \times \frac{22}{7} \times (14)^3 \times \frac{1}{3} = \frac{22}{7} \times \left(\frac{35}{2}\right)^2 \times \left(\frac{h}{3}\right)$$

$$4(14)^3 = h\left(\frac{35}{2}\right)^2$$

h = 35.84 cm

Answer is (b)

9. A = 2(xy + yz + zx)

V = xyz

$$\frac{A}{V} = \frac{2(xy + yz + zx)}{xyz}$$

$$= 2\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$$

Answer is (c)

10. Radius of cylinder and hemisphere $= \frac{7}{2} = 3.5\text{cm}$

Height of cylinder = 19- $(3.5 \times 2) = 12$ cm

Total surface area of solid $= 2\pi rh + 4\pi r^2 = 2 \times 3.14 \times 3.5 \times 12 + 4 \times 3.14 \times (3.5)^2 = 418 \text{ cm}^2$

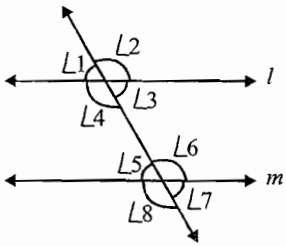
Answer is (a)

CHAPTER

11

Geometry

About parallel lines :



Suppose l and m are two parallel lines and n is a transversal line, then

(i) vertically opposite angle are equal

$\angle 1 = \angle 3, \quad \angle 2 = \angle 4$

$\angle 5 = \angle 7, \quad \angle 6 = \angle 8$

(ii) correspondence angles are equal

$\angle 1 = \angle 5, \quad \angle 2 = \angle 6$

$\angle 3 = \angle 7, \quad \angle 4 = \angle 8$

(iii) alternate interior angle are equal

$\angle 4 = \angle 6, \quad \angle 3 = \angle 5$

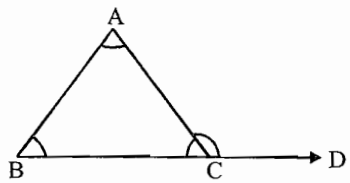
(iv) sum of interior angle of same side is equals to 180°

$\angle 4 + \angle 5 = \angle 3 + \angle 6 = 180^\circ$

TRIANGLE

1. **Exterior angle theorem :** In any triangle, the exterior angle of any angle is equals to sum of remaining two interior angle of that triangle.

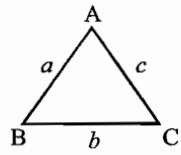
$\angle ACD = \angle ABC + \angle BAC$



2. In any triangle, the sum of two sides is always greater than the third side.

OR

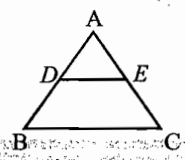
In any triangle, the difference of two sides is always less than the third side.



$AB + BC > AC$ $\quad$ $AB - BC < AC$

$a + b > c$ $\quad$ $a - b < c$

Fundamental theorem : In any triangle, the line joining mid point of two sides is always parallel to third side and half of it.



Let D and E are midpoints of AB and AC, then

$DE \,||\, BC$ and $DE = \frac{1}{2}BC$

• If D and E are midpoint, the line DE divides AB and AC in the same ratio.

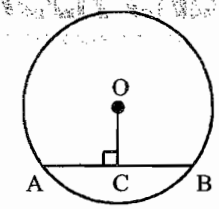
$$\frac{AD}{BD} = \frac{AE}{EC}$$

CIRCLE

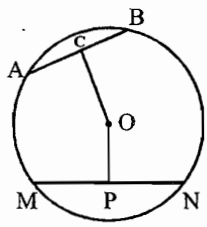
1. The perpendicular line draw from centre of circle on chord divides it in two equal parts.

If $OC \perp AB$

$\Rightarrow$ $\quad AC = BC$

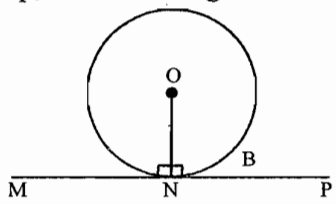


2. Two chords located from same distance of centre of circle have same length.



Now if OP = OC $\quad \Rightarrow AB = MN$

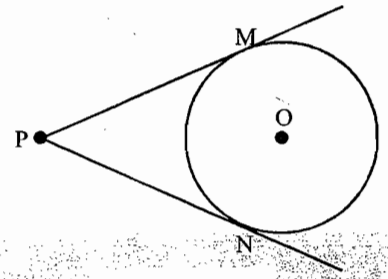
3. The radius of the circle is always perpendicular on tangent line.



$\Rightarrow \quad ON \perp MP$

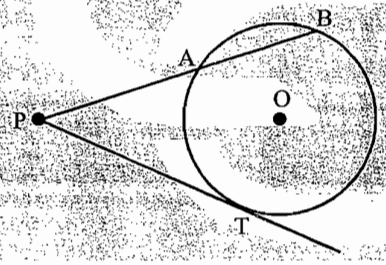
$\Rightarrow \quad \angle ONM = \angle ONP = 90^\circ$

4. The two tangent line draw from same point on any circle have equal length.



here PM = PN

5. If PAB is a secant line and PT is a tangent line of any circle, then
$PA \times PB = PT^2$



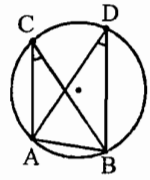
CAREER ENDEAVOUR

6. The two arc of the same length make same angle on the centre of circle

here $\overset{\frown}{AB} = \overset{\frown}{MN}$

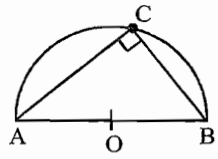
$\Rightarrow \quad \angle AOB = \angle MON$

7. Angles of same segments are equals



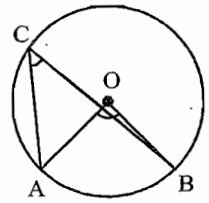
here C and D are two points of same segments. Then
$\angle ACB = \angle ADB$

8. An angle of semi circle is always right angle.



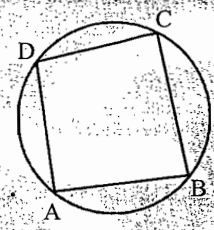
Here, $\angle ACB = 90°$

9. Angle made by any arc at the centre is always twice to the angle made by same arc at any point of remaining segments.



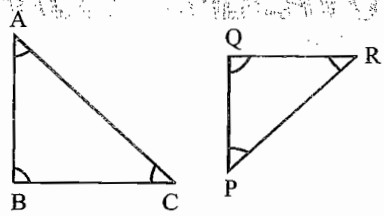
$$\angle ACB = \frac{1}{2} \times \angle AOB$$

Cyclic Quadrilateral : If the vertices of any quadrilateral is located on circumference of any circle, then it is called cyclic quadrilateral. In cyclic quadrilateral sum of opposite angles is equals to 180°.



$$\angle A + \angle C = \angle B + \angle D = 180°$$

Similarity of triangles : Two triangle are said to be similar if there all angles are equal



Here, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$

Hence, $\Delta ABC \sim \Delta PQR$

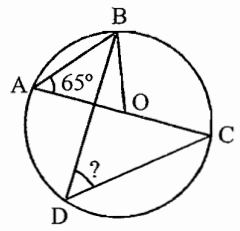
Result : In two similar triangles, their respective sides are proportionally.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

and $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

SOLVED EXAMPLES

1. In given circle, O is the centre of circle and $\angle OAB = 65°$ then find $\angle BDC$



(a) 50° (b) 65° (c) 30° (d) none of these

Soln. First method:

We know that angle at two point on same segment are equal. Here point A and D are in same segment for arc $\overset{\frown}{BC}$. Hence,

$$\angle BAC = \angle BDC = 65°$$

Second Method: In ΔOAB

$$OA = OB = r \text{ (radius of circle)}$$

$\Rightarrow \angle OAB = \angle ABO = 65°$

Now, $\angle ABO + \angle BAO + \angle AOB = 180°$

then $\angle AOB = 180 - 65 - 65 = 50°$

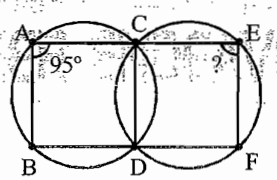
again $\angle AOB + \angle BOC = 180°$

$\Rightarrow \quad \angle BOC = 180° - 50 = 130°$

also, angle made by any arc at the centre is twice to the angle made by same arc at any point of the remaining segment. So,

$$\angle BDC = \frac{1}{2} \times \angle BOC = \frac{1}{2} \times 130 = 65°$$

2. In given figure two circles are intersecting at point C and D. If $\angle A = 95°$ then find $\angle E = ?$



(a) 95° (b) 85° (c) 110° (d) none of these

Soln. Since ABCD is a cyclic quadrilateral

$\Rightarrow \quad \angle A + \angle D = 180°$

$\angle BDC = 180° - 95° = 85°$

Now, $\angle BDC + \angle CDF = 180°$

$\Rightarrow \quad \angle CDF = 180° - 85° = 95°$

Also, CDEF is a cyclic quadrilateral

$\Rightarrow \quad \angle CDF + \angle CEF = 180°$

$\Rightarrow \quad \angle CEF = 180 - 95 = 85°$

TWO DIMENSIONAL GEOMETRY

1. Distance between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

distance of (x_1, y_1) from origin (0, 0) is $d = \sqrt{x_1^2 + y_1^2}$

2. Point (x, y) which divides the join of two given points (x_1, y_1) and (x_2, y_2) in a given ratio $m_1 : m_2$ (Internally and extranally)

Internally $$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$$

$$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

Externally $$x = \frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}$$

$$y = \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2}$$

Co-ordinates of any point on the join of (x_1, y_1) and (x_2, y_2) can be taken as

$$\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right)$$

Mid point: This point divides the given line in the ratio 1 : 1.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

3. Area of a triangle whose vertices are

$(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3) = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Note that if one vertex (x_3, y_3) is at the origin (0, 0) then

Area $= \frac{1}{2}(x_1 y_2 - x_2 y_1)$

Particular case :

(a) If the two vertices on x-axis say (a, 0) (b, 0) and third vertex is (h, k) then area

$$\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times (a - b)k$$

(b) Similarly if the two vertices be on y-axis say (0, c) (0, d) and third vertex is (h, k), then area $= \frac{1}{2}(c - d).h$ as on above.

(c) ΔOAB where O is (0, 0), A is (a, 0) on x-axis and B is (0, b) on y-axis then area $= \frac{1}{2}ab$

(d) If the area of the triangle be zero, then the three points will be collinear.

4. Co-ordinates of standard point

(a) **Centroid of a triangle:** The point of the intersection ofthe medians. This point divides each median in the ratio 2 : 1. Its co-ordinates is

$$G=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

(b) **Circum centre of triangle:** This is a point which is equidistant from the three vertices of triangle. It is the also the point of intersection of perpendicular bisectors of the sides of the triangle. It is the center O that passes through the vertices of the triangle. If the triangle be a right angled one, the circumcentre be the mid point of the hypotenuse.

(c) **Incentre of a triangle:** This is the centre of the circle which touches the sides of a given triangle. It is the point of intersection of the internal bisectors of the angles of the triangle. Its co-ordinates are given by the formula.

$I=(x,y)$

$$x=\frac{ax_1+bx_2+cx_3}{a+b+c},\qquad y=\frac{ay_1+by_2+cy_3}{a+b+c}$$

where a, b, c are the lengths of the sides of triangle ΔABC

5. Orthocentre of a triangle: This point H is the intersection of the altitudes. In a right angle ΔABC, $\angle C=90°$, the orthocentre H is the vertex C of the triangle.

6. The points O, G and H are collinear and G divides OH in the ratio 1 : 2.

7. In an equilateral triangle the points G, H and I are concide.

8. The internal bisector of the angle of a triangle divides the opposite sides in the ratio of the arms of the angle.

9. If the triangle be isosceles, then perpendicular from vertex to base bisects it.

Straight line (first degree) equations:

1. $Ax+By+C=0$ (General form)
2. $x=0$, $y-$axis
3. $y=0$, $x-$axis
4. $x=a$, parallel to y-axis.
5. $y=b$, parallel to x-axis
6. The distance of a point (x,y) from x-axis is $|y|$ and from y-axis is $|x|$.
7. $y=mx+c$, line which cuts off an intercept C on y-axis and makes an angle θ with the positive direction (anticlockwise) of x-axis and $\tan\theta=m$ is called its slope or gradient
8. $y=mx$, any line through the origin.
9. $y-y_1=m(x-x_1)$, equation of a line through a given point (x_1,y_1) and having slope i.e., $\tan\theta=m$

$\frac{y-y_1}{x-x_1}=m$. If $m=0$, i.e. the line is parallel to x-axis, then its equation will be $y-y_1=0$.

If $m=\infty$, i.e. the line is perpendicular to y-axis then its equation will be

$x-x_1=0$

10. Equation of a line passing through two given points (x_1,y_1) and (x_2,y_2) is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$$

Its slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

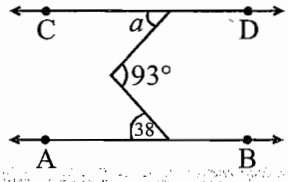
11. Angle θ between two given lines: It is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{A_1B_2 - A_2B_1}{A_1A_2 + B_1B_2}\right|$$

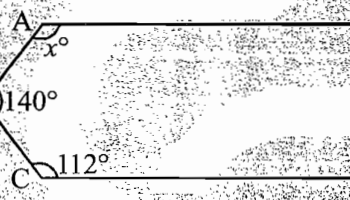
where, $A_1x + B_1y + c_1 = 0$ and $A_2x + B_2y + c_2 = 0$

EXERCISE

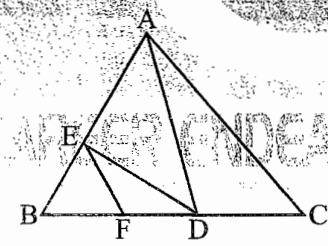
1. In figure AB ||CD. Determine $\angle a$.



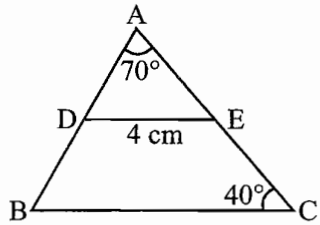
2. In figure AB ||CD. Find the value of x.



3. In figure, EF ||AD and ED ||AC. If BF = 4 cm, FD = 6 cm and BE = 8 cm, Find BC.



4. In figure, D and E are the mid points of sides AB and AC respectively of ΔABC. If $\angle A = 70°$ and $\angle C = 40°$ and DE = 4 cm. Find $\angle EDB$ and BC.



5.

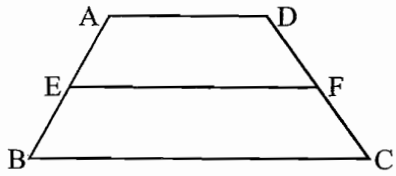
6.

7.

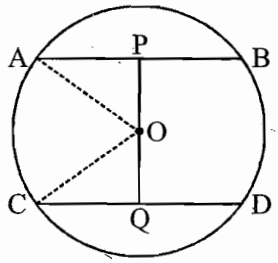
8.

9.

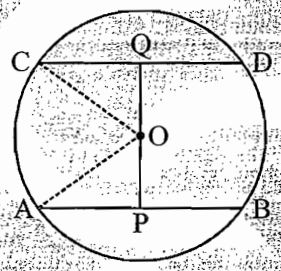
5. In figure AD ||EF ||BC. If EB = 2AE and DF = 1.5 cm. Find the length of CD.



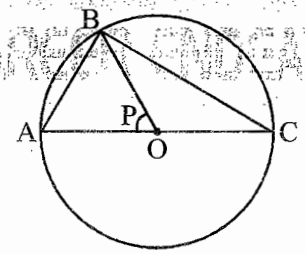
6. In given figure O is the centre of the circle with radius 5 cm. OP ⊥ AB and OQ ⊥ CD, AB = 6 cm and CD = 8 cm. Determine PQ.



7. AB and CD are two parallel chords of a circle such that AB = 10 cm and CD = 24 cm. If the chords on the opposite sides of the centre and the distance between them is 17 cm. Then the radius of the circle is what.

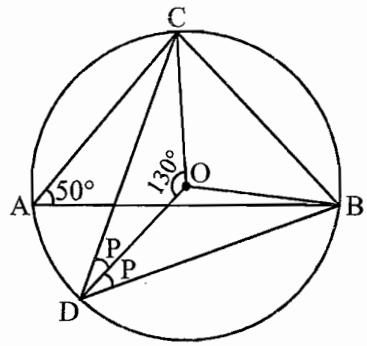


8. In figure, length of arc AB = $\frac{1}{2}$ × length of arc BC. Find ∠AOB.

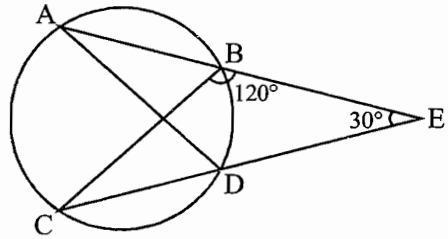


40°

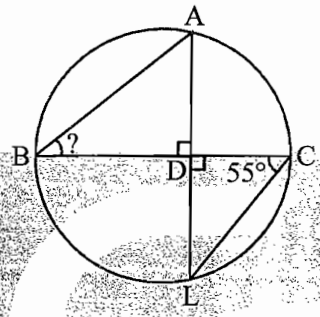
9. In figure, O is the centre of the circle. Find ∠ODB and ∠ODC.



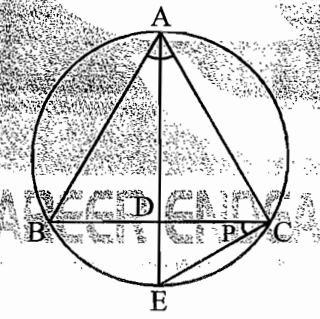
10. Two chords AB and CD of a circle intersect at a point E out side the circle as in given figure. If $\angle E = 30°$ and $\angle CBE = 120°$. Find $\angle EAD$.



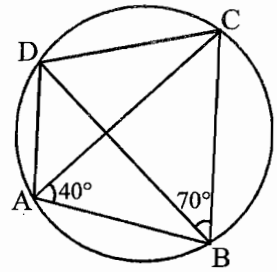
11. ΔABC is inscribed in a circle and $AD \perp BC$ meets BC in D and AD produced meets the circle at L in given figure. If $\angle BCL = 55°$ the find $\angle ABC$.



12. In figure ΔABC is inscribed in a circle and the bisector of $\angle A$ meets BC in D and the circle in E. If $\angle BAC = 50°$ find $\angle BCE$.



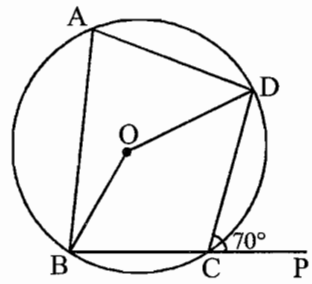
13. In given figure if $\angle DBC = 70°$ and $\angle BAC = 40°$. Find $\angle BCD$.



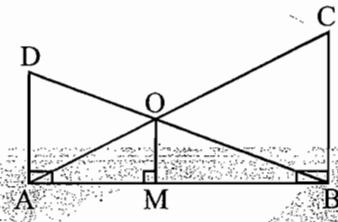
14.

15.

14. In figure, O is the centre of the circle and BC is produced to P. If $\angle DCP = 70°$. Then what is the angle made by arc BD on the centre.



15. In given figure $DA \perp AB$, $CB \perp AB$ and $OM \perp AB$. If $OA = 5.4$ cm, $OC = 7.2$ cm and $OB = 6$ cm. Find OD.



ANSWER KEY

1. (55°)	**2. (108°)**	**3. (25cm)**	**4. (BC = 8cm, $\angle EDB = 110°$)**	**5. (4.5 cm)**
6. (7 cm)	**7. (13 cm)**	**8. (60°)**	**9. (25°)**	**10. (30°)** **11. (35°)**
12. (25°)	**13. (70°)**	**14. (140°)**	**15. (4.5cm)**	

CAREER ENDEAVOUR

CHAPTER

12

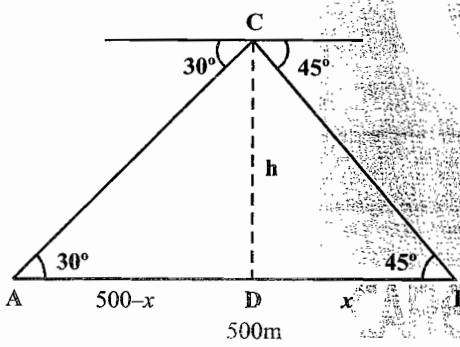
Height and Distance

SOLVED EXAMPLES

1. If two stones are 500 meters apart. The angle of depressions being 30° and 45° as seen by aeroplane. What is the altitude the plane is flying

(a) $250\left(\sqrt{3}+1\right)$ (b) $250\sqrt{3}$ (c) $250\left(\sqrt{3}-1\right)$ (d) none of these

Soln.



In ΔADC, $\tan 30° = \dfrac{h}{500-x}$

$$\frac{1}{\sqrt{3}} = \frac{h}{500-x}$$

$$\sqrt{3}\,h = 500 - x \quad \text{... (i)}$$

in ΔBDC

$$\tan 45° = \frac{h}{x}$$

$$h = x \quad \text{... (ii)}$$

From (i) & (ii)

$$\sqrt{3}h = 500 - h$$

$$\left(\sqrt{3}+1\right)h = 500$$

$$h = \frac{500\left(\sqrt{3}-1\right)}{\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)} = \frac{500\left(\sqrt{3}-1\right)}{2} = 250\left(\sqrt{3}-1\right)$$

2. The angle of elevation of a cloud from a point x meter above a lake is 'A' and the angle of depression of its reflection in the lake is 45°. The height of the cloud is

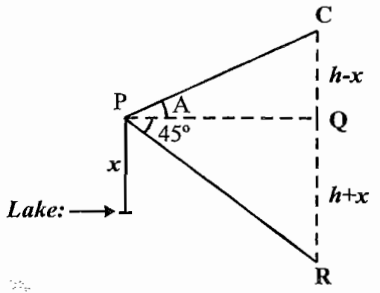
(a) $x\tan A$ (b) $x\tan(45°)$ (c) $x\tan(A+45°)$ (d) $x\cot(A+45°)$

Soln. Let height of cloud is h.

In ΔPQR

$$\tan 45° = \frac{h+x}{PQ} \Rightarrow PQ = h+x$$

In ΔPQC,



$$\tan A = \frac{h-x}{h+x} \qquad \Rightarrow \frac{1}{\tan A} = \frac{h+x}{h-x}$$

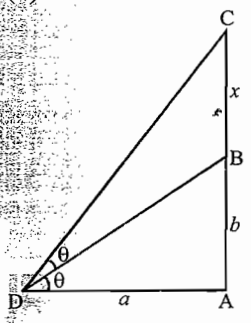
$$\Rightarrow \frac{h}{x} = \frac{1+\tan A}{1-\tan A} \qquad \Rightarrow \frac{\tan 45° + \tan A}{1-\tan 45°\tan A}$$

$$\Rightarrow \frac{h}{x} = \tan(A+45°) \qquad \Rightarrow h = x\tan(A+45°)$$

3. At a distance of a from foot of a tower AB of known height b, a flagstaff BC and the tower subtend equal angles. The height of the flagstaff is

(a) $a\dfrac{(b^2+a^2)}{(a^2-b^2)}$ (b) $\dfrac{b(a^2+b^2)}{a^2-b^2}$ (c) $\dfrac{a^2(a^2+b^2)}{a^2-b^2}$ (d) none of the above

Soln. In ΔABC, $\tan\theta = \dfrac{b}{a}$, In ΔDAC, $\tan 2\theta = \dfrac{x+b}{a}$



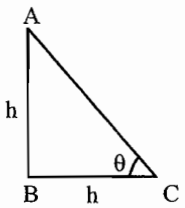
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2.\frac{b}{a}}{1-\left(\frac{b}{a}\right)^2} \Rightarrow \frac{b+x}{a} = \frac{2ab}{a^2-b^2}$$

$$\Rightarrow b+x = \frac{2a^2b}{a^2-b^2} \qquad \Rightarrow x = \frac{2a^2b-a^2b+b^3}{a^2-b^2} = \frac{a^2b+b^3}{a^2-b^2} = \frac{b(a^2+b^2)}{a^2-b^2}$$

4. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is

(a) 30° (b) 45° (c) 60° (d) 75°

Soln.



$$\tan\theta = \frac{h}{h} = 1$$

$$\tan\theta = \tan 45°$$

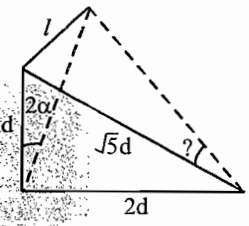
$$\theta = 45°$$

5. A cat knows the three exits A, B and C of a mouse hole. Where should the cat sit so that the distance to the farthest exist is the minimum.
(a) At the centre of the circle passing thorugh A, B and C.
(b) At the point D, where the perpendicular from C intersects line AB
(c) At the pont D inside the $\Delta DABC$, such that the area of $\Delta DABD$ = area of $\Delta DBDC$ = area of $\Delta DCDA$
(d) none of these

6. A person standing of the bank of a river observes that the angle α subtended by the tree on the opposite bank is twice the angle subtended by it when moves away a distance twice as much as the width of the river. Angle α is:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{10}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

Soln. Let l be the length of the tree and d be the width of the river. So, $\tan\alpha = \frac{\ell}{\sqrt{5}d}$ and $\tan 2\alpha = \frac{\ell}{d}$.



$$\frac{\tan 2\alpha}{\tan\alpha} = \sqrt{5}$$

$$\frac{2\tan\alpha}{1-\tan^2\alpha} = \sqrt{5}\tan\alpha$$

$$\sqrt{5}\left(1-\tan^2\alpha\right) = 2$$

$$\sqrt{5}\tan^2\alpha = \sqrt{5}-2$$

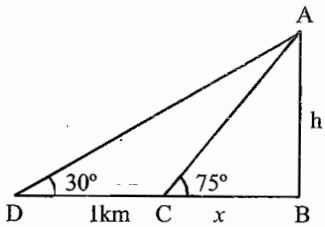
$$\Rightarrow \quad \tan^2\alpha = \frac{\sqrt{5}-2}{\sqrt{5}}$$

$$\Rightarrow \quad \tan\alpha = \sqrt{\frac{\sqrt{5}-2}{\sqrt{5}}} \qquad \Rightarrow \alpha = \frac{\pi}{10}$$

7. A person walking along a straight road observes that at two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The height of the pole is

(a) $250\left(\sqrt{3}+1\right)m$ (b) $250\left(\sqrt{3}-1\right)m$ (c) $225\left(\sqrt{2}-1\right)m$ (d) $225\left(\sqrt{2}+1\right)m$

Soln.



$$\tan 75° = \frac{h}{x}$$

$$\tan(45+30)° = \frac{\tan 45+\tan 30}{1-\tan 45.\tan 30} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{h}{x} \qquad \Rightarrow x = \frac{\sqrt{3}-1}{\sqrt{3}+1}h$$

and $\tan 30° = \frac{h}{x+1}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+1} \qquad \Rightarrow x+1 = \sqrt{3}h$$

$$\Rightarrow \qquad \frac{\sqrt{3}-1}{\sqrt{3}+1}h + 1 = \sqrt{3}h$$

$$\Rightarrow \qquad h\left(\sqrt{3} - \frac{\sqrt{3}-1}{\sqrt{3}+1}\right) = 1$$

$$\Rightarrow \qquad h\left(\frac{4}{\sqrt{3}+1}\right) = 1$$

$$\Rightarrow \qquad h = \frac{\left(\sqrt{3}+1\right)\times 1}{4} km$$

$$h = 250\left(\sqrt{3}+1\right) m$$

CHAPTER

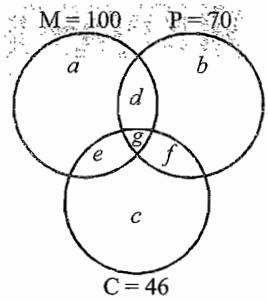
13

Venn Diagram

SOLVED EXAMPLES

1. A class has 175 students. The following table shows the number of students studing one or more of the following subjects in this case.
Maths — 100
Physics — 70
Chemistry — 46
Math and Physics — 30
Math and Chemistry — 28
Physics and Chemistry — 23
Math, Physics and Chemistry — 18
How many students are enrolled in maths alone, physics alone and chemistry alone and how many students are there who have not been offered anyone of these subject.

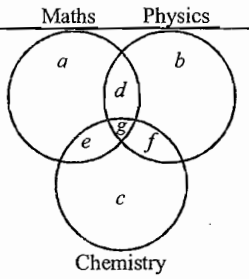
Soln. $a + d + e + g = 100$
$b + d + f + g = 70$
$c + e + f + g = 46$
$d + g = 30$
$e + g = 28$
$f + g = 23$
$g = 18$
after solving we get,
$f = 5$, $e = 10$, $d = 12$, $c = 13$, $b = 35$, $a = 60$
Maths only = 60
Physics only = 35
Chemistry only = 13
Now $a + b + c + d + e + f + g = 153$
Number of students who have not been offered any one of these subjects = 175 – 153 = 22



2. Out of 120 students, 80 students have been taken mathmatics, 60 students have taken physics, 40 students have taken chemistry, 30 students have taken both physics and maths, 20 students have taken chemistry and maths and 15 students have taken both physics and chemistry. If every student has taken at least one course, then how many students have taken all three courses.
(a) 5 (b) 25 (c) 15 (d) 10

Soln. $a + d + e + g = 80$
$b + d + f + g = 60$

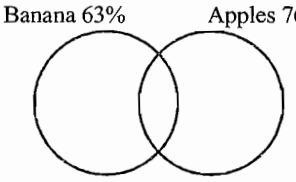
$c + e + f + g = 40$
$d + g = 30$
$e + g = 20$
$f + g = 15$
$g = ?$



$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
$120 = 80 + 60 + 40 - (30 + 20 + 15) + x$
$120 = 180 - 65 + x$
$x = 120 - 115 = 5$

3. A survey shows that 63% of Indians like banana where as 76% like apples. If x% of Indians like both banana and apples, then
(a) $x = 39$ (b) $x = 63$ (c) $39 \le x \le 63$ (d) None of these

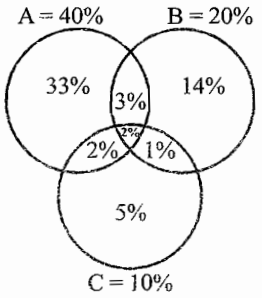
Soln.



$n(A) + n(B) - n(A \cup B) \le n(A \cap B) \le \min(n(A), n(B))$
$63 + 76 - 100 \le n(A \cap B) \le \min(63, 76)$
$39 \le x \le 63$

4. In a town of 10,000 families, it was found 40% buy newspaper A, 20% buy newspaper B and 10% buy newspaper C. 5% of the families buy A and B, 3% buy B and C, 4% buy A and C. If 2% buy all the three newspaper, then the number of families which buy none of the newspaper A, B and C is
(a) 1400 (b) 6000 (c) 3300 (d) 4000

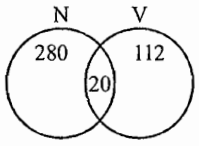
Soln.



By the above Venn diagram, we have 40% people not reading either of newspaper A, B or C.
Hence $10000 \times 40\% = 4000$

5. In a dinner attended by 432 people, 300 choose non-vegetarian and 132 choose vegetarian dishes. If 20 choose both, then how many choose exactly one kind of food.
(a) 412 (b) 392 (c) 402 (d) 372

Soln.



$280 + 112 = 392$

5. The enrollments of the 2^{nd} year chemistry students of a college in three elective papers namely AA, AOS and ACN are as follows 30 students have taken both AA and AOS, 20 students have taken both AOS and ACN, 30 students have taken both ACN and AA, 50 students have taken AA, 60 students have taken ACN and 70 students have taken AOS, 5 students have taken all the 3 subjects. If each students in the class has taken at least one of AOS, AA and ACN, then the total no. of students in the class is

(a) 75 (b) 95 (c) 100 (d) 105

Soln. We know that

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$= 50 + 60 + 70 - 30 - 20 - 30 + 5$$
$$= 105$$

6. Each student in a class takes at least one elective out of the 3 available electives. Each one of the electives is taken by 100 students. The number of students who have taken any 2 electives is either 50 or 51, and the number of students who have taken all 3 electives is 34. The total number of students in the class is bounded by

(a) 150 and 153 (b) 180 and 183 (c) 181 and 184 (d) 179 and 182

Soln. If x is the number of students then

If 50 students took 2 electives, then

$34 \times 3 + 50 \times 2 + (x - 84)\,1 = 300$

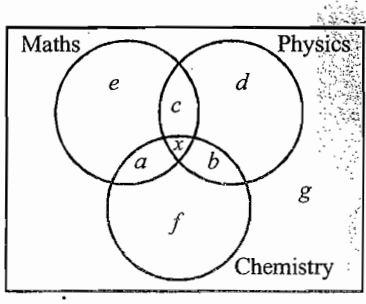
$102 + 100 + x - 84 = 300$

$\Rightarrow \quad x = 182$

7. From 50 students taking examination in Maths, Physics and Chemistry, 37 passed Maths , 24 in Physics and 43 in Chemistry. At most 19 passed Maths and Physics, at most 29 in Maths and Chemistry and at most 20 in physics and Chemistry. The largest possible number that could have passed all three examination is

(a) 10 (b) 12 (c) 9 (d) None of these

Soln.



Let x be the number of possible students who have passed all the papers.

We have,

$a + b + c + d + e + f + g + x = 50$ — (i)

$b + d + f = 13$ — (ii)

$a + e + f = 26$ — (iii)

$c + d + e = 7$ — (iv)

$c + x \le 19$

$a + x \le 29$

$b + x \le 20$

Equation (ii) and (iii) and (iv)

$a + b + c + d + e + f + (d + e + f) = 46$

$\Rightarrow d + e + f + 50 - g - x = 46$

$\Rightarrow g + x = 4 + d + e + f$

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$50 = 37 + 24 + 43 - 19 - 29 - 20 + x$

$x = 14$

CHAPTER

14

Permutations & Combinations

(i) Permutation is all related to the act of permuting (rearranging) objects or values.
(ii) Combination is a way of selecting several things out of a larger group, where (unlike permutations) order does not matter.

(i) Factorial Notation "!" or "$\lfloor$".

$\lfloor n = n(n-1)(n-2).....3.2.1$

Or, $n! = n(n-1)(n-2).....3.2.1$

= product of n consecutive integers starting from 1 to the number n.

(ii) $0! = 1$

(iii) Factorials; of only natural numbers are defined.

$n!$ is defined only for $n \geq 0$; $n!$ is not defined for $n < 0$.

(iv) ${}^nC_r = 1$ when $n = r$.

(v) Combinations (represented by ${}^nC_r$) can be defined as the number of ways in which r things at a time can be "selected" from amongst "n" things available for selection.
The keyword here is "selected". Please understand here that the order in which the r things are selected has no importance in the counting of combinations.

${}^nC_r$ = Number of combinations (selections) of n things taken "r" at a time.

$${}^nC_r = \frac{n!}{r!(n-r)!}$$; where $n \geq r$ (n is greater than or equal to r).

Some typical situations where selection/combination is used:

(i) Selection of people for a team, a party, a job, an office and so on (e.g. selection of a cricket team of 11 from 16 members).
(ii) Selection of a set of objects (like letters, hats, shirts) from amongst another larger set available for selection.
In other words, any selection in which the order of selection holds no importance is counted by using combinations.

(iii) Permutations (represented by ${}^nP_r$) can be defined as the number of ways in which r things at a time can be "selected" and "arranged" at a time from amongst n things.

The keyword here is "arrangement". Hence, please understand here that the order in which the "r" things are arranged has critical importance in the counting of permutations.
In other words, permutations can also be referred to as an "ordered selection".

${}^{n}P_{r}$ = number of permutations (arrangements) of n things taken r at a time.

$${}^{n}P_{r} = \frac{n!}{(n-r)!}; n \geq r!$$

Some typical situations where ordered selection/permutations are used:
(a) Making words and numbers from a set of available letters and digits respectively.
(b) Filling posts with people.
(c) Selection of batting order of a cricket team of 11 from 16 members.
(d) Putting distinct objects/people in distinct places. For example, making people sit, putting letters in envelops, finishing order in horse race etc.

(v) The relationship between Permutation and Combination: When we look at the formulae for permutations and combinations and compare the two, we see that ${}^{n}P_{r} = r! \times {}^{n}C_{r} = {}^{n}C_{r} \times {}^{r}P_{r}$

(vi) Important Results: The following results are important as they help in problem-solving.

(i) Number of permutations (or arrangements) of n different things taken all at a time = $n!$

(ii) Number of permutations of n things out of which P_1 are alike and are of one type, P_2 are alike and are of a second type and P_3 are alike and are of a third type and the rest are all different $= \frac{n!}{P_1!P_2!P_3!}$

SOLVED EXAMPLES

1. The number of words formed with the letters of the word Allahabad.
Total number of letters = 9 of which A occurs four times, L occurs twice and the rest are all different.
Total number of words formed = 9!/(4!2!1!).

(iii) Number of permutations of "n" different things taken "r" at a time when repetition is allowed $n \times n \times n \times \ldots\ldots$ (r times) = n^r.

2. In how many ways can 4 rings be worn in the index, ring finger and middle finger if there is no restriction of the number of rings to be worn on any finger?

Soln. Each of the 4 rings could be worn in 3 ways either on the index, ring or middle finger.
So, 4-rings could be worn in $3 \times 3 \times 3 \times 3 = 3^4$ ways.

(iv) Number of selections of r things out of n identical things = 1.

3. In how many ways 5 marbles can be chosen out of 100 identical marbles?

Soln. Since, all the 100 marbles are identical number of ways to select 5 marbles = 1.

(v) Total number of selections of zero or more things out of "k" identical things = k + 1.
This includes the case when zero articles are selected.

(vi) Total number of selections of zero or more things out of n different things =

${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \ldots\ldots + {}^{n}C_{n} = 2^{n}$.

(vii) The number of selections of 1 or more things out of n different thing ${}^{n}C_{1} + {}^{n}C_{2} + \ldots\ldots + {}^{n}C_{n} = 2^{n} - 1$.

(viii) Number of ways of dividing m + n different things in two groups containing m and n things respectively

$$= {}^{m+n}C_{n} \times {}^{m}C_{m} = \frac{(m+n)!}{m!n!} \quad \text{Or} \quad {}^{m+n}C_{m} \times {}^{n}C_{n} = \frac{(m+n)!}{n!m!}$$

(ix) Number of ways of dividing "2n" different things in two groups containing n things $= \frac{2n!}{n!n!2!}$

(x) ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$

(xi) ${}^{n}C_{x} = {}^{n}C_{y} \Rightarrow x = y$ or $x + y = n$

(xii) ${}^{n}C_{r} = {}^{n}C_{n-r}$

(xiii) Number of selections of r things out of n different things.

(a) When k particular things are always included $= {}^{n-k}C_{r-k}$.

(b) When k particular things are excluded $= {}^{n-k}C_{r}$.

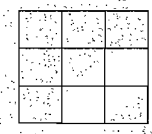
(c) When all the k particular things are not together in any selection $= {}^{n}C_{r} - {}^{n-k}C_{r-k}$.

(viii) Important Results:

(i) Number of ways of arranging n people on a circular track (circular arrangement =(n–1)!

(ii) When clockwise and anti-clockwise observation are not different then number of circular arrangements of n different things $= (n-1)!/2$.

4. Thus the number of squares and rectangles in the following figure are given by



Number of squares $= 1^2 + 2^2 + 3^2 = 14$.

Number of rectangles $= 1^3 + 2^3 + 3^3 = 36$ for rectangle.

5. In how many ways 3 prizes can be given away to 7 boys when each boy is eligible for ay of the prizes.

Soln. $\left.\begin{matrix}1^{st}\text{ prize}\\ 2^{nd}\text{ prize}\end{matrix}\right] \rightarrow$ same boy; $\left.\begin{matrix}1^{st}\text{ boy}\\ 2^{nd}\text{ boy}\end{matrix}\right] \rightarrow$ same prize

Two prizes can be given to the same boy but two boys can not get the same prize, therefore, we must start with prize.

Each of the three prizes can be given away to anyone of the 7 boys in 7 ways.

Therefore, required number $= 7 \times 7 \times 7 = 343$

6. The letters of the word figment are to be arranged in the manner such that vowels at first and last positions. How many words can be formed?

Soln. For vowels at first and last positions, first place can be taken by I and last by E, or vice versa.

Remaining 5 positions can be filled by 5 letters is 5! ways.

So, total words formed $= 5! \times 2! = 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 1 = 240$

7. A candidate is required to answer six out of ten questions which are divided into groups, each containing five questions, and he is not permitted to attempt more than 4 from any group. In how many ways can be make up his choice?

(a) 60 (b) 300 (c) 150 (d) 200

8. 4 married couples and 1 single women want to play tennis while vacationing in a resort. How many different mixed doubles tennis match can be organized if no man is willing to play if his wife is participating in the same match? (In the mixed double tennis match, each of the two teams consists of one male and one female player).

(a) 36 (b) 18 (c) 54 (d) 12

9. A license plate has 3 letters and 3 digits in that order. A witness to a hit and run accident saw the first 2 letters and the last digit. If the letters and digits can be repeated, how many license plates must be checked by the police to find the culprit?
(a) 480 (b) 960 (c) 260 (d) none of these

10. A prospective buyer can choose between a fixed and a variable interest rate and can also choose a payment period of 24 months, 36 months or 48 months. How many total outcomes are possible?

Soln. This experiment is made up of two steps: choosing an interest rate and selecting a loan payment period. There are two outcomes (a fixed or a variable interest rate) for the first step and three outcomes (a payment period of 24 months, 36 months or 48 months) for the second step.
Hence, total outcomes $= 2 \times 3 = 6$

EXERCISE

1. Find the number of ways of arrangement of 10 person in four chairs.
(a) 5020 (b) 5040 (c) 5000 (d) 5060

2. How many word can be formed by using letter of ward "MADAM"
(a) 10 (b) 20 (c) 30 (d) 40

3. How many words can be formed form letter of ENGINEER
(a) 3300 (b) 3360 (c) 3600 (d) 3000

4. How many numbers between 2000 and 3000 can be formed with the digits 0,1,2,3,4,5,6,7 (repitition of digit not allowed)
(a) 42 (b) 440 (c) 336 (d) 210

5. If there are 11 players to be selected from a team of 16 has many ways this can be done.
(a) $^{11}C_{10}$ (b) $^{16}C_{11}$ (c) $^{16}P_{11}$ (d) $^{11}P_{10}$

6. Of the different words that can be formed from the letter "BEGINS", how many begin with "B" end with "S".
(a) $4! \times 3!$ (b) $4! \times 4!$ (c) $4! \times 6!$ (d) $4!$

7. How many letting orders are possible for the Indian cricket team if there is a square of 15 to choose from such that captain Mr. Dhoni is always chosen ?
(a) $1001 \times 11!$ (b) $364 \times 11!$ (c) $11!$ (d) $15.11!$

Direction: Using the following information: In the English alphabet there are 11 symmetric letters that appear the same when looked at in a mirror. Other 15 letters in the alphabet are asymmetric letters.

8. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?
(a) 7920 (b) 330 (c) 14640 (d) 419430

9. In how many different ways can the letters of the work 'LEADING' be arranged in such a way that the vowels are always placed next to each other?
(a) 360 (b) 480 (c) 720 (d) 5040

10. How many 5 digit's number can be formed with the help of digits 1, 2, 3, 4, 5 ?

11. How many 3 digits different even number can be formed by using the above digits ?

12. How many five digit number can be formed with the help of digit 0, 1, 2, 3, 4, 5 which is also divisible by 3 ?

13. How many different messages can be generated with the help of 5 zeroes and 4 ones ?

14. How many four digit numbers are there between 1000 and 2000 such that every digit is either 0, or 1, or 2 ?

15. How many different words can be formed by using the letter of word 'GLORY'.

16. How many different words can be formed by using the letter of word 'Preparation'.

CAREER ENDEAVOUR

17. How many different words can be formed by using letter of word 'Banana' in which all vowels come together?
18. How many different words can be form by using the letter of word 'NATURE' in which all vowels come in odd places ?
19. How many words can be formed by using the letter of word 'FIGURE' in which all vowels and all consonents come alternate ?
20. How many different ways 3 pens can be put in 4 pockets ?
21. There are 4 questions in any examination and every question has 5 different options. In how many different ways can a boy not give the correct answer for all questions
22. From 7 English and 4 americans a team of 6 to be formed. In how many ways it can be done if -
    (i) Team contains exactly 5 English. (ii) Team contains at least 2 Americans.
    (iii) Team contains No English.
23. In how many different ways can a playing 11 be chosen out of 15 players if -
    (i) a particular player is always chosen. (ii) a particular player is never chosen.
24. In how many ways 12 different things can be distributed equally among 3 persons ?
25. Ram has 5 friends. In how many ways he can invite one or more of them to a dinner party ?
26. How many different triangles can be formed by joining the vertices of an octagon.
27. How many different diagonals can be formed in any heptagon.
28. There are 10 points in a plane out of which 7 are collinear. How many different triangles can be formed.
29. In a party every person shakes hands with every other persons. It there are a total of 105 hand shakes in the party. Find the number of persons present in the party.
30. There are 10 lamps in a hall. Each one of them can be switched on independently. Find the number of ways in which the hall can be illuminated ?
31. In how many ways 100 runs can be scored with the help of fours and six's.
32. How many number of signals can be generated by using 6 differently colour flags ?
33. It is a small town railway station and there are 25 stations on at line. At each of the 25 station the passenger different types of ticket you think the booking clerk has to keep ?
34. A single tournament of tennis is held in which 30 men participate. If a player is eliminated as soon as he loses a match. How many matches are required to determine the winner ?
35. There are 10 cards numbered 1, 2, 3, ..., 10 in a bag. 4 cards are drawn together. Find how many ways we get the sum of written number in card even ?

HINTS & SOLUTIONS

1. ${}^{10}P_4 = \frac{10!}{6!} = 5040$
   Answer is (b)

2. In word "MADAM" we have 2 – M's, 2 – A's. So total number of ways of forming words are $= \frac{5!}{2!2!} = 30$
   Answer is (c)

3. Here "ENGINEER" contains 3 R's and 2 N's, one each G,I and R. So total number of words $= \frac{8!}{3!2!} = 3360$
   Answer is (b)

4. Number between 2000 and 3000 are four digits number in which "2" must be its first digit

2			
d_1	d_2	d_3	d_4

Next three boxes (d_2, d_3, d_4) can be filled in ${}^7P_3$ ways $\Rightarrow \frac{7!}{3!4!} = 210$ ways

Answer is (d)

5. This is problem of selection, thus combination formula is applicable $= {}^{16}C_{11}$

Answer is (b)

6. B and S are fixed at the start and the end position. Hence, we have to arrange E, G, I and N among them solves.

B					S
	①	②	③	④	

Four places four letter can arrange is 4 ! ways
Answer is (d)

7. ${}^{14}C_{10} \times 11! = 1001 \times 11!$
Answer is (a)

8.

11	10	9	8

Hence, number of passwards $= 72 \times 110 = 7920$
Answer is (a)

9. LEADING
Vowels = EAI = α
then L, D, N, G, α can make words.
$\lfloor 5 \times \lfloor 3 = 120 \times 6 = 720$
Answer is (c)

10.

5	4	3	2	1

$5 \times 4 \times 3 \times 2 \times 1 = 120$ or ${}^5P_5 = \lfloor 5 = 120$

11.

4	3	2

; $4 \times 3 \times 2 = 24$

12. Sum of the digits should be divisible by 3. Then group of 5 digit (1, 2, 3, 4, 5) and (1, 2, 0,4, 5)

By (1, 2, 3, 4, 5) the total number of 5 digit =

5	4	3	2	1

$= 120$

By (1, 2, 0, 4, 5) the total number of 5 digit =

4	4	3	2	1

$= 96$

Hence, total number = 120 + 96 = 216

13. $\frac{\lfloor 9}{\lfloor 5 \times \lfloor 4} = \frac{9 \times 8 \times 7 \times 6 \times \lfloor 5}{\lfloor 5 \times 4 \times 3 \times 2 \times 1} = 126$

14. The thousands place can be filled by only 1.

1	3	3	3

$\Rightarrow 1 \times 3 \times 3 \times 3 = 27$

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But number is between 1000 to 2000. Hence in these 27 number > 1000 is also included. So total number is 27–1 = 26

15. GLORY contains 5 letters hence number of words = $\lfloor\underline{5} = 120$

16. PREPARATION has 11 letters in which P comes 2 times, R comes 2 times, A comes 2 times.

Hence, number of words = $\frac{\lfloor\underline{11}}{\lfloor\underline{2}\times\lfloor\underline{2}\times\lfloor\underline{2}} = \frac{11\times10\times9\times8\times7\times\lfloor\underline{6}}{2\times2\times2} = 110\times63\times720$

17. In BANANA, here are 3 vowels
B, N, N, AAA = α

Hence, now total letter is 4. Then number of words = $\frac{\lfloor\underline{4}}{\lfloor\underline{2}}\times\frac{\lfloor\underline{3}}{\lfloor\underline{3}} = 4\times3 = 12$

[Since, N is repeated 2 times and A is repeated three times]

18. In NATURE, there are 6 letters , hence 6 places

3	3	2	2	1	1
1st		3rd		5th	

Now vowel is AUE, and 3 consonent.
Hence, total word = 3×3×2×2 = 36

19. In word FIGURE, there are 3 consonent and 3 vowel.
It means word can be start either vowel or consonent.

C	V	C	V	C	V
3	3	2	2	1	1

+

V	C	V	C	V	C
3	3	2	2	1	1

[Answer is 72]

36 + 36 = 72

20.

P_1	P_2	P_3	P_4

First pen can be put in anyone pocket i.e., 4 ways , second pen = 4 ways and third pen = 4 ways
Hence, number of ways = 4×4×4 = 4^3 = 64

21. Every question can be answered by 5 different ways
hence for , I II III IV
5 5 5 5

Hence, boy can answered = 5^4 ways but there is only one way in which his all answer is right.
So, 5^4–1 = 625 – 1 = 624 is the way when a boy can not give correct answer for all questions.
Answer is 624

22. (i) If team contains 5E then there will be 1 American

hence, $^7C_5 \times {}^4C_1 = \frac{7\times6}{2\times1}\times4 = 84$

(ii) At least 2 American can be choose of the following ways
(i) 2A and 4E (ii) 3A and 3E (iii) 4A and 2E

$^4C_2 \times {}^7C_4 + {}^4C_3 \times {}^7C_3 + {}^4C_4 \times {}^7C_2$

$\frac{4\times3}{2\times1}\times\frac{7\times6\times5}{3\times2\times1} + \frac{4}{1}\times\frac{7\times6\times5}{3\times2\times1} + 1\times\frac{7\times6}{2\times1} = 210+140+21 = 371$

(iii) Zero, because there are only 4 American and team has 6 persons.

23. (i) When a particular player is always choosen means in 11 player, 1 is fixed. So, we have to select only 10 player out of 14

$\therefore \quad {}^{14}C_{10} = {}^{14}C_4 = \frac{14\times13\times12\times11}{4\times3\times2\times1} = 91\times11 = 1001$

(ii) When a particular player never choosen, it means we have to choose 11 player out of 14.

Hence, ${}^{14}C_{11} = {}^{14}C_3 = \frac{14\times13\times12}{3\times2\times1} = 182\times2 = 364$

24. Because things are distributed equally, hence every person get 4 things.

I II II

${}^{12}C_4 \quad {}^{8}C_4 \quad {}^{4}C_4$

Hence, total number of ways $= {}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4$ (solve ownself)

25. Ram can invite either 1 or 2 or 3 or 4 or 5.

Hence, ${}^{5}C_1 + {}^{5}C_2 + {}^{5}C_3 + {}^{5}C_4 + {}^{5}C_5 = 5 + 10 + 10 + 5 + 1 = 31$

26. Octagone has 8 vertices.

Hence, number of triangles $= {}^{8}C_3 = \frac{8\times7\times6}{3\times2\times1} = 56$

For triangle 3 vertices are needed.

27. In any figure, number of diagonals $= {}^{n}C_2 - n$

Heptagone contain 7 vertices or points.

Hence, ${}^{7}C_2 - 7 = \frac{7\times6}{2\times1} - 1 = 21 - 7 = 14$

28. If all points are non collinear then number of triangles $= {}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 120$

But 7 are collinear, they are not making any triangle.

Hence, ${}^{7}C_3 = \frac{7\times6\times5}{3\times2\times1} = 35$

Hence, number of triangles $= 120 - 35 = 85$.

29. ${}^{n}C_2 = 105 \quad \Rightarrow \frac{n(n-1)\lfloor n-2}{\lfloor 2\times\lfloor n-2} = 105 \quad \Rightarrow n(n-1) = 105\times2 = 210$

Hence, $n = 15$.

30. Hall can be illuminated if only one lamp in ON.
Every lamp has 2 process ON/OFF. So there are number of option $= 2^{10} = 1024$
There is only one way when all lamp is OFF.
Hence, number of ways when hall is illuminated $= 1024 - 1 = 1023$

31. 100 runs can be scored by 4's and 6's like this way

4's	6's	Runs
25	0	100
22	2	100
19	4	100
16	6	100
13	8	100
10	10	100
7	12	100
4	14	100
1	16	100

Number of possible way = 9

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32. There are 6 colours.

Hence number of signals = ${}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6 = 6 + 15 + 20 + 15 + 6 + 1 = 63$

33. A booking Clerk has ${}^{25}C_2$ tickets for upward and ${}^{25}C_2$ tickets for downward journey

$$= 2 \times {}^{25}C_2 = \frac{2 \times 25 \times 24}{2 \times 1} = 600$$

34. If tournament is knock out hence match are required less than one to number of players.
Hence, 30–1 = 29

35. Sum is even when there take off
(i) 4E (ii) 40 (iii) 2E and 20

$${}^5C_4 + {}^5C_4 + {}^5C_2 \times {}^5C_2 = 5 + 5 + \frac{5 \times 4}{2 \times 1} \times \frac{5 \times 4}{2 \times 1} = 110$$

Answer is 110

CAREER ENDEAVOUR

CHAPTER

15

Probability

SOLVED EXAMPLES

1. What is the probability of getting a number greater than 2, in a throw of a normal unbiased dice having 6 faces?

Soln. The event is defined as getting 3 or 4 or 5 or 6. The individual probabilities of each of these are 1/6. 1/6, 1/6 and 1/6 respectively.
Hence, the required probability is $1/6 + 1/6 + 1/6 + 1/6 = 4/6 = 2/3$.

2. If we have the probability of A hitting a target as 1/3 and that of B hitting the target as 1/2 , then the probability that both hit the target if one shot is taken by both of them is got by.
A hits the target "and" B hits the target.

$$P(A) \times P(B) = 1/3 \times 1/2 = 1/6.$$

(Note that since we use the conjunction "and" in the definition of the event here, we multiplying the individual probabilities that are connected through the conjunction "and").

3. If we have the probability of "A" winning a race as 1/3 and that of "B" winning the race as ½, then the probability that either A or B win a race is got by

Soln. A wins "or" B wins.

$$P(A) + P(B) = 1/3 + 1/2 = 5/6.$$

(Note that since we use the conjunction "or" in the definition of the event here, we add the individual probabilities that are connected through the conjunction "or".)

(i) Random Experiment: An experiment whose outcome has to be among a set of events that are completely known but whose exact outcome is unknown is a random experiment (e.g. Throwing of a dice, tossing of a coin.) Most questions on probability are based on random experiments.

(ii) Sample Space: This is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment. [e.g. sample space when a coin is tossed is (Head, Tail). Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6).]

(iii) Event: The set representing the desired outcome of a random experiment is called the event. Note that the event is a subset of the sample space.

(iv) Non-event: The outcome that is opposite the desired outcome is the non-event. Note that if the event occurs, the non-event does not occur and vice versa.

(v) Impossible event: An event that can never occur is an impossible event. The probability of an impossible event is 0. e.g. (Probability of the occurrence of 7 when a dice with 6 faces numbered 1-6 is thrown).

(vi) Mutually Exclusive events: A set of events is mutually exclusive when the occurrence of any one of them means that the other events cannot occur. (If head appears on a coin, tail will not appear and vice versa).

(viii) Expectation: The expectation of an individual is defined as = Probability of winning × Reward of winning.

$$\text{Hence, probability of event } P(E) = \frac{n(E)}{n(S)}$$

4. A man holds 20 out of the 500 tickets to a lottery. If the reward for the winning ticket is Rs. 1000, find the expectation of the man.

Soln. $\text{Expectation} = \text{Probability of winning} \times \text{Reward of winning} = \frac{20}{500} \times 1000 = \text{Rs. } 40.$

5. At a car park there are 100 vehicles, 60 of which are cars, 30 are vans and the remainder are lorries. If every vehicle is equally likely to leave, find the probability of a car leaving second if either a lorry or van had left first.
(a) 12/33 (b) 14/33 (c) 17/33 (d) 8/33

EXERCISE

[LEVEL-I]

1. Two balls are to be drawn from a bag containing 8 grey and 3 blue balls. Find the chance that they will both be blue.
(a) $\frac{3}{55}$ (b) $\frac{1}{5}$ (c) $\frac{11}{15}$ (d) $\frac{14}{45}$

2. In a singlet throw of two dice, what is the probability of a doublet ?
(a) $\frac{5}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{18}$ (d) $\frac{1}{9}$

3. Two dice are tossed. The probability that the total score is a prime number is:
(a) $\frac{1}{6}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) $\frac{7}{9}$

4. A box contains 20 electric bulbs, out of which 4 are defective. Two bulbs are chosen at random from box. The probability that atleast one of these is defective is
(a) $\frac{12}{19}$ (b) $\frac{21}{95}$ (c) $\frac{7}{19}$ (d) $\frac{4}{19}$

5. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green ?
(a) $\frac{8}{21}$ (b) $\frac{9}{21}$ (c) $\frac{1}{3}$ (d) $\frac{3}{4}$

6. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The chance that exactly 2 of them are children is.
(a) $\frac{10}{21}$ (b) $\frac{1}{12}$ (c) $\frac{1}{5}$ (d) $\frac{1}{9}$

7. A box contains 10 black and 10 white balls. The probability of drawing two balls of the same colour is.

(a) $\frac{5}{19}$ (b) $\frac{10}{19}$ (c) $\frac{9}{38}$ (d) $\frac{9}{19}$

8. A speaks the truth in 70 percent cases and B in 80 percent cases. The probability that they will contradict each other in describing a single event is

(a) 0.36 (b) 0.38 (c) 0.40 (d) 0.42

9. A problem is given to three students A, B and C whose chances of solving the problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that the problem will be solved is

(a) $\frac{11}{12}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{24}$

10. Three coins are tossed together. What is the probability to getting exactly two tail.

11. Two dice are thrown randomly. What is the probability to getting sum more than 7.

12. Find the probability to getting triplet on throwing three dice randomly.

13. On a pack of 52 cards 2 cards are drawn randomly. Find the probability that drawn cards are -

(a) black colour card. (b) Both are Ace. (c) One queen. (d) Honour cards

(e) Either both are queen or both are red colour card.

14. In a bag, there are 6 Red, 5 Green and 4 Black balls colour balls. 3 balls are picked out from bag. What is the probability that

(a) All balls are of different colours. (b) All balls from same colour.

(c) No ball of green colour. (d) At least two balls from red colour

(e) At most two balls from black colour.

15. There are two bags containing 3 Red and 4 Black balls in first bag and 4 Red and 2 Black balls in Bag II. One - one ball drawn from each bag. What is the probability that

(a) Both balls are red colour. (b) Both balls are green colour

(c) One ball is red and other is black

16. In leap year, what is the probability that there are 53 Sundays.

Conditional Probability :

17. The probability that Rahul Dravid will make a century is $\frac{1}{4}$ and sachin will make a centaury is $\frac{1}{3}$. Then find the probability that In a match -

(a) Both will make centaury. (b) None will make centaury.

(c) Exactly one of them make a centaury. (d) At least one centaury made in match.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15.

EXERCISE

[LEVEL-II]

1. There are 3 envelopes corresponding to 3 letters. If the letters are placed in the envelopes at random. What is the probability that none of the letters are placed in the right envelop ?

2. The probability of horse A winning a race is 1/3 and that of horse B winning the same race is 1/5. Find the probability that
(a) Either of them will win (b) None of them will win.

3. A letter is taken at random from the letters of the word "STATISTICS" and another letter is taken at random from the letters of the word "ASSISTANT" what is the probability that they are the same letter.

4. A number is chosen from each of the two sets {1,2,3,4,5,6,7,8,9} and {1,2,3,4,5,6,7,8,9}. If P_1 denotes the probability that the sum of the two numbers be 10 and P_2 the probability that their sum be 8, then $(P_1 + P_2)$ is

5. A coin is biased so that probability of head $= \frac{1}{4}$. The coin is tossed 5 times. Find the probability of obtaining two heads and three tails with heads occurring in succession.

6. What is the probability that a person tossing three fair coins will get together all heads or all tails for the second time on the 5^{th} toss.

7. The probabilities that three men hit target are $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ respectively. Each shoots once the target. What is the probability that exactly one of them hits the target.

8. What is the probability of getting 7^{th} head in the tenth toss of an unbiased coin ?

9. A group of 5 is to be chosen from a group of 9 people. What is the probability that a certain married couple will either serve together or not at all ?

10. From a box containing 10 cards, numbered 1, 2, 3, ..., 10 four cards are drawn together. what is the probability that their sum is even ?

11. A fair dice is rolled repeatedly until 6 shows up 3 times. What is the probability that exactly 5 tosses are needed.

12. In a tank containing 3 fish A, B, C, pellets of food are frequently placed. Each time a pellet is dropped, the fish compete for it. Suppose that over a long period of time it is observed that either A or B is successful $\frac{1}{2}$ of the time and that either A or C is successful $\frac{3}{4}$ of the time. What is the probability that A is successful.

13. There are 2 groups of subjects, one of which consists of 5 science and 3 engineering subjects and the other consists of 3 science and 5 engineering subjects, an unbiased dice is cast. If the number 3 or 5 tums up a subject is selected at random from the first group. What is the probability that an engineering subject is ultimately selected ?

14. A coin is tossed until a head appears or until the coin has been tossed 5 times. If a head does not occur on the first two tossed, then the probability that the coin will be tossed 5 times is what ?

15. A and B play 12 games of chess of which 6 are won by A, 4 are won by B and 2 end in a tie. They agree to play a tournament consisting of 3 games. What is the probability that A and B win alternatively ?

16. The probability of getting a defective floppy in three boxes A, B and C are 1/3, 1/6 and 3/4 respectively. A box is selected randomly and a floppy is drawn from it. The probability that the floppy is defective and is drawn from box A is what ?

17. Two letters are chosen one after another without replacement from the English alphabet. What is the probability that the second letter chosen is vowel ?

18. A cow is tied with a pole by a 100 meter long rope. What is the probability that at some point of time the cow is at least 60 meters away from the pole ?

19. A and B play a game of dice. A throws the dice first. The person who first gets a 6 is the winner. What is the probability that A win ?

20. Two teams A and B play a series of four matches. If the probability that team A wins a match is $\frac{2}{3}$, then the probability that team A wins three matches, loses one and the third win occurs in the fourth match is what ?

21. A player is going to play a match either in the morning or in the afternoon or in the evening all possibilities being equally likely. The probability that he wins the match is 0.6, 0.1 and 0.8 according as if the match is played in the morning, afternoon or in the evening respectively. Given that he has won the match, the probability that the match was played in the afternoon is what ?

22. Three persons play a game by tossing a fair coin each independently. The game ends in a trial if all of them get the same outcome in that trial, otherwise they continue to the next trial. What is the probability that the game ends in an even number of trials ?

23. If a student is likely choose any of the four choices with equal probability in a multiple choice examination with 5 questions then the probability that the student answer at least 4 question correctly is what ?

24. If the letters of the word 'REGULATION' be arranged at random what is the probability that there will be exactly 4 letters between R and E.

25. The last 3 digits of a telephone number have been erased and all we know is that the number was 25785???. Assuming that all possibilities are equally likely, the probability that the missing digits are all equal to each other is what ?

HINTS & SOLUTIONS

[LEVEL-I]

1. Probability that all are blue $= \frac{{}^{3}C_{2}}{{}^{11}C_{2}} = \frac{3}{55}$

Answer is (a)

2. "Doublet" means that both dice show the same number favourable outcome ϕ are (1,1), (2,2), (3,3), (4,4), (5,5), (6,6). Hence probability $= \frac{6}{36} = \frac{1}{6}$

Answer is (b)

3. Event that sum is a prime number
{ (1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)}

Total events favourable for prime number = 15. Probability $= \frac{15}{36} = \frac{5}{12}$

Answer is (b)

4. Probability (No defective piece) $= \frac{^{16}C_2}{^{20}C_2} = \frac{12}{19}$. Probability of atleast one defective piece $= 1 - \frac{12}{19} = \frac{7}{19}$

Answer ic (c)

5. Total number of balls $= (8 + 7 + 6) = 21$. Let E = event that the ball drawn is neither blue nor green = event that the ball drawn is red n(E) = 8. Probability $= \frac{8}{21}$

Answer is (a)

6. Number of ways choosing 4 persons out of nine $= {}^9C_4 = \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} = 126$

Number of ways of choosing two children out of 4 and 2 persons out of (3+2) persons $= {}^4C_2 \times {}^5C_2 = 60$

Require probability $= \frac{60}{126} = \frac{10}{21}$

Answer is (a)

7. Two ball either both black or both white can drawn in $^{10}C_2 + {}^{10}C_2 = 90$ ways

Answer is (d)

8. They contradict each other when A tells truth and B tells lie or A tells lie and B tells truth.

$P(A) = 70\% = 0.7,\ P(\bar{A}) = 0.3$

$P(B) = 80\% = 0.8,\ P(\bar{B}) = 0.2$

Hence, required probability $= P(A).P(\bar{B}) + P(\bar{A}).P(B) = 0.7 \times 0.2 + 0.3 \times 0.8 = 0.14 + 0.24 = 0.38$

Answer is (b)

9. Problem will be solved if one of the student can solve the problem.

So, required probability = 1 – no one solve the problem $= 1 - P(\bar{I}).P(\bar{II}).P(\bar{III})$

$P(\bar{I}) = \frac{1}{2},\ P(\bar{II}) = \frac{2}{3}, P(\bar{III}) = \frac{3}{4} = 1 - \left(\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}\right) = \frac{3}{4}$

Answer is (c)

10. $n(s) = 2^3 = 8\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(E) = 3 \quad \Rightarrow P(E) = \frac{3}{8}$

11. n(s) = 6× 6 = 36

Sum more than 7

8 = (2, 6) (3, 5) (4, 4) (5, 3) (6, 2)

9 = (3, 6) (4, 5) (5, 4) (6, 3)

10 = (6, 4) (5, 5) (4, 6)

11 = (6, 5) (5, 6)

12 = (6, 6)

$n(E) = 15 \quad \therefore \quad P(E) = \frac{15}{36} = \frac{5}{12}$

12. $n(s) = 6 \times 6 \times 6 = 216$

$n(E) = 6\{(1, 1, 1)\ (2, 2, 2)\ (3, 3, 3)\ (4, 4, 4)\ (5, 5, 5),\ (6, 6, 6)\}$

$$P(E) = \frac{6}{6\times6\times6} = \frac{1}{36}$$

13. $n(s) = {}^{52}C_2 = \frac{52\times51}{2\times1} = 26\times51$

(a) $n(E) = {}^{26}C_2 = \frac{26\times25}{2\times1} = 13\times25 \quad \Rightarrow P(E) = \frac{13\times25}{26\times51} = \frac{25}{102}$

(b) $n(E) = {}^{4}C_2 = P(E) = \frac{{}^{4}C_2}{{}^{52}C_2}$

(c) $n(E) = {}^{4}C_1 \times {}^{5}C_1 \quad \Rightarrow P(E) = \frac{{}^{4}C_1 \times {}^{4}C_1}{{}^{52}C_2} = \frac{2}{13}$

(d) $n(E) = {}^{16}C_2 \quad \Rightarrow P(E) = \frac{{}^{16}C_2}{{}^{52}C_2}$ ($\because$ there are 16 honour cards)

(e) $n(E) = {}^{26}C_2 + {}^{4}C_2 - {}^{2}C_2 \quad \Rightarrow P(E) = {}^{26}C_2 + {}^{4}C_2 - \frac{{}^{2}C_2}{{}^{52}C_2}$

14. 6R, 5G and 4B, $T = 15$

$n(s) = {}^{15}C_3 = \frac{15\times14\times13}{3\times2\times1} = 35\times13 = 455$

(a) All balls are different colours if one-one ball come from each group, $n(E) = {}^{6}C_1 \times {}^{5}C_1 \times {}^{4}C_1$

$$P(E) = \frac{6\times5\times4}{455} = \frac{24}{91}$$

(b) All balls from same colour if all ball are red or green or black.

$$n(E) = {}^{6}C_3 + {}^{5}C_3 + {}^{4}C_3 = \frac{6\times5\times4}{3\times2\times1} + \frac{5\times4}{2\times1} + 4 = 34$$

$$P(E) = \frac{34}{455}$$

(c) $n(E) = {}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 120$ (d) $\frac{31}{91}$ (e) $\frac{451}{455}$

$\therefore \quad P(E) = \frac{120}{455}$

15. $P(R_I) = \frac{3}{7} \qquad P(R_{II}) = \frac{4}{6}$

$P(B_I) = \frac{4}{7} \qquad P(B_{II}) = \frac{2}{6}$

(a) $P(R_I).P(R_{II}) = \frac{3}{7}\times\frac{4}{6} = \frac{2}{7}$

(b) Zero, since bag do not contain any green ball.

(c) $P(R_I).P(B_{II})+P(B_I).P(R_{II})=\frac{3}{7}\times\frac{2}{6}+\frac{4}{7}\times\frac{4}{6}=\frac{6}{42}+\frac{16}{42}=\frac{22}{42}=\frac{11}{21}$

16. In leap year, there are 52 weeks and 2 extra days
These can be (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thur), (Thur, Fri), (Fri, Sat), (Sat, Sun)

$$n(s)=7;\ n(E)=2;\ P(E)=\frac{2}{7}$$

17. $P(R)=\frac{1}{4}, \quad P(\bar{R})=\frac{3}{4}$

$P(s)=\frac{1}{3}, \quad P(\bar{s})=\frac{2}{3}$

(a) $P(R).P(s)=\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$

(b) $P(\bar{R}).P(\bar{s})=\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$

(c) $P(R).P(\bar{s})+P(\bar{R}).P(s)=\frac{1}{4}\times\frac{2}{3}+\frac{3}{4}\times\frac{1}{3}=\frac{5}{12}$

(d) $P(R).P(s)+P(R).P(\bar{s})+P(\bar{R}).P(s)=\frac{1}{12}+\frac{5}{12}=\frac{6}{12}=\frac{1}{2}$

HINTS & SOLUTIONS

[LEVEL-II]

1. 3 letters can be put on $\lfloor 3$ ways. $n(s)=\lfloor 3=6$
Let us suppose the A, B and C are envelops and 1 in A,
2 in B and 3 in C is correct order. Letter can be placed in envelop

A	B	C
1	2	3
1	3	2
2	1	3
2	3	1
3	1	2
3	2	1

There is only 2 ways when no one letter can correct envelope.
Hence, $n(E)=2$

$$\therefore \quad P(E)=\frac{2}{6}=\frac{1}{3}$$

2. $P(A)=\frac{1}{3}$ and $P(B)=\frac{1}{5}$
Since event are mutually exclusively or dependent
Hence, $P(A\cap B)=0$

(i) $P(A \cup B) = P(A) + P(B) = \frac{1}{3} + \frac{1}{5} = \frac{5+3}{15} = \frac{8}{15}$

(ii) $P(A \cup B)' = 1 - P(A \cup B) = 1 - \frac{8}{15} = \frac{7}{15}$

3. There are 4 letter common in both words S, T, A, I

STATISTICS

$P(s) = \frac{3}{10}, P(T) = \frac{3}{10}, P(A) = \frac{1}{10}, P(I) = \frac{2}{10}$

ASSISTANT

$P(s) = \frac{3}{9}, P(T) = \frac{2}{9}, P(A) = \frac{2}{9}, P(I) = \frac{1}{9}$

Hence, required probability, $\frac{3}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{1}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{1}{9} = \frac{9}{90} + \frac{6}{90} + \frac{2}{90} + \frac{2}{90} = \frac{19}{90}$

4. $n(s) = 9 \times 9 = 81$

$E_1 = \text{sum be } 10 = (1,9)(2,8)(3,7)(4,6)(5,5)(6,4)(7,3)(8,2)(9,1)$

$P_1 = \frac{9}{81}$ $E_2 = \text{sum be } 8 = (1,7)(2,6)(3,5)(4,4)(5,3)(6,2)(7,1)$

$P_2 = \frac{7}{81}$ $\therefore$ $P_1 + P_2 = \frac{9}{81} + \frac{7}{81} = \frac{16}{81}$

5. We get head in succession but in which tosses. So, first we have to select succession of toss.

I	II	III	IV	V
H	H			
	H	H		
		H	H	
			H	H

There are 4 ways in which 1 is selected in ${}^4C_1$ way.

Now, $P(H) = \frac{1}{4}$, hence $P(T) = 1 - \frac{1}{4} = \frac{3}{4}$

Then probability $= {}^4C_1 \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{256}$

6. Probability of getting all heads or all tails on tossing three fair coins.

n(s) = 8{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

n(E) = 2

$P(E) = \frac{2}{8} = \frac{1}{4}$ $P(\bar{E}) = 1 - \frac{1}{4} = \frac{3}{4}$

the event is happend 2nd time in in 5th toss means in previous 4 tosses it happened only 1 time but in which toss. So, first we have to select a toss. So, required probability

$${}^4C_1 \times \frac{1}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{27}{256}$$

7. $P(I) = \frac{1}{6}, P(II) = \frac{1}{4}, P(III) = \frac{1}{3}$

8.

9.

10.

11.

12.

$P(\overline{I})=\frac{5}{6}, P(\overline{II})=\frac{3}{4}, P(\overline{III})=\frac{2}{3}$

Hence required probability

$$P(I),P(\overline{II})P(\overline{III})+P(\overline{I})P(II)P(\overline{III})+P(\overline{I})P(\overline{II})P(III)$$

$$\Rightarrow \quad \frac{1}{6}\times\frac{3}{4}\times\frac{2}{3}+\frac{5}{6}\times\frac{1}{4}\times\frac{2}{3}+\frac{5}{6}\times\frac{3}{4}\times\frac{1}{3} \quad \Rightarrow \quad \frac{6}{72}+\frac{10}{72}+\frac{15}{72}=\frac{31}{72}$$

8. $P(H)=\frac{1}{2},\ P(\overline{H})=P(T)=\frac{1}{2}$

We get 7th head in 10th toss it means in previous 9 tosses. We get already 6 head. Hence probability,

$$^9C_6\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{9\times8\times7}{3\times2\times1}\times\frac{1}{2^{10}}=\frac{21}{2^8}=\frac{21}{256}$$

9. $n(s)=^9C_5=\frac{9\times8\times7\times6}{4\times3\times2\times1}=126$

(i) When married couple always choosen then number of ways $=^7C_3$.

(ii) When not at all then $-^7C_5$

$$n(E)=^7C_3+^7C_5=\frac{7\times6\times5}{3\times2\times1}+\frac{7\times6}{2\times1}=56$$

Hence, probability $=\frac{56}{126}=\frac{28}{63}$

10. There are 10 cards in which 5 is odd and 5 is even

Now, if 4 cards is even then sum is even $^5C_4=5$

If 4 card is odd then sum is even $^5C_4=5$

If 2 card is odd and 2 is even then sum is even $^5C_2\times{}^5C_2=10\times10=100$

$$n(s)=^{10}C_4=\frac{10\times9\times8\times7}{4\times3\times2\times1}=210$$

Hence, $P=\frac{5+5+100}{210}=\frac{110}{210}=\frac{11}{21}$

CAREER ENDEAVOUR

11. Getting 6 of fair dice rolled $P(E)=\frac{1}{6},\ P(\overline{E})=\frac{5}{6}$ because 5 tosses are needed it means third show in 5th, rolled.

So, previous 4 rolled it becomes two times.

Hence, probability $=^4C_2\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=6\times\frac{1}{6}\times\frac{1}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{1296}$

12. $P(A\cup B)=\frac{1}{2},\ P(A\cup C)=\frac{3}{4}$

$P(A\cup B\cup C)=1$

$P(A\cup B\cup C)-P(A\cup B)=1-\frac{1}{2} \quad \Rightarrow P(C)=\frac{1}{2}$

$P(A\cup B\cup C)-P(A\cup C)=1-\frac{3}{4} \quad \Rightarrow P(B)=\frac{1}{4}$

$\because \quad P(A)+P(B)+P(C)=1$

$\therefore \quad P(A)+\frac{1}{4}+\frac{1}{2}=1 \quad \Rightarrow P(A)=\frac{1}{4}$

13. Probability to become 3 or 5

$$P(3\cup 5)=\frac{2}{6}=\frac{1}{3}, \qquad P(\overline{3\cup 5})=\frac{2}{3}$$

Probability of engineering subject from group $A = P(E_A)=\frac{3}{8}$

$$P(E_B)=\frac{5}{8}$$

Hence, required probability $=\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{5}{8}=\frac{3}{24}+\frac{10}{24}=\frac{13}{24}$

14.

I	II	III	IV	V
×	×	½	½	–

If head not appears in first two tosses and 5 tosses are needed then it is cleared that it not comes in 3rd and 4th tosses also. So, probability depends in 3rd and 4th tosses.

So, required probability $=\frac{1}{2}\times\frac{1}{2}\left(\text{for } 3^{rd} \text{ \& } 4^{th} \text{ tosses}\right)=\frac{1}{4}$

15. $P(A)=\frac{6}{12}=\frac{1}{2}, \; P(B)=\frac{4}{12}=\frac{1}{3}$

They can win alternatively such that $P(A).P(B).P(A)+P(B).P(A).P(B)$

$$\frac{1}{2}\times\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{2}\times\frac{1}{3}=\frac{1}{12}+\frac{1}{18}=\frac{5}{36}$$

16. At first we have to choose box A, then get defective floppy from there. So, probability for box $A=\frac{1}{3}$

So, probability of getting defective floppy from box $A=\frac{\frac{1}{3}\cdot\frac{1}{3}}{\frac{1}{3}\left[\frac{1}{3}+\frac{1}{6}+\frac{3}{4}\right]}=\frac{4}{15}$

17. Second letter vowel can be choosen on the following way.

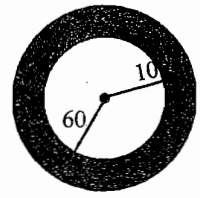
	I	II		I	II
(i)	C &	V	or	V &	V

Hence, probaility $=\frac{21}{26}\times\frac{5}{25}+\frac{5}{26}\times\frac{4}{25}=\frac{125}{26\times 25}=\frac{5}{26}$

18. Since cow is tied from 100m long rope.

Hence, $n(s)=\pi R^2=\pi 100^2$

At any time cow is at least 60 m away from pole if it is located on shaded area



$n(E)=\pi 100^2-\pi 60^2$

$$\therefore \quad P(E)=\frac{\pi 100^2-\pi 60^2}{\pi 100^2}=\frac{\pi(100+60)(100-60)}{\pi 100^2}=\frac{160\times 40}{100\times 100}=\frac{16}{25}$$

19. Probability of getting 6

$$\Rightarrow \quad P(E)=\frac{1}{6}$$

$$\therefore \quad P(\bar{E})=1-\frac{1}{6}=\frac{5}{6}$$

Since A throw dice first and also A will win.
So, A can win in Ist or 3rd or 5th or 7th .....so on throw.

So, probability $= P(E)=P(A)+P(\bar{A}).P(\bar{B}).P(A)+P(\bar{A}).P(\bar{B}).P(\bar{A}).P(\bar{B}).P(A)+.....+\infty$

$$=\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+.....\infty$$

$$=\frac{1}{6}\left[1+\left(\frac{5}{6}\right)^2+\left(\frac{5}{4}\right)^4+.....+\infty\right]$$

The series is G.P. whose first term is $a=1$, $r=\left(\frac{5}{6}\right)^2=\frac{25}{36}$

So, sum $S_\infty=\frac{a}{1-r}=\frac{1}{1-\frac{25}{36}}$

$$\therefore \quad P(E)=\frac{1}{6}\times\frac{36}{11}=\frac{6}{11}$$

20. 3rd win occurs in 4th match means in previous 3 matches.
A has win 2 times but in which match. So, first we select 2 match from 3 matches i.e. $^3C_2$.

Now, $P(A)=\frac{2}{3}$ $\qquad P(\bar{A})=1-\frac{2}{3}=\frac{1}{3}$

Hence, probability $= {}^3C_2\times\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\times\frac{2}{3}=\frac{8}{27}$

21. Same as Q. No. 17

$$P(E)=\frac{\frac{1}{3}\times 0.1}{\frac{1}{3}[0.6+0.1+0.8]}=\frac{0.1}{1.5}=\frac{1}{15}$$

22. They get same out come if they all get HHH or TTT.

Hence, $P(E)=\frac{2}{8}=\frac{1}{4}$, $\qquad P(\bar{E})=\frac{3}{4}$

Now, game ends in even trials means in 2nd or 4th or 6th.....so on trial.

Hence, probability $= P(\bar{I}).P(II)+P(\bar{I}).P(\bar{II}).P(\bar{III}).P(IV)+.....$

$$=\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}+.....$$

$$=\frac{3}{16}\left[1+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^4+.....\right]$$

It is a G.P.

$$=\frac{3}{16}\left[\frac{1}{1-\frac{9}{16}}\right]=\frac{3}{16}\times\frac{16}{7}=\frac{3}{7}$$

23. At least 4 question can correct in following ways.
(i) 4 correct and 1 wrong or (ii) 5C

$$P(C)=\frac{1}{4},\qquad P(\bar{C})=\frac{3}{4}$$

$$P(E)={}^5C_4\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{4^5}[15+1]=\frac{4^2}{4^5}=\frac{1}{4^3}=\frac{1}{64}$$

24. REGULATION contains 10 letters. So, there are exactly 4 letter between R and E. If R and E located in $(I^{st}\ \&\ 6^{th}),(2^{nd}\ \&\ 7^{th}),(3^{rd}\ \&\ 8^{th}),(4^{th}\ \&\ 9^{th})$ and $(5^{th}\ \&\ 10^{th})$ position.

Now, we have to select a position i.e. ${}^5C_1$.

Now, R and E can be arranged in $\underline{|2}$ ways and remaining 8 letters can be arranged in $\underline{|8}$ ways.

So, $n(E)={}^5C_1\times\underline{|2}\times\underline{|8}$

$n(s)=\underline{|10}$

$\therefore\quad P(E)=\dfrac{5\times2\times\underline{|8}}{\underline{|10}}=\dfrac{5\times2\times\underline{|8}}{10\times9\times\underline{|8}}=\dfrac{1}{9}$

25. $n(s)=10\times10\times10=1000$
Since missing digit are all equal.

$\therefore\quad n(E)=10\{000,111,222,333,444,555,666,777,888,999\}$

$$P(E)=\frac{10}{1000}=\frac{1}{100}$$

CAREER ENDEAVOUR

1.

2.

CHAPTER

16

Function and Graph

Function: Function is the chapter of mathematics that studies the dependence between variable quantities in the process of their changes. For instances, with a change in the side of a square, the area of the square also varies. The question of how the change in the side of the square affects the area is answered by a mathematical relationship between the area of the square and the side of the square.

Function is symbolized as $y = f(x)$ where f denotes the rule by which y varies with x, where "x" is independent variable and "y" is dependent variable.

(i) Analytical Representation of Function: This is essentially representation through a formula. This representation could be a uniform formula in the entire domain, for example, $y = 3x^2$ or by several formulae which are different parts of the domain.

EXAMPLES

1. $y = 3x^2$ if $x < 0$ and $y = x^2$ if $x > 0$

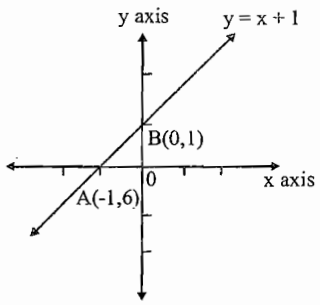
(ii) Tabular Representation of Functions: For representing functions through table, we simply write down a sequence of values of the independent variable "x" and then write down the corresponding value of the dependent variable "y". Thus, we have tables of logarithms, trigonometric values and so forth, which are essentially tabular representations of functions.

2.

Time (minutes)	Speed (m/s)
1	1.5
2	3.0
3	4.5
4	6.0
5	7.5

(iii) Graphical Representation of functions: In this way of representation function is produced in "xy" coordinate plane for every value of x from the domain D of the function, a point P(x, y) is constructed whose abscissa is x and whose ordinate y is got by putting the particular value of x in the formula representing the function.

3. $y = x + 1$



(iv) Even Functions: Let a function $y = f(x)$ be given in a certain interval. The function is said to be even if for any value of x.

$$f(x) = f(-x).$$

Properties of even functions:

(a) The sum, difference, product and quotient of an even function is also an even function.

(b) The graph of an even function is symmetrical about the y-axis.

Examples: $y = x^2$, $y = x^4$.

(v) Odd functions: Let a function be given in a certain interval. The function is said to be odd if for any value of x. If $f(-x) = -f(x)$

Properties of odd functions:

(a) The sum and difference of an odd function is an odd function.

(b) The product and quotient of an odd function is an even function

Examples: Not all functions need be even or odd. However, every function can be represented as the sum of an even function and an odd function.

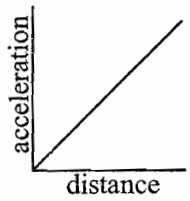
EXERCISE

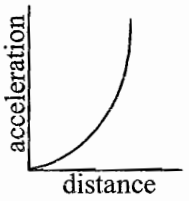
1. The speed of a car increases every minute as shown in the following table. The speed at the end of the 19th minute would be

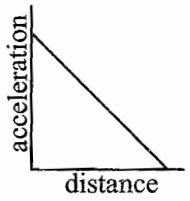
Time /minutes	Speed (m/sec)
1	1.5
2	3.0
3	4.5
24	36.0
25	37.5

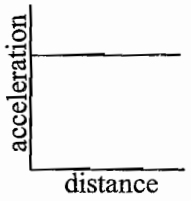
(a) 96.5 (b) 98.5 (c) 27.0 (d) 28.5

2. A ball is dropped from a height h above the surface of the earth. Ignore air drag, the curve that best represents its variation of acceleration is

a) 

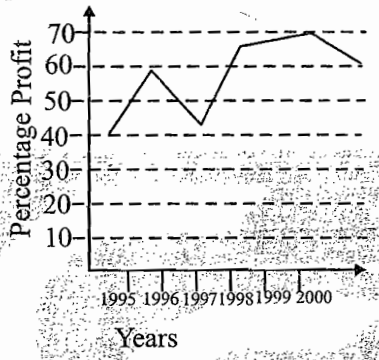
(b) 

(c) 

(d) 

3. Which of the following straight line passes through the point (1,1) ?
(a) $y = 2x + 3$ (b) $2y = x - 6$ (c) $x = 1$ (d) $y = x$

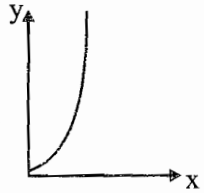
4. The following line graph gives the annual percent profit earned by a company during a period 1995 - 2000, study the line-graph and answer the questions that are based on it.

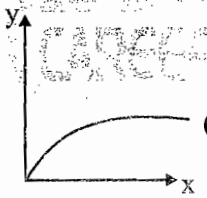


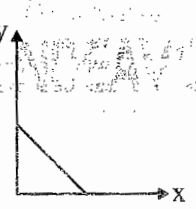
(i) In the income in 1998 was Rs. 264 corores, what was the expenditure in 1998 ?
(a) Rs. 104 corors (b) Rs. 14 corors (c) Rs. 160 corors (d) Rs. 184 corors.

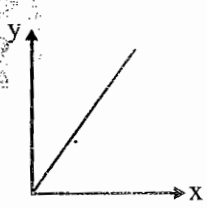
(ii) If the profit in 1992 wa Rs. 4 corors, what was the profit in 2000 ?
(a) Rs. 6.3 corors (b) Rs. 6 corors (c) Rs. 4 corors (d) Cannot be determine

5. If $y=e^x$ then graph of this function is

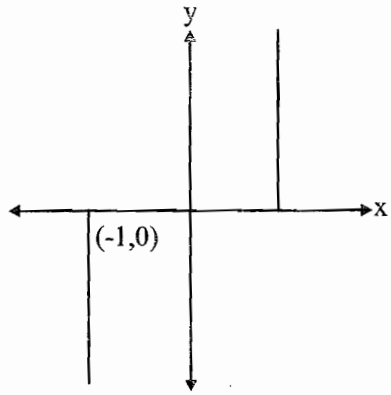
(a) 

(b) 

(c) 

(d) 

6. Which one of the following is true about the given graph ?



(a) $x=\frac{|y|}{y}, y\neq 0$ (b) $y=\frac{|x|}{x}, x\neq 0$ (c) $x=\frac{-|y|}{y}, y\neq 0$ (d) $y=\frac{-|x|}{x}, x\neq 0$

7. Which of the following straight line posses through the point (3, 2)
(a) $y=3x+4$ (b) $y=x+7$ (c) $y=x+1$ (d) $y=x-1$

8. If A#B = A+B +AB. If for any A there is a number C such that A#C = A, then A = ?
(a) –1 (b) 1 (c) –2 (d) 0

9. What is the value of x if $|x-1|+|x|=2$
(a) $-\frac{1}{2}$ (b) $\frac{3}{2}$ (c) both (a) and (b) (d) none of these

10. If a + b = 20 then maximum value of ab is
(a) 19 (b) 75 (c) 100 (d) 120

HINTS & SOLUTIONS

1. From the table we can draw the conclusion that speed = 1.5 (time)
Thus speed (After 19th minute) = $1.5\times 19 = 28.5$ m/sec
Answer is (d)

2. Newton third law $s=ut+\frac{1}{2}at^2$ where u(Initial velocity) = 0 (Ball just dapped)
t(time), a(acceleration), s(distance cover). Thus, $s=\frac{1}{2}at^2$
but for free falling body gravity must be constant equal to 'g', $s=\frac{1}{2}gt^2$
Then, Conclusion: g constant, s gradually increasing, only (d) graph hold both conditions
Answer is (d)

3. Put the value of x=1 in all equation the equation which satisfy the point will must pqrs through the equation.
(a) $y=2x+3=2+3=5$ No. (b) $y=\frac{x-6}{2}=\frac{1-6}{2}=-5/2$ No.
(c) x = 1 for all value of y thus No. (d) y = 1
Answer is (d)

4. (i) Let the expenditure is 1998 be Rs. x carores. Then $65=\frac{264-x}{x}\times 100=\frac{65}{100}=\frac{264}{x}-1$
$x=\frac{264\times 100}{165}=160$
Answer is (c)
(ii) From the line graph we obtain information about the percentage profit only. To find profit in 2000 we must have data for the income or expenditure in 2000. Therefore, the profit for 2000 cannot be determined.
Answer is (d)

5. For function $y = e^x$ If $x = 1$ then $y = e = 2.718$

If $x = 2$ then $y = e^2 = 7.382$

So (c) is not possible and increasing of value of y is much faster than n thus only (a) can be the possible graph of the function.

Answer is (a)

6. Mode (1×1) means the absolute value of the "x" thus it must be always positive. We have two point (-1,0) × (1,0), it these point must be the satisfy the graph.

Answer is (a)

7. Put the value of x in all equation.

(a) $y = 9+4 = 13$ (b) $y = 3+2 = 5$ (c) $y = 3+1 = 4$ (d) $y = 3-1 = 2$

Answer is (d).

8. A#C = A We get. A+C+AC = A

C(1+A) = 0, C = 0 or A = -1

Answer is (a)

9. Put $x = \frac{-1}{2}$ then $= |x-1| + |x| = \left|\frac{-1}{2} - 1\right| + \left|\frac{-1}{2}\right| = \frac{3}{2} + \frac{1}{2} = 2$

Put $x = \frac{3}{2}$ then $= |x-1| + |x| = \left|\frac{3}{2} - 1\right| + \left|\frac{3}{2}\right| = \frac{1}{2} + \frac{3}{2} = 2$

Answer is (c)

10. Maximum value of (ab) when value of "a" and "b" are equal to each other. Thus a = b = 10. Then ab = 100

Answer is (c)

(ii) Sample Space: This is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment. [e.g. sample space when a coin is tossed is (Head, Tail). Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6).]

(iii) Event: The set representing the desired outcome of a random experiment is called the event. Note that the event is a subset of the sample space.

(iv) Non-event: The outcome that is opposite the desired outcome is the non-event. Note that if the event occurs, the non-event does not occur and vice versa.

CHAPTER

17

Calendar, Clocks

(1) Calendar: The year consists of 365 days, 5 hours, 48 minutes (52 weeks and 1 odd day). An extra day is added once in every fourth year which was called the leap year, which has 366 days (52 weeks and 2 odd days).

Important Concept of Calendar:

(i) Odd Days: In a given period the number of days more than the complete weeks are called odd days.

(ii) Leap Year: Every year divisible by 4 is a leap year if it is not a century. Every 4th century is a leap year and no other century is a leap year. A leap year has 366 days.

(i) Each of the years 1948, 2004, 1676 etc. is a leap year.

(ii) Each or the years 400, 800, 1200, 1600, 2000 etc. is a leap year.

(iii) None of the years 2001, 2002, 2003, 2005, 1800, 2100 etc. is a leap year.

(iii) Ordinary Year: The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

CAREER ENDEAVOUR

(iv) Counting of Odd Days:

1 ordinary year = 365 days = (52 weeks + 1 day).

$\therefore$ 1 ordinary year has 1 odd day.

1 leap year = 366 days = (52 weeks + 2 days). Therefore, 1 leap year has 2 odd days.

100 years = 76 ordinary years + 24 leap years = $(76 \times 1 + 24 \times 2)$ odd days = 124 odd days

= (17 weeks + 5 days) $\equiv$ 5 odd days.

Number of odd days in 100 years = 5 odd days.

Number of odd days in 200 years = $(5 \times 2) = 3$ odd days.

Number of odd days in 300 years = $(5 \times 3) = 1$ odd day.

Number of odd days in 400 years = $(5 \times 4 + 1) = 0$ odd day.

Similarly each one of 800 years, 1200 years, 1600 years, 2000 years etc. has zero odd day.

Day of the Week Related to Odd Days:

No. of days	0	1	2	3	4	5	6
Day	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

SOLVED EXAMPLES

1. What was the day on 4th June 2002:

Soln: $2001 = 1600 + 400 + 1$

$\downarrow \quad \downarrow \quad \downarrow$

Odd days : 0 0 1

Up to 4th June number of days are equal to 31+28+31+31+30 +4=155 days =22 weeks+1 day= 1odd day
So total number of odd days is 2 and hence day is Tuesday.

(2) Clock: The face or dial of a watch is a circle whose circumference is divided into 60 equal parts called minute spaces. A clock has two hands the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand. The minute hand moves around the whole circumference of clock once in 1 hour while hour hand moves around the whole circumference of clock in twelve hours.
So minute hand is twelve times faster than hour hand.

Important points about clock:

(i) Angle traced by minute hand in 60 min. = 360° & Angle traced by hour hand in 12 hrs = 360°.

(ii) In one minute hand moves by 6 degree while hour hand moves in 1 minute by 0.5 degree. So in one minute, minute hand gains $5\frac{1}{2}$ degree more than hour hand.

(iii) In 60 minutes the minute hand gains 55 minutes over the hour hand or in terms of angle by 330 degree.

(iv) In every hour both hands coincide means both are 0 degree apart.

(v) In every hour both the hands coincide once means they are 0 degree apart. In fact if both hands start moving together from the same position both the hands will coincide after $\frac{360\times 2}{11} = 65\frac{5}{11}\text{min}$

(vi) When the two hands are at right angles they are 15 minute spaces apart $15\times 6 = 90^0$ this happens twice in every hour.

(vii) When the two hands are in opposite direction they are 30 minutes spaces apart $30\times 6 = 180^0$ this happens once in every hour.

(viii) The hands are in same straight line when they are coincident or opposite to each other or together.

(ix) When hands are together they are 0 degree apart.

2. Clock A losses 2 minute a day and clock B gains 3 minutes per day. If clock B is 10 minutes ahead of clock A, how many days will it take clock B to be 30 minutes ahead of clock A.

Soln. B must be ahead of A by (30–10) = 20 minutes
Since clock A loses 2 minutes and B gains 3 minutes
B gains over A = (3 + 2) = 5 minutes
Therefore, required number of days = 20/5 = 4 days

3. A watch which gains 5 seconds in 3 minutes was set right at 7 : 00 am. In the afternoon of the same day, when the watch indicated quarter past 4 'O' clock. Find the true time.

Soln. Time from 7 : 00 to quarter past 4 on the same day = 9 hours 15 minutes = 555 minutes
3 minutes and 5 seconds of this clock = 3 minute of the correct clock.

555 minutes of this clock $= \left(3\times\frac{12}{37}\times 555\right) = 540$ minutes = 9 *hrs* of of the correct clock

hence correct time will be 9 hrs. after 7 : 00 a.m. The true time is 4 : 00 p.m.

4. A watch gains uniformly. It was observed that it was 5 min. Slow at 12 'O' clock in the night on sunday. On thursday of 6 : 00 p.m. it was 5 minute 48 sec. fast when was it correct?

Soln. The time between 12 'O' clock on sunday right and 6 pm on thursday = 90 hours.

Now, the watch gains $5+\frac{29}{5}=\frac{54}{5}$ minutes in 90 hours

So, the watch gains 5 minutes in $=90\times\frac{5}{54}\times5=\frac{125}{3}$ *hours* . $=41$ hours 40 minuts

Thus watch is correct at 5 : 40 on tuesday.

5. A watch slows down by 3 minutes in one day after one day, the percentage error in time shown by the watch is

(a) 5/24% (b) 6/24% (c) 50/3% (d) 20/3%

Soln. Error in 24 hours = 3 minutes
Error in (24×60) minutes = 3 minutes

hence percentage error $=\frac{3}{24\times60}\times100=\frac{5}{24}\%$

6. Clock X loses one minute a day and clock Y gains 3 minutes per day. If clock Y is 15 minutes ahead of clock X. How many days will it take clock Y to be 35 minutes ahead of clock X

(a) 2 (b) 3 (c) 4 (d) 5

Soln. Y must be ahead of X by 35–15 = 20 minutes
Y gains over clock X = (3 + 1) = 4 minutes
Therefore, required days = 20/4 = 5 days.

7. A clock is set right at 1 pm. It gains one minute an hour, when the clock indicates 6 p.m. on the same day. The true time is

(a) 55 minutes past 5 pm (b) $55\frac{5}{61}$ minute past 5 pm.

(c) $33\frac{5}{61}$ minutes past 5 pm (d) $5\frac{5}{143}$ minutes past 5 pm

Soln. Time from 1 pm to 6 pm on the same day = 5 hours
1 hour 1 minute of this clock = 1 hour of the correct clock

$\frac{61}{60}$ hour of this clock = 1 hour of the correct clock

5 hour of this clock $=\left(\frac{60}{61}\times5\right)$ hours of the correct clock = 4 hour $55\frac{5}{61}$ minutes of correct clock

Therefore, true time is 4 hour $55\frac{5}{61}$ minutes after 1 : PM

This is $55\frac{5}{61}$ minutes past 5 : 00 PM.

8. How much does a watch gain or lose per day if its hands coincide every 66 minutes (approx)

(a) loses 12 min (b) loses 14 min (c) loses 10 min (d) none of these

Soln. The hands coincide $65\frac{5}{11}$ minutes

which means it is slow by $\frac{6}{11}$

In 66 min, watch loses by $\frac{6}{11}$ minutes

Therefore, (24×60) min, watch loses by $=\frac{6}{11}\times24\times60\,\text{min}=11.90=12\,\text{min}\,(\text{approx.})$

(3) Required time and required angle in degree between hour hand and minute hand:

If Initial position of an hour hand is "H" and then the time at which both hour hand and minute hand make angle a degree in one hour is $\theta=\frac{1}{2}\left|60\times H-11M\right|$

EXERCISE

[LEVEL-I]

1. What was the day of the week on 17th June, 1998 ?
(a) Monday (b) Tuesday (c) Wednesday (d) Thursday

2. What was the day of the week on 28th may 2006 ?
(a) Thursday (b) Friday (c) Saturday (d) Sunday

3. On 8th Feb 2005 it was Tuesday. What was the day of the week on 8th Feb 2004 ?
(a) Tuesday (b) Monday (c) Sunday (d) Wednesday

4. The angle between the minute hand and hour hand of a clock when time is 4:20 is ?
(a) 0° (b) 10° (c) 5° (c) 20°

5. How much does a watch lose per day, if its hands coincide every 64 minute.
(a) $32\frac{8}{11}$ min (b) $36\frac{5}{11}$ min (c) 90 min (d) 96 min

6. At what time between the "4" and "5" o'clock will the hands of a watch point in opposite directions.
(a) 45 minute past 4 (b) 40 minute past 4 (c) $50\frac{4}{11}$ minute past 4 (d) $54\frac{6}{11}$ minute past 4

7. While a wall was being a wall clock got hedged and displaced in such a way be a workman that 2 o'clock in the afternoon, the hour hand was pointing roughly to the south west direction. Then at which point of time will the minute hand heat point towards the north east direction ?
(a) At around 2:40 p.m. (b) At 3:45 p.m. (c) At 4:10 p.m. (d) At 6:50 p.m.

8. The train for Lucknow leaves every two and half hours from New Delhi Railway Station. An announcement was made at the station that the train for Lucknow had left forty minutes ago and next train will leaves at 18:00 hours. At what time was the announcement made?
(a) 15:30 hours (b) 17:10 hours (c) 16:00 hours (d) None of these

9. If the seventh day of a month is three days earlier than Friday, what day it will be on the nineteenth day of a month?
(a) Sunday (b) Monday (c) Wednesday (d) Friday

10. A calendar year on the hypothetical planet. Dektron has a year of 1000 days. Each year is divided into 7-day weeks just like ours. If their New Year falls on a Sunday this year, what day will it be 55 years from now.
(a) Sunday (b) Monday (c) Friday (d) Saturday
(e) None of the above.

11. In a leap year, one of the months started with a Sunday. What is the shortest time gap between two such months? For example, if February and March both started on a Sunday, then the time gap is 1 month.
(a) 6 months (b) 3 months (c) 2 months (d) 1 month
(e) None of the above.

12. In any year, if April 1 is a Wendesday, then so is
(a) January 1 (b) July 1 (c) October 1 (d) December 1

13. On Planet X, a year ha 400 days with a leap year of 401 days every 4 years. Also, a year ending in '00' is a leap year only if the year is divisible by 400, e.g. 2000 is a leap year but 3000 is not. Such a calender is exact and needs no more corrections.
The length of the year on Planet X is
(a) 400.2425 days (b) 400.2475 days (c) 400.25 days (d) None of the above.

HINTS & SOLUTIONS

1. 17th June, 1998 = (1997 years + period from 1.1.1998 to 17.6.1998)
odd days in 1600 years = 0 odd days in 300 years = $5 \times 3 = 1$
It years has 24 leap years + 73 ordinary years.
Number of odd days in the 91 years = $24 \times 2 + 73 = 121 = 2$ odd days.
Jan + Feb + March + April + May + June
(31 + 28 + 31 + 30 + 31 + 17) = 168 days= 24 weeks= 0 odd day
Total number of odd day = 0 + 1 + 2 + 0 = 3 Given day is Wednesday
Answer is (c)

2. 28 may, 2006 = (2005 years + period from 1.1.2006 to 28.5.2006)
odd days in 1600 years = 0; odd days in 400 years = 0
5 years = (4 ordinary years + 1 leap years) = $(4 \times 1 + 1 \times 2)$ odd days = 6 odd days
Jan + Feb + March + April + May
(31 + 28 + 31 + 30 + 28) = 148 days= 21 weeks 1 day = 1 odd day
Total number of odd days = (0 + 0 + 6 + 1) = 7 = 0 odd day
Answer is (d)

3. The year 2004 is a leap year. It has two odd days
The day on 8th February 2004 is 2 days before the day on 8th February 2005. Hence this day is Sunday
Answer is (c)

4. Angle traced by hour hand in $\frac{13}{3}$ hrs $= \left(\frac{260}{12} \times \frac{13}{3}\right)^\circ = 130^\circ$

Angle traced by minute in 20 minute $= \left(\frac{360}{60} \times 20\right)^\circ = 120^\circ$

Required angle = 130 – 120 = 10°
Answer is (b)

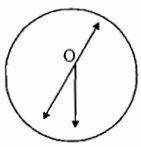
5. 55 minute spaces are covered in 60 minute. 60 minute spaces are covered in $\left(\frac{60}{55} \times 60\right) = 60\frac{6}{11}$ minute

Loss in 64 minute $= \left(65\frac{5}{11} - 64\right) = \frac{16}{11}$ minute. Loss in 2 hour $= \frac{16}{11} \times \frac{1}{64} \times 24 \times 60 = 32\frac{8}{11}$ minute

Answer is (a)

6. At 4 o'clock the hands of the watch are 20 minute spaces apart. To be in opposite directions, they must be 30 minute spaces apart. Minute hand will have to gain 50 minute spaces.
55 minute spaces are gained in 60 minute

50 minute spaces are gained in $\left(\frac{60}{55} \times 50\right) = 54\frac{6}{11}$ minute Answer is (d).

7. 

Answer is (a)

8. Answer is (d) 9. Answer is (c)
10. Answer is (b) 11. Answer is (b) 12. Answer is (b) 13. Answer is (a)

[LEVEL-II]

1. A watch which gains uniformly is 2 minutes low at noon on Monday and is 4 min. 48 sec fast at 2 pm on the following Monday. When was it correct?

Soln. Time from 12 pm on Monday to 2 pm on the following Monday = 7 days 2 hours = 170 hours.

Therefore, watch gains $\left(2+4\frac{4}{5}\right)\text{min} = \frac{34}{5}\text{min}$ in 170 hrs

Now, $\frac{34}{5}\text{min}$ are gained in 170 hrs.

2 min. gained in $\left(170\times\frac{5}{34}\times 2\right)\text{hrs} = 50\text{ hrs}$

Therefore, watch is correct 2 days 2 hrs. after 12 p.m. on Monday i.e., it will be correct at 2 p.m. on wednesday.

2. A clock loses 1% time during the first week and then gains 2% time during the next one week. If the clock was set right at 12 noon on a Sunday, what will be the time that the clock will show exactly 14 days from the time it was set right?

Soln. The clock loses 1% time during the first week.
In a day there are 24 hours and in a week there are 7 days. Therefore, there are 7×24 = 168 hours in a week.
If the clock loses 1% time during the first week, then it will show a time which is 1% of 168 hours less than 12 Noon a the end of the first week = 1.68 hours less.

Subsequently, the clock gains 2% during the next week. The second week has 168 hours and the clock gains 2% time = 2% of 168 hours = 3.36 hours more than the actual time.
As it lost 1.68 hours during the first week and then gained 3.36 hours during the next week, the net result will be a –1.68 + 3.36 = 1.68 hour net gain in time. So, the clock will show a time which is 1.68 hours more than 12 noon two weeks from the time it was set right. 1.68 hours = 1 hour and 40.8 minutes = 1 hour + 40 minutes + 48 seconds. That is 1 : 40 : 48 p.m.

3. Sumedha would like to complete all her homework before 10 p.m. in order to watch important final cricket match between India and Pakistan on TV. She has 40 minutes assignment in each of her five prepared subjects. What is the latest time at which she can start and still complete her homework in time for the final cricket match?

Soln. Time needed for Sumedha to complete all her 5 assignment = (40 × 5) miutes = 200 minutes
= 3 hours 20 minutes
Hence, the required time is 3 hours 20 minutes before 10 p.m. so, she can start work at 6.40 p.m.

4. Neetu exactly remembers that her brother's birthday is after twelfth but before the sixteenth of February whereas her sister correctly remembers that their brother's birthday is after fourteen but before twentieth february. On which day was definitely her brother's birthday?

Soln. According to Neetu her brother's birthday may be on 13th or 14th or 15th February.
According ot Neetu's sister her brother's birthday may be on 15th or 16th or 18th or 19th February.
Hence, common date is 15th February.

5. Which cannot be the last day of a century?

Soln. 100 years contain 5 odd days.
Therefore, last day of first century is Friday
200 years contain (5×2) = 10 ≡ 3 odd days.
Therefore, last day of second century is Wednesday.
300 years contain (5×3) = 15 ≡ 1 odd day.

Therefore, last day of third century is Monday
400 years contain 0 odd day.
Therefore, last day of fourth century is Sunday.
This cycle is repeated.
Therefore, last day of century cannot be Tuesday or Thursday or Saturday.

6. There are two clocks, both set to show the correct time at 10 pm. One clock gains one minute in an hour while the other gains 2 minute in one hour, then by how many minutes do the two clocks differe at 10 a.m. on the next day?
(a) 8 minutes (b) 10 minutes (c) 14 minutes (d) 12 minutes

Soln. Correct option is (d)

7. A watch, which gains uniform ally, was observed to be 5 minutes, slow at 10 a.m. on a Tuesday. On the next day at 11 a.m. it was noticed that watch was 5 minutes fast. When did the watch show the correct time?
(a) 10 : 15 p.m. Tuesday (b) 10 : 30 p.m. Tuesday
(c) 11 : 30 p.m. Tuesday (d) 12 : 30 p.m. Tuesday137

Soln. Correct option is (b)

CAREER ENDEAVOUR

CHAPTER

18

Series & missing characters

1. Series:

When we check any series in reasoning then we have to follow these special rules:

(1) Are the given series a special series or not.

e.g. (i) Series of prime numbers-2, 3, 5, 7, 11, ...?

Ans. 13

(ii) Series of N^2- 1, 4, 9, 16, 25, ...?

Ans. 36.

(iii) Series of $N^2 \pm N$ - 0, 2, 6, 12, 20, ...?

Ans. 30 (Here, put the value of N = 1, 2, 3, 4, 5 & 6 respectively and use negative sign.)

(iv) Series of $N^2 \pm N$ in alternate form-0, 2, 8, 14, 24, ...?

Ans. 34 (This series is based on N^2-1/N^2-2 form alternatively. As 1^2-1, 2^2-2, 3^2-1, 4^2-2, 5^2-1. Hence 6^2-2 is our answer)

(iv) $N^3/N^3 \pm N$ type series.

(v) Series based on sum of multiplication of previous terms-2, 3, 5, 8, 13, ...?

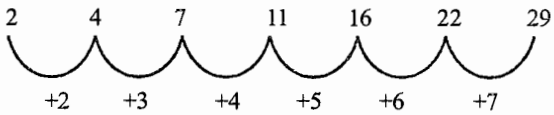
Ans. 21 (Every next term is based on sum of last two previous terms)

(vi) Series based on product of digit in given terms-66, 36, 18, ...?

Ans. 8 (Here every next terms is based on multiplication of digit in previous terms as $6\times6=36$, $3\times6=18$. Hence $1\times8=8$ our answer)

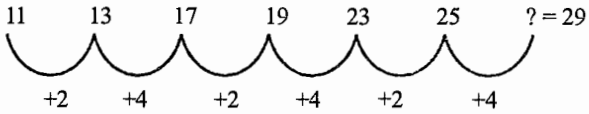
(2) If a given series is not a special series then to find our answer. We subtract every next term from his previous terms in each step. By this process we get our answer easily in two or three steps.

e.g. (i) 2, 4, 7, 11, 16, 22, ?



Answer is 29.

(ii) 11, 13, 17, 19, 23, 25, ?

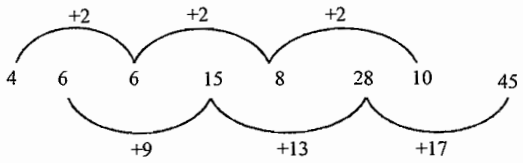


Answer is 29.

(3) Some times given series is based on mixup two series. In this form terms of written in alternative order.

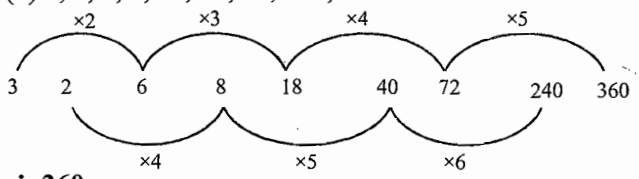
e.g. (i) 4, 6, 6, 15, 8, 28, 10, ?

Ans. (here the difference of terms are vary irregular and at a first look the series is not a special series. Hence, it is a mixed series.



Answer is 45.

(ii) 3, 2, 6, 8, 18, 40, 72, 240, ?



Answer is 360.

Identify next number in the series:

1. 4, 9, 20, 43, ...
   (a) 90 (b) 84 (c) 96 (d) 95
2. 4, 9, 19, 39, 79, ...
   (a) 169 (b) 159 (c) 119 (d) 139
3. 3, 2, 7, 6, 11, ...
   (a) 13 (b) 8 (c) 4 (d) 10
4. 6, 5, 24, 25, 144, ...
   (a) 155 (b) 160 (c) 170 (d) 175
5. 3, 6, 12, 15, 30, ...
   (a) 33 (b) 35 (c) 45 (d) 60
6. 0, 9, 3, 7, 6, 5, 9, 3, ?, 1
   (a) 6 (b) 1 (c) 12 (d) 8
7. 0, 2, 6, 12, ?
   (a) 20 (b) 36 (c) 24 (d) 18
8. 2, 5, 12, 27, ?
   (a) 54 (b) 58 (c) 36 (d) 40
9. 121, 225, 361, ?
   (a) 529 (b) 484 (c) 441 (d) 729
10. 9, 16, 30, 58, ?
    (a) 104 (b) 114 (c) 118 (d) 116

Identify wrong number in the series:

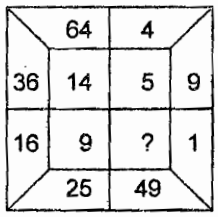
11. 336, 210, 120, 62, 24, 6, 0
    (a) 24 (b) 62 (c) 6 (d) 0
12. 1, 2, 9, 37, 65, 126, 217
    (a) 2 (b) 9 (c) 37 (d) 65
13. 0, 3, 8, 15, 27, 35, 48
    (a) 27 (b) 35 (c) 15 (d) 48

14. 6, 12, 24, 14, 24, 48, 24, 48, 96
(a) 6 (b) 12 (c) 24 (d) 14

15. 97, 86, 73, 58, 45, 34
(a) 58 (b) 73 (c) 34. (d) none of these

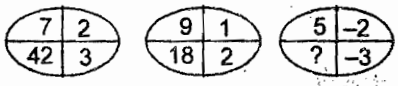
Graphical Missing Numbers:

16.



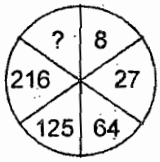
(a) 11 (b) 7 (c) 8 (d) 9

17.



(a) 18 (b) 13 (c) 30 (d) – 30

18.



(a) 729 (b) 343 (c) 305 (d) 4

Identify the Missing Term:

19. DKM, FJP, HIS, JHV, ____
(a) HGY (b) IGZ (c) IGY (d) LGY

20. AND, COF, FPI, JQM, ____
(a) ORS (b) ORR (c) QRS (d) PRT

21. A, Z, D, X, G, V, J, T, M, ____, ____
(a) S, N (b) P, R (c) N, S (d) R, P

22. B, D, G, K, P, _____
(a) V (b) W (c) T (d) S

Answer Key

Q	1	2	3	4	5	6	7	8	9	10
Ans.	a	b	d	d	a	c	a	b	a	b
Q	11	12	13	14	15	16	17	18	19	20
Ans.	b	c	a	d	d	c	c	b	d	b
Q	21	22								
Ans.	d	a								

CHAPTER

19

Syllogism

Syllogism

Directions : In each question below are three statements followed by two conclusions numbered I and II. You have to take the three given statements to be true even if they seem to be at variance from commonly known facts and then decide which of the given conclusions logically follows from the three statements disregarding commonly known facts.

Given answer (a) if only conclusion I is follows

Given answer (b) if only conclusion II is follows

Given answer (c) if either I or II is follows

Given answer (d) if neither conclusion I nor conclusion II are follows

Given answer (e) if both conclusion I and conclusion II are follows.

(01–02): Statments :

Some stars are moons

All moons are planets.

No planet is universe

1. **Conclusions:**

(I) All moons being stars is a possibility

(II) No universe is star.

2. **Conclusions:**

(I) At least some planets are stars.

(II) No moon is universe

(3–4): Statements:

All sticks are plants

All plants are insects

All insects are amphibians.

3. **Conclusions:**

(I) At least some ampibians are plants.

(II) All sticks are insects

4. **Conclusions:**

(I) All amphibians are sticks

(II) All plants are amphibians.

(5–6) : Statements:

All apartments are huts

No hut is a building

All buildings are cottages

Conclusions:
(I) No cottage is an apartment
(II) Some cottage are apartmetns

6. **Conclusions:**
(I) No apartment is a cottage
(II) Some building being apartment is a possibility

7. **Conclusions:**
(I) Some cottage being apartment is a possibility
(II) No cottage is a hut.

Directions: In each of question below are given four statements followed by three conclusions numbered I, II and III. You have to take the given statements to be true even if they seem to be at variance from commonly known facts. Read all the conclusions and then decide which of the given conclusions logically follows from the given statements disregarding commonly known facts

8. **Statements:**
All coins are glasses.
Some glasses are cups
All boxes are pins.
Conclusions:
(I) Some coins are cups
(II) Some pins are glasses.
(III) Some cups are pins
(a) None follows (b) Only I follows (c) Only III follows (d) Only II and III follow
(e) None of these

9. **Statements:**
Some pens are pencil
All pencils are caps
All caps are buses
Some buses are trains.
Conclusions:
(I) Some trains are caps
(II) Some pens are buses
(III) Some pencil are trains.
(a) Only I follows (b) Only II follows (c) Only I and III follow (d) none follows
(e) All I, II and III follow

10. **Statements:**
All shirts are skirts
All skirts are banks
All banks are roads
All roads are brushes
Conclusion:
(I) All banks are skirts
(II) All roads are banks
(III) Some brushes are shirt
(a) Only I follows (b) Only III follows (c) Only I and III follows (d) All I, II and III follow
(e) none of follows

Answer Key

Q.	1	2	3	4	5	6	7	8	9	10
Ans.	a	e	e	b	c	e	a	a	b	b

CHAPTER

20

CSIR-NET/JRF PREVIOUS YEARS' SOLVED PAPERS

CSIR-NET/JRF-Dec. 2012 Paper

SET-I | SOLVED

1. Which of the following numbers is the largets?

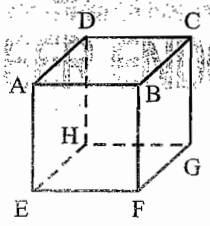
$2^{3^4}, 2^{4^3}, 3^{2^4}, 3^{4^2}, 4^{2^3}, 4^{3^2}$

(a) 2^{3^4} (b) 3^{4^2} (c) 4^{3^2} (d) 4^{2^3}

Soln. $2^{3^4} = 2^{81}$; $2^{4^3} = 2^{64}$; $3^{2^4} = 3^{16}$; $3^{4^2} = 3^{16}$, $4^{2^3} = 4^8 = 2^{16}$; $4^{3^2} = 4^9 = 2^{18}$

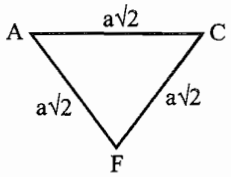
Correct answer is (a)

2. The cube ABCDEFGH in the figure has each edge equal to a. The area of the triangle with vertices atA, C and F is:



(a) $\frac{\sqrt{3}}{4}a^2$ (b) $\frac{\sqrt{3}}{2}a^2$ (c) $\sqrt{3}\,a^2$ (d) $2\sqrt{3}\,a^2$

Soln.



Area of equilated triangle ACF $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}\left(a\sqrt{2}\right)^2 = \frac{\sqrt{3}}{2}a^2$

Correct answer is (b)

3.

Soln.

4.

Soln.

5.

Soln.

6.

What is the number of distinct arrangements of the letters of the word UGCCSIR so that U and I cannot come together?

(a) 2520 (b) 720 (c) 1520 (d) 1800

Soln. Total number of arrangements $= \frac{7!}{2!} = 2520$

Total number of arrangment when UZ comes together $= \frac{6!}{2!} \times 2 = 720$

Desire arrangement $= 2520 - 720 = 1800$

Correct answer is (d)

4. Suppose the sum of the seven positive numbers is 21. What is the minimum possible value of the average of the squares of these numbers?

(a) 63 (b) 21 (c) 9 (d) 7

Soln. Lets number are a, b, c, d,e, f, g

Given: $a + b + c + d + e + f + g = 21$

We know that Arithmetic Progression $\geq$ Geometrical Progression.

Arithmetic Progression = Geometrical Progression (When all the numbers are equal)

Two or more positive numbers which sum is constant, then the square of numbers minimum when the numbers are equal.

$a + b + c + d + e + f + g = 21 \quad \Rightarrow \frac{a + b + c + d + e + f + g}{7} = 3$

For the given requirement $a = b = c = d = e = f = g = 3 \quad \Rightarrow \frac{a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2}{7} = 9$

Correct answer is (c)

Let $A = \frac{1^{13} + 2^{13} + 3^{13} + \ldots\ldots + 100^{13}}{100}, B = \frac{1^{13} + 3^{13} + 5^{13} + \ldots\ldots + 99^{13}}{50}, C = \frac{2^{13} + 4^{13} + 6^{13} + \ldots\ldots + 100^{13}}{50}$

Which of the following is true?

(a) $B < C < A$ (b) $A < B < C$ (c) $B < A < C$ (d) $C < A < B$

Soln. $50B = 1^{13} + 3^{13} + 5^{13} + \ldots\ldots + 99^{13}$ ... (i)

$50C = 2^{13} + 4^{13} + 6^{13} + \ldots\ldots + 100^{13}$ ... (ii)

Each term of (ii) is greater to term of (i)

Then $50C > 50B$. So, $C > B$

$100A = 1^{13} + 2^{13} + 3^{13} + \ldots\ldots 10^{13}$

$100A = 50B + 50C; \quad 2A = B + C$

Thus, $2A < 2B$ and $2A > 2C$

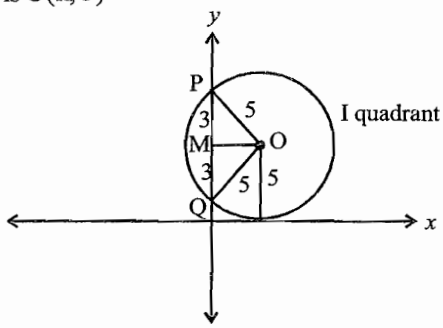
$\boxed{A < B} \qquad \boxed{A > C}$

Correct answer is (c)

A circle of radius 5 units in the XY plane has its centre in the first quadrant, touches the x-axis and has a chord of length 6 units on the y-axis. The coordinates of its centre are

(a) 4, 6 (b) 3, 5 (c) 5, 4 (d) 4, 5

m. Let co-ordinate of the centre is O(h, 5)



Applying Pythagoras theorem is in right angle triangle ΔQOM

$OQ^2 = OM^2 + MQ^2$; $5^2 = h^2 + 3^2$; $h^2 = 25 - 9$; $h = \pm 4$

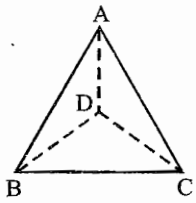
Cente is lie in first quadrant then negative value is not possible.

Correct anwer is (d)

7. A wire of length 6cm is used to make a tetrahedron of each edge 1m, using only one strand of wire for each edge. The minimum number of times the wire has to be cut is

(a) 2 (b) 3 (c) 1 (d) 0

Soln.



It required atlest a single cut. First wire ABCDB = 5 cm, second wire AD = 1 cm

Correct answer is (c)

. If the sum of the next two terms of the series below is x, what is the value of $\log_2 x$?

2, –4, 8, –16, 32, –64, 128, ...

(a) 128 (b) 10 (c) 256 (d) 8

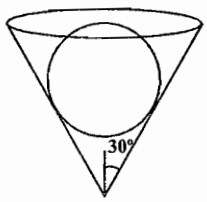
oln. It is a geometrical series

Next two terms are –256, 512

$x = -256 + 512 = 256$

$\log_2 x = \log_2 256 = \log_2 2^8 = 8 \log_2 2 = 8 \times 1 = 8$

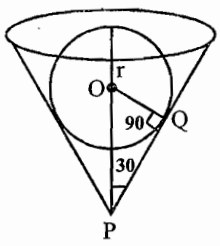
Hence, option (d) is correct.



A conical vessel with semi-vertical angle 30° and height 10.5 cm has a thin lid. A sphere kept inside it touches the lid. The radius of the sphere in cm is

(a) 3.5 (b) 5 (c) 6.5 (d) 7

Soln.



Let radius of sphere is 'r' then $OP = 10.5 - r$

In triangle ΔPOQ; $\sin 30 = \frac{OQ}{OP}$; $0.5 = \frac{r}{10.5 - r}$; $r = 3.5$

Correct answer is (a)

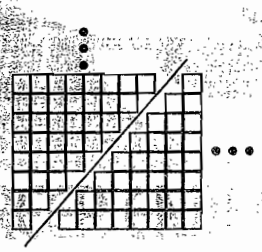
10. Amar, Akbar and Anthony are three friends, one of whom is a doctor, another is an engineer and the third is a professor. Amar is not an engineer. Akbar is the shortest. the tallest person is a doctor. The engineer's height is the geometric mean of the heights of the other two. Then which of the following is true?
(a) Amar is a doctor and he is the tallest (b) Akbar is a professor and he is the tallest
(c) Anthony is an engineer and he is shortest (d) Anthony is a doctor and he is the tallest

Soln. Correct answer is (a)

11. If 100 cats catch mice in 100 minutes, then how long will it take for 7 cats to catch 7 mice?
(a) 100/7 minutes (b) 100 minuts (c) 49/100 minutes (d) 7 minutes

Soln. Correct answer is (b)

12. What does this diagram demonstrate?



(a) $1+2+3+....+n = \frac{n(n+1)}{2}$ (b) $1^2+2^2+3^2+....+n^2 = \frac{n(n+1)(2n+1)}{6}$

(c) $1+3+.....+(2n+=-1) = n^2$ (d) $2^2+4^2+....+(2n)^2 = \frac{2.n(n+1)(2n+1)}{3}$

Soln. The each half diagram demonstrate gradually increasing of unit square by one unit.
Correct answer is (a)

13. Suppose therea are socks of N different colors in box. If you take out one sock at a time, what is the maximum number of socks that you have to take out before a matching pair is found? Assume that N is na even number.
(a) N (b) N + 1 (c) N–1 (d) N/2

Soln. Socks in pair atleast 'n+1' number of socks would be taken out. But before matching pair N socks are required.
Correct answer is (a)

14. At what time after 4 O' clock, the hour and the minute hands will lie opposite to each other?
(a) 4–50'–30" (b) 4–52'–51" (c) 4–53'–23" (d) 4–54'–33"

Correct answer is (d)

15. Which of the following curves just touches the 'x' axis?

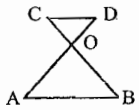
(a) $y = x^2 - x + 1$ (b) $y = x^2 - 2x + 2$ (c) $y = x^2 - 10x + 25$ (d) $y = x^2 - 7x + 12$

Soln. Curve touch the 'x' axis where $y = 0$ and for that condition atleast a real 'x' should be exists.

Equation, $y = x^2 - 10x + 25 = 0$, $(x-5)^2 = 0$, $x = 5$.

Correct answer is (c)

16.



If AB is parallel to CD and AO = 2OD, then the area of triangle OAB is bigger than the area of triangle OCD by a factor of

(a) 1 (b) 3 (c) 4 (d) 8

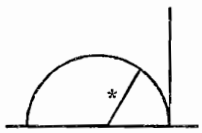
Soln. Let OD = x; CD || AB. Thus, $\angle COD = \angle AOB$ & $\angle DCO = \angle ABO$

ΔOCD & ΔOBA are symmetric triangle.

Thus, $\dfrac{\text{Area of triangle } \Delta OAB}{\text{Area of triangle } \Delta OCD} = \left(\dfrac{2}{1}\right)^2 = 4$

Correct answer is (c).

17.



A semi-circular arch of radius R has a vertical pole put on the ground together with one of its legs. An ant on the top of the arch finds the angular height of the tip of the pole to be 45°. The height of the pole is

(a) $\sqrt{2}$ R (b) $\sqrt{3}$ R (c) $\sqrt{4}$ R (d) $\sqrt{5}$ R

Soln. **Correct answer is (c)**

18. Suppose we make N identical smaller spheres from a big sphere. The total surface area of the smaller spheres is X times the total surface area of the big sphere, where X is

(a) $\sqrt{N}$ (b) 1 (c) $N^{1/3}$ (d) N^3.

Soln. Let big sphere is 'R' and small sphere radius is 'r'.

Thus, $\dfrac{4}{3}\pi R^3 = N\dfrac{4}{3}\pi r^3$; $R^3 = Nr^3$; $\left(\dfrac{r}{R}\right)^3 = \dfrac{1}{N}$

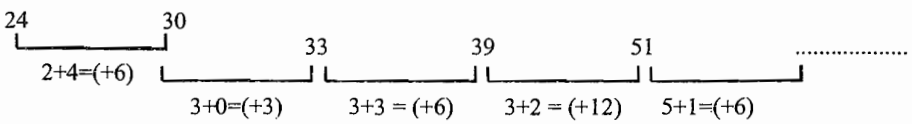
$$4\pi R^2 X = N\left(4\pi r^2\right);\ R^2 X = Nr^2;\ X = N\left(\frac{r}{R}\right)^2 = N\left(N^{-2/3}\right) = N^{1/3}$$

Correct answer is (c)

19. What is the next number in the sequence 24, 30, 33., 39, 51, ...?

(a) 57 (b) 69 (c) 54 (d) 81

Soln.



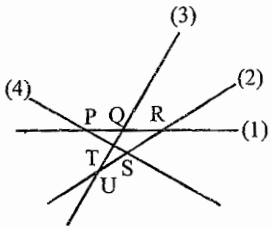
Thus the next number must be $51 + 6 = 57$

Correct answer is (a)

20. Four lines are drawn on a plane with no two parallel and no three concurrent. Lines are drawn joining the points of intersection of the previous four lines. The number of new lines obtained this way is:

(a) 3 (b) 5 (c) 12 (d) 2

Soln.



New lines, PU, QS, RT

Correct answer is (a)

SET-II [SOLVED]

. A granite block of 2m × 5m ×3 m size is cut into 5 cm thick slabs of 2m×5m size. These slabs are laid over a 2 m wide pavement. What is the length of the pavement that can be covered with these slabs?

(a) 100 m (b) 200 m (c) 300 m (d) 500 m

oln. Let number of slabs is n.

Thus, $2\times5\times3 = n\times\frac{5}{100}\times2\times5; \quad \therefore \ n = 60$

Area covered by 60 slabs = $60\times2\times5\ m^2$.

Length of pavement = $\frac{60\times2\times5\ m^2}{2\ m} = 300\ m$

Correct option is (c)

Which is the least among the following?

$0.33^{0.33}$, $0.44^{0.44}$, $\pi^{-1/\pi}$, $e^{-1/e}$

(a) $0.33^{0.33}$ (b) $0.44^{0.44}$ (c) $\pi^{-1/\pi}$ (d) $e^{-1/e}$

In. $(0.33)^{0.33} = \left(\frac{1}{3}\right)^{1/3}$; $(0.44)^{0.44} = \left(\frac{4}{9}\right)^{4/9}$; $(\pi)^{-1/\pi} = \left(\frac{1}{\pi}\right)^{1/\pi}$; $(e)^{-1/e} = \left(\frac{1}{e}\right)^{1/e}$

All terms are of the form: $y = x^x$

$\Rightarrow \quad \ell ny = x\ell nx \quad \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ell nx + 1 \quad \Rightarrow \frac{dy}{dx} = x^x(\ell nx + 1)$

For maxima or minima, $\frac{dy}{dx} = 0 \quad \Rightarrow \ell nx + 1 = 0 \quad \Rightarrow x = \frac{1}{e}$

Since, $\left.\frac{d^2y}{dx^2}\right|_{x=\frac{1}{e}}$ = positive

So, $f(x) = x^x$ has minima at $x = \frac{1}{e}$

Correct option is (d)

What is the next number in this "see and tell" sequence?

1 11 21 1211 111221 _____

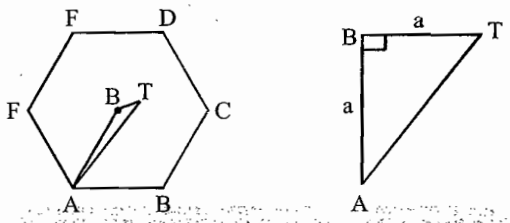
(a) 312211 (b) 1112221 (c) 1112222 (d) 1112131

Soln. **Correct option is (a)**

4. A vertical pole of length 'a' stands at the centre of a horizontal regular hexagonal ground of side 'a'. A rope that is fixed taut in between a vertex on the ground and the tip of the pole has a length.

(a) a (b) $\sqrt{2}$ a (c) $\sqrt{3}$ a (d) $\sqrt{6}$ a

Soln. Using pythogorus theorem,



$AT^2 = AB^2 + BT^2$; $AT^2 = 2a^2$; $AT = \sqrt{2}\,a$

Correct option is (b).

5. A peacock perched on the top of a 12 m high tree spots a snake moving towards its hole at the base of the tree from a distance equal to thrice the height of the tree. The peacock flies towards the snake in a straight line and they both move at the same speed. At what distance from the base of the tree will the peacock catch the snake?

(a) 16 m (b) 18 m (c) 14 m (d) 12 m

Soln. Distance travelled by peacock and snake must be same.

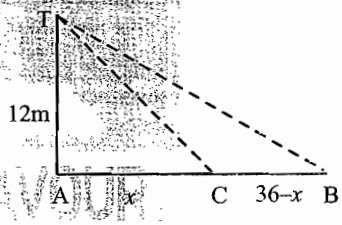
$TC = BC$

$12^2 + x^2 = (36 - x)^2$

$144 + x^2 = 1296 - 72x + x^2$

$72x = 1152$; $x = 16$

Correct option is (a).



6. The cities of a country are connected by intercity roads. If a city is directly connected to an odd number of other cities, it is called an odd city. If a city is directly connected to an even number of other cities, it is called an even city. Then which of the following is impossible?

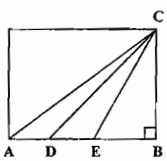
(a) There are an even number of odd cities (b) There are an odd number of odd cities.

(c) There are an even number of even cities (d) There are an odd number of even cities.

Soln. **Correct option is (b).**

7. In the figure $\angle ABC = \frac{\pi}{2}$, $AD = DE = EB$

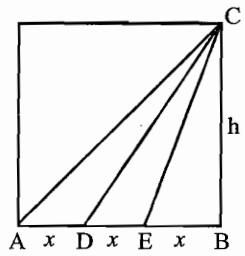
What is the ratio of the area of triangle ADC to that of triangle CDB?



(a) 1:1 (b) 1:2 (c) 1:3 (d) 1:4

Soln. Area of ΔADC (Pythagorus theorem) = area of ABC – area of DBC

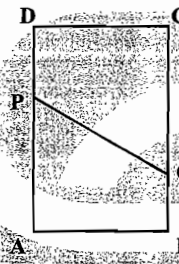
$$= \frac{1}{2}h(2x-x) = \frac{1}{2}hx$$



Area of $\Delta CDB = \frac{1}{2}(2x)h = xh$

Correct option is (b).

8. A rectangular sheet ABCD is folded in such a way that vertex A meets vertex C, thereby forming a line PQ. Assuming AB = 3 and BC = 4, find PQ. NOte that AP = PC and AQ = QC.



(a) 13/4 (b) 15/4 (c) 17/4 (d) 19/4

Soln. **Correct option is (b).**

9. A string of diameter 1 mm is kept on a table in the shape of a close flat spiral i.e. a spiral with no gap between the turns. The area of the table occupied by the spiral is 1 m^2. Then the length of the string is

(a) 10 m (b) 10^2 m (c) 10^3 m (d) 10^6 m.

Soln. Let the length of string is l m.

Thus, $(\ell)\left(\frac{1}{1000}\right) = 1 \qquad \therefore \ \ell = 1000$ m

Correct option is (c)

10. 25% of 25% of a quantity is x% of the quantity where 'x' is

(a) 6.25% (b) 12.5% (c) 25% (d) 50%

Soln. Let the given quantity is y.

$$\frac{25}{100}x\frac{25}{100}y = \left(\frac{X}{100}\right)y; \ \frac{25}{4} = X \qquad \therefore \ X = 6.25$$

Correct option is (a)

In sequence $\{a_n\}$ every term is equal to the sum of all its previous terms.

If $a_0 = 3$, then $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ is:

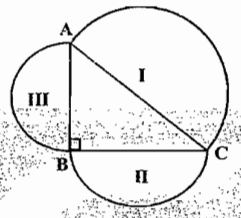
(a) 3 (b) 2 (c) 1 (d) e

Soln. $a_0 = 3;\ a_1 = a_0 = 3;\ a_2 = a_1 + a_0 = 6;\ a_3 = a_0 + a_1 + a_2 = (a_0 + a_1) + a_2 = 12.$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{a_n + (a_0 + a_1 + a_3 + \ldots\ldots a_{n-1})}{a_n} = \lim_{n\to\infty} \frac{a_n + a_n}{a_n} = \lim_{n\to\infty} \frac{2a_n}{a_n} = 2$$

Correct option is (b).

2. In the figure below, angle ABC = π/2. I, II, III are the areas of semicircles on the sides opposite angles B, A, and C respectively. Which of the following is always true?



(a) $II^2 + III^2 = I^2$ (b) II + III = I (c) $II^2 + III^2 > I^2$ (d) II + III < I

Soln. Let length of AB is k and length of BC is h.

Thus, $AC = \sqrt{h^2 + k^2}$

$$\text{Area of I} = \frac{\pi}{2}\left(\frac{\sqrt{h^2+k^2}}{2}\right) = \frac{\pi}{2}\left(\frac{h^2+k^2}{4}\right)$$

$$\text{Area of II} = \frac{\pi}{2}\left(\frac{h}{2}\right)^2 = \frac{\pi}{2}\left(\frac{h^2}{4}\right)$$

$$\text{Area of III} = \frac{\pi}{2}\left(\frac{k}{2}\right)^2 = \frac{\pi}{2}\left(\frac{k^2}{4}\right)$$

$$\text{Area of II + area of III} = \frac{\pi}{2}\left(\frac{h^2}{4} + \frac{k^2}{4}\right) = \frac{\pi}{2}\left(\frac{h^2+k^2}{4}\right) = \text{area of I}$$

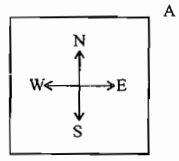
Correct option is (b).

3. What is the minimum number of days between one Friday the 13th and the next Friday the 13th? (Assume that the year is a leap year).

(a) 28 (b) 56 (c) 91 (d) 84

Soln. Correct option is (c)

4. Suppose a person A is at the North-East corner of a square (see figure below). From that point he moves along the diagonal and after covering 1/3rd portion of the diagonal, he goes to his left and after sometime he stops, rotates 90° clockwise and moves straight. After a few minutes he stopes, rotates 180° anticlockwise. Towards which direction he is facing now?



(a) North-East (b) North-West (c) South-East (d) South-West

Soln. **Correct option is (a)**

15. Cucumber contains 99% water. Ramesh buys 100 kg of cucumbers. After 30 days of storing, the cucumbers lose some water. They now contain 98% water. What is the total weight of cucumbers now?
(a) 99 kg (b) 50 kg (c) 75 kg (d) 2 kg.

Soln.

	Water	Cucumber	Total weight
At the time of purchasing	(99%) 99 kg	(1%) 1 kg	100 kg
After 30 days	(98%) 49 kg	(2%) 1 kg	50 kg

Correct option is (b)

16. In a museum there were old coins with their respective years engraved on them, as follows
(a) 1837 AD (b) 1907 AD (c) 1947 AD (d) 200 BC

Soln. Option (d) is wrong because coins are not invented in 200 BC.
Correct option is (d).

17. A student observes the movement of four snails and plots the graphs of distance moved as a function of time as given in figure (A), (B), (C) and (D).

Which of the following is not correct?
(a) Graph (A) (b) Graph (B) (c) Graph (C) (d) Graph (D)

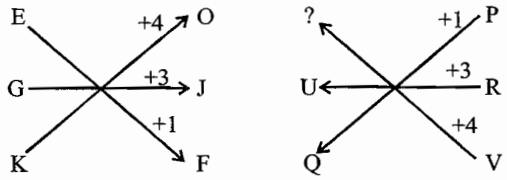
Soln. For a given time more than one distance as in graph C is not possible
Correct option is (c).

18. Find the missing letter:

A	EGK	C
?		P
U		R
Q		V
B	OJF	D

(a) H (b) L (c) Z (d) Y

Soln. Letters are arranged as



So, the required letter is (22 + 4) letter of english alphabet i.e. Z.

Correct option is (c).

19. Consider the following equation

$x^2+4y^2+9z^2=14x+28y+42z-147$

where x, y and z are real numbers. Then the value of $x+2y+3z$ is

(a) 7 (b) 14 (c) 21 (d) Not unique.

Soln. $x^2+4y^2+9z^2=14x+28y+42z-147$

$x^2-14x+4y^2-28y+9z^2-42z+147=0$

$\Rightarrow (x^2-2.x.7+49)+(4y^2-2.2y.7+49)+(9z^2-2.3z.7+49)=0$

$\Rightarrow (x-7)^2+(2y-7)^2+(3z-7)^2=0$

Thus, each term must be separately zero.

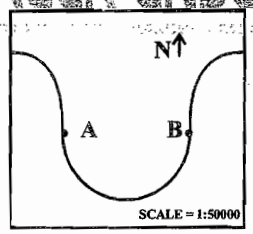
$x-7=0 \Rightarrow x=7;\ 2y-7=0 \Rightarrow y=1;\ 3z-7=0 \Rightarrow z=\frac{7}{3}$

$x+2y+3z=7+2\left(\frac{7}{2}\right)+3\left(\frac{7}{3}\right)=21$

Correct option is (c).

20. The map given below shows a meandering river following a semi-circular path, along which two villages are located at A and B, the distance between A and B along the east-west direction in the map is 7 cm. What is the length of the river between A and B in the ground?

CAREER ENDEAVOUR



(a) 1.1 km (b) 3.5 km (c) 5.5 km (d) 11.0 km

Soln. Actual distance between A and B = (7×50000) cm = 3.5 km

Length of the river from A to B in the ground $=\pi\times\left(\frac{3.5}{2}\right)$ km = 5.5 km

Correct option is (c)

CSIR-NET/JRF -JUNE 2013 Paper

1. During an evening party, when Ms. Black, Ms. Brown and Ms. White met, Ms. Brown remarked, "it is interesting that our dresses are white, black or brown, but for each of us the name does not match the colour of the dress!". Ms. White replied, "But your white dress does not suit you!". Pick the correct answer

(a) Ms White's dress was brown (b) Ms. black's dress was white.

(c) Ms. White's dress was black (d) Ms. Black's dress was black.

Soln. (a) Miss Brown – white

(b) Miss White – Black

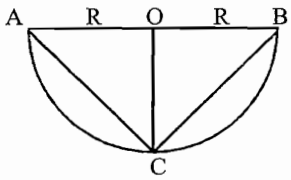
(c) Miss Black – Brown

Correct answer is (c)

2. Of all the triangles that can be inscribed in a semicircle of radius R with the diameter as one side, the biggest one has the area

(a) R^2 (b) $R^2\sqrt{2}$ (c) $R^2\sqrt{3}$ (d) $2R^2$

Soln.



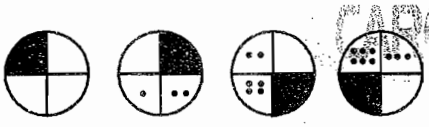
$$\text{Area of triangle} = \frac{1}{2} \times AB \times OC = \frac{1}{2} \times 2R \times R = R^2$$

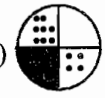
Correct answer is (a)

3. A square pyramid is to be made using a wire such that only one strand of wire is used for each edge. What is the minimum number of times that the wire has to be cut in order to make the pyramid?

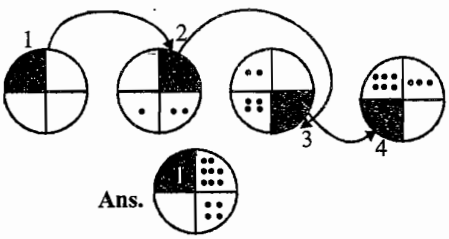
(a) 3 (b) 7 (c) 2 (d) 1

Soln. Correct answer is (d)



(a) (b) (c) (d) 

Soln.

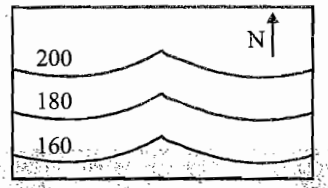


Correct answer is (a)

5. In a customer survey conducted during Monday to Friday, of the customers who asked for child care facilites in super markets, 23% were men and the rest, women. Among them, 19.9% of the women and 8.8% of the men were willing to pay for the facilities.
(A) What is the ratio of the men to women customers who wanted child care facilities?
(B) If the survey had been conducted during the weekend instead, how will the result change?
With the above data,
(a) Only A can be answered (b) Only B can be answered
(c) Both A and B can be answered (d) Neither A nor B can be answered.

Soln. Correct answer is (a)

6. The map given below shows contour lines which connect points of equal ground surface elevation in a region. Inverted 'V' shaped portions of contour lines represent a valley along which a river flows. What is the down-stream direction of the river?

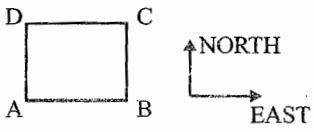


(a) North (b) South (c) East (d) West

Soln. Correct answer is (b)

7. During a summer vacation, of 20 friends from a hostel, each wrote a letter to each of all others. The total number of letters written was
(a) 20 (b) 400 (c) 200 (d) 380

Soln. Every student will write 19 letters to his friends.
Hence, number of letters = $20 \times 19 = 380$
Correct answer is (d)

8. 

A person has to cross a square field by going from A to C. The person is only allowed to move towards the east or towards the north or use a combination of these movements. The total distance travelled by the person
(a) depends on the length of each step
(b) depends on the total number of steps
(c) is different for different paths
(d) is the same for all paths.

Soln. Correct answer is (d)

9. A crow is flying along a horizontal circle of radius R at a height R above the horizontal ground. Each of a number of men on the ground found that the angular height of the crow was a fixed angle $\theta(<45°)$ when it was closest to him. Then all these men must be on a circle on the ground with a radius.
(a) $R + R\sin\theta$ (b) $R + R\cos\theta$ (c) $R + R\tan\theta$ (d) $R + R\cot\theta$

Soln. Correct answer is (d)

10. How many pairs of positive integers have gcd 20 and lcm 600?
(gcd = greatest common divisor, lcm = least common multiple)
(a) 4 (b) 0 (c) 1 (d) 7

Soln. Since, GCD = 20 and LCM = 600

Hence, number = 20a and 20b $\Rightarrow$ 20a× 20b = 20×600 = ab = 30

Hence, number of pair = (1, 30) (2, 15) (3, 10) (5, 6)

Correct answer is (a)

11. Two integers are picked at random from the first 15 positive integers without replacement. What is the probability that the sum of the two numbers is 20?

(a) $\frac{3}{4}$ (b) $\frac{1}{21}$ (c) $\frac{1}{105}$ (d) $\frac{1}{20}$

Soln. $n(s) = {}^{15}C_2 = \frac{15\times14}{2\times1} = 105$

n(E) = sum of number is 20 {(15, 5) (14, 6) (13, 7) (12, 8) (11, 9)}

n(E) = 5, $\Rightarrow$ $P(E) = \frac{5}{105} = \frac{1}{21}$

Correct answer is (b)

12. A daily sheet calendar of the year 2013 contains sheets of 10×10 cm size. All the sheets of the calendar are spread over the floor of a room of 5m×7.3m size. What percentage of the floor will be covered by these sheets?

(a) 0.1 (b) 1 (c) 10 (d) 100

Soln. Area of all daily sheet = 365×10×10 cm².

Area of floor = 500×730 cm².

Hence, percentage = $\frac{365\times10\times10}{500\times730}\times100 = 10\,\%$

Correct answer is (c)

3. How many rectangles (which are not squares) are there in the following figure?

(a) 56 (b) 70 (c) 86 (d) 100

oln. Number of rectangle = $\Sigma n^3 = 4^3 + 3^3 + 2^3 + 1^3 = 100$

Number of squares = $\Sigma n^2 = 4^2 + 3^2 + 2^2 + 1^2 = 30$

Hence number of rectangles = 100–30 = 70

Correct answer is (b)

4. Define $a \otimes b = \text{lcm}(a,b) + \text{gcd}(a,b)$ and $a \oplus b = a^b + b^a$. What is the value of $(1\oplus2)\otimes(3\oplus4)$? Here lcm = least common multiple and gcd = greatest common divisor.

(a) 145 (b) 286 (c) 436 (d) 572

oln. $1\oplus2 = 1^2 + 2^1 = 1+2 = 3$

$3\oplus4 = 3^4 + 4^3 = 31+64 = 145$

$145\otimes3 = \text{LCM}(145,3) + \text{GCD}(145,3) = 435+1 = 436$

Correct answer is (c).

. There is a an equilateral triangle in the XY plane with its centre at the origin. The distance of its sides from the origin is 3.5 cm. The area of its circumcircle in cm² is:

(a) 38.5 (b) 49 (c) 63.65 (d) 154

Soln. **Correct answer is (a)**

16. What is the value of $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots$ to ∞

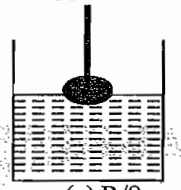
(a) 2/3 (b) 1 (c) 2 (d) ∞

Soln. $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots\ldots\ldots\ldots+\infty$

$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\ldots\ldots\ldots\ldots+\left(\frac{1}{\infty-1}-\frac{1}{\infty}\right)$ $\Rightarrow$ $1-\frac{1}{\infty}=1-0=0$

Correct answer is (b)

17. A sphere of iron of radius R/2 fixed to one end of a string was lowered into water in a cylinderical container of base radius R to keep exactly half the sphere dipped. The rise in the level of water in the container will be



(a) R/3 (b) R/4 (c) R/8 (d) R/12

Soln. Volume of hemisphere $=\frac{2}{3}\pi\left(\frac{R}{2}\right)^3=\frac{2}{3}\pi\frac{R^3}{8}=$ volume of water

Then increased in height $=\pi R^2H=\frac{2}{3}\pi\frac{R^3}{8}$

$\therefore$ $H=\frac{R}{12}$

Correct answer is (d)

18. Choose the largest number

(a) 2^{500} (b) 3^{400} (c) 4^{300} (d) 5^{200}

CAREER ENDEAVOUR

Soln. $2^{500}, 4^{300}$ $3^{400}, 5^{200}$

$2^{500}, \left(2^2\right)^{300}$ $3^{399}\times3^1, \left(5^3\right)^{100}$

$2^{500}, 2^{600}$ $\left(3^3\right)^{133}\times3^1, 25^{100}$

$2^{500}<4^{300}$ $(27)^{133}\times3^1>(25)^{100}$

$3^{400}, 4^{300}$ $3^{400}>5^{200}$

$\left(3^2\right)^{200}, \left(2^2\right)^{300}$

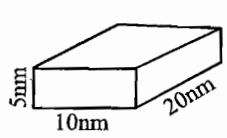
$9^{200}, 2^{600}=\left(2^3\right)^{200}$

$9^{200}>8^{200}$

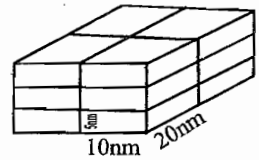
$2^{500} < 4^{300} < 3^{400} > 5^{200}$

Correct answer is (b)

19. A crystal grows by stacking of unit cells of 10×20×5 nm size as shown in the diagram given below. How many unit cells will make a crystal of 1 cm^3 volume?



Unit cell (not to scale)



Crystal (not to scale)

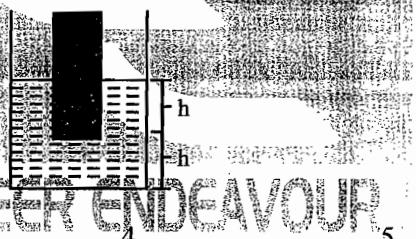
(a) 10^6 (b) 10^9 (c) 10^{12} (d) 10^{18}

Soln. $1\ m^3 = 10^9$ nm
$100\ cm = 10^9$ nm
$10^2\ cm = 10^9$ nm
$1\ cm = 10^7$ nm
Volume of crystal = 1cm × 1 cm × 1 cm

Hence, number of unit cell $= \frac{10^6 \times 10^6 \times 10^6}{5 \times 10 \times 20} = 10^{18}$

Correct answer is (d)

20. A solid cylinder of basal area A was held dipped in water in a cylindrical vessel of basal area 2A vertically such that a length 'h' of the cylinder is immersed. The lower tip of the cylinder is at a height 'h' from the base of the vessel. What will be the height of water in the vessel when the cylinder is taken out?



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(a) 2 h (b) $\frac{3}{2}$h (c) $\frac{4}{3}$h (d) $\frac{5}{4}$h

Soln. Volume of small cylinder sinked in water $= \pi R^2 \times H = A \times h = Ah$

Height of this water in dinder (Big) $= \pi r^2 \times H^1 = Ah$

$\Rightarrow 2A \times H^1 = Ah \quad \Rightarrow H^1 = \frac{h}{2}$

Hence, decreased in water level $= 2h - \frac{h}{2} = \frac{3}{2}h$

Correct answer is (b)

CSIR-NET/JRF -DEC 2013 Paper

Solved

1. Three fishermen caught fishes and went to sleep. One of them woke up, took away one fish and 1/3rd of the remainder as his share, without others knowledge. Later, the three of them divided the remainder equally. How many fishes were caught?
(a) 58 (b) 19 (c) 76 (d) 88

Soln. We will solve it by hit and trail method. According to condition after subtracting 1, remaining number should be exactly divisible by 3 and also $\frac{2}{3}$ rd part of number again divisible by 3. From option (2)

$$19-1=\frac{18}{3}=6 \text{ and } (18-6)=\frac{12}{3}=4$$

Correct answer is (b)

2. What is the arithmetic mean of $\frac{1}{1\times2},\frac{1}{2\times3},\frac{1}{3\times4},\frac{1}{4\times5},\ldots\ldots,\frac{1}{100\times101}$?

(a) 0.01 (b) $\frac{1}{101}$ (c) 0.00111... (d) $\frac{\frac{1}{49\times50}+\frac{1}{50\times51}}{2}$

Soln. The given number can be written as

$$\left(1-\frac{1}{2}\right),\left(\frac{1}{2}-\frac{1}{3}\right),\left(\frac{1}{3}-\frac{1}{4}\right),\ldots\ldots\left(\frac{1}{100}-\frac{1}{101}\right)$$

$$\text{Hence, Average} = \frac{1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots\ldots+\frac{1}{100}-\frac{1}{101}}{100}$$

$$\Rightarrow \left(1-\frac{1}{101}\right)\div100=\frac{100}{101\times100}=\frac{1}{101}$$

Correct answer is (b)

3. Every time a ball falls to ground, it bounces back to half the height it fell from. A ball is dropped from a height of 1024 cm. The maximum height from the ground to which it can rise after the tenth bounce is
(a) 102.4 cm (b) 1.24 cm (c) 1 cm (d) 2 cm

Soln. The question is related to G.P.

If ball is dropped from height of 1024 cm, then after bounce it will jumped $1024\times\frac{1}{2}=512$, on second jump $=512\times\frac{1}{2}=256$, on third $=256\times\frac{1}{2}=128$ ..... and on 10th bounce $=2\times\frac{1}{2}=1\,cm$

Correct answer is (c)

4. A circle of radius 7 units lying in the fourth quadrant touches the x-axis at (10, 0). The centre of the circle has coordinates.
(a) (7, 7) (b) (–10, 7) (c) (10, –7) (d) (7, –7)

Soln. Since, circle is located at 4th quardrant. Hence value of 'x' is negative and y is positive. Therefore, only (c) option satisfies the condition.

Correct answer is (c)

5. A cylinder of radius 1 cm and height 1 cm is broken into three pieces. Which of the following MUST be true?
(a) At least one pieces has volume equal to 1 cm^3.
(b) At least two pieces have equal volumes.
(c) At least one piece has volume less than 1 cm^3.
(d) At least one piece has volume greater than 1 cm^3.

Soln. Radius of cylinder = 1 cm
height = 1 cm

Hence, volume $= \pi \times 1^2 \times 1 = 3.14\ cm^3$

We can divide it 3 equal part then volume of one part $= \frac{3.24}{3} = 1.046\ cm^3$

Then, it is clear that at least one piece has volume greater than 1 cm^3.

Correct answer is (d)

6. One of the four A, B, C and D committed a crime. A said, "I did it," B said, "I didn't." C said, "B did it." D said, "A did it." Who is lying?
(a) A (b) B (c) C (d) D

Soln. The statement of A is satisfied by B and D but not satisfied by C. So, C is lying.

Correct answer is (c)

7. Consider the sequence of ordered sets of natural numbers:
{1}, {2, 3}, {4, 5, 6}, ...
What is the last numbers in the 10^{th} set?
(a) 10 (b) 19 (c) 55 (d) 67

Soln. Since first term contain one digit, second term contain two digit, 3rd three.

Hence, it is A.P. i.e. 1, 2, 3, ...10^{th} term

then $S_{10} = \frac{10}{2}\left[2\times1+(10-1)1\right] = \frac{10}{2}\times11 = 55$

Hence, last digit of 10^{th} term = 55

Correct answer is (c)

8. If $a+b+c+d+e = 10$ (all positive numbers), then the maximum value of $a\times b\times c\times d\times e$ is
(a) 12 (b) 32 (c) 48 (d) 72

Soln. If we take different values of a, b, c, d, e according to condition. We see that the value is maximum when value of all are equal as $2\times2\times2\times2\times2 = 32$

Correct answer is (b)

9. A farmer gives 7 full, 7 half-full and 7 empty bottles of honey to his three sons and asks them to share these among themselves such that each of them gets the same amount of honey and the same number of bottles. In how many ways can this be done? (bottles cannot be distinghuished otherwise, they are sealed and cannot be broken)
(a) 0 (b) 1 (c) 2 (d) 3

Soln. Every son get 7 bottle and 3.5 unit honey.

I	II	III
1/2	1/2	1/2
1	1	1
1	1	1
1	1/2	1/2
0	1/2	1/2
0	0	0
0	0	0

Since II and III are same. So, distribution is possible only 3 different ways.

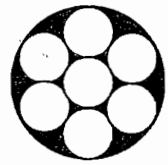
Correct answer is (d)

10. A car is moving aloing a straight track. Its speed is changing with time as shown above.
Which of the following statement is correct?

(a) The speed is never zero
(b) The acceleration is zero once on the path
(c) The distance covered initially increases and then decerases.
(d) The car comes back to its initial position once.

Soln. Since graph firstly increasing and then decreased. Then it is clear that the acceleration is zero once on the path.
Correct answer is (b)

11. A circle circumscribes identical, close packed circles of unit diamter as shown in the figure above. What is the total area of the shaded portion?



(a) 2 (b) 2π (c) $1/2$ (d) $\pi/2$

Soln. Radius of circum circle $= \frac{1}{2}+1=\frac{3}{2}$ unit

Radius of smaller circle = 1/2

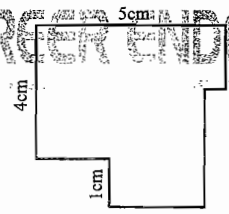
Since, smaller circle are identical. Then area of circles must be same.

Then $\text{Area}_c = \pi\times\left(\frac{1}{2}\right)^2\times 7=\frac{7\pi}{4}$ and $\text{Area}_{cc} = \pi\times\left(\frac{3}{2}\right)^2=\frac{9\pi}{4}$

Hence, shaded portion $= \frac{9\pi}{4}-\frac{7\pi}{4}=\frac{2\pi}{4}=\frac{\pi}{2}$

Correct answer is (d)

12. What is the perimeter of the given figure as above, where adjacent sides are at right angles to each other?



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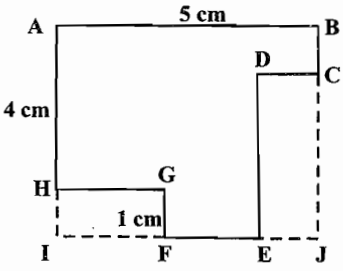
(a) 20 cm (b) 18 cm (c) 21 cm (d) cannot be determined.

Soln.

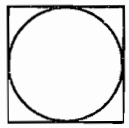


AB = IF + FE + EJ = IJ (since, CD = EJ)
AH + HI = AI = BC + CJ = BJ (since, DE = CJ
AB = IJ = AI = BJ = 5cm (since, HG = IF)

Hence, perimeter = 20

from given figure.
Correct answer is (a)

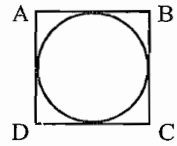
13. What does the diagram above establish?



Note: The diagram is a circle inside a square.

(a) $\pi > 3$ (b) $\pi \geq 2\sqrt{2}$ (c) $\pi < 4$ (d) π is closer to 3 than to 4.

Soln. Let side of square = 'a' unit



then radius of circle = 9/2 unit

Hence, area of square = a^2

and area of circle = $\pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

Since, circle is incircle. Hence, area of square > area of circle.

$$a^2 > \frac{\pi a^2}{4} \quad \Rightarrow a^2 > \pi a^2 \quad \Rightarrow 4 > \pi$$

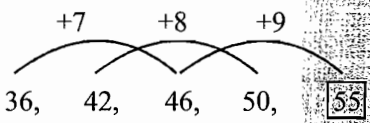
Correct answer is (c)

14. What is the next number in the following sequence?

39, 42, 46, 50, ...

(a) 52 (b) 53 (c) 54 (d) 55

Soln.



given series is alternative increasing.

Correct answer is (d)

15. $(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3) =$

(a) 61 (b) 22 (c) 41/24 (d) 16

Soln. By BODMAS

$(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3)$

$(5 + 3 - 8) + (64 - 3)$

$0 + 61 = 61$

Correct answer is (a)

16. A student buys a book from an online shop at 20% discount. His friend buys another copy of the same book in a book fair for Rs. 192 paying 20% less than his friend. What is the full price of the book?

(a) Rs. 275 (b) Rs. 300 (c) Rs. 320 (d) Rs. 392

Soln. Let the price of book = x rs.

then price for student = $x \times \frac{(100-20)}{100} = x \times \frac{80}{100} = \frac{4x}{5}$ rs

and price for his friend = $\frac{4x}{5} \times \frac{(100-20)}{100} = 1920$

$$\Rightarrow \quad x = \frac{192 \times 5 \times 100}{80 \times 4} = 300 \; rs$$

Correct answer is (b)

17. How many nine-digit positive integers are there, the sum of squares of whose digits are 2?
(a) 8 (b) 9 (c) 10 (d) 11

Soln. Since sum is equal to 2. Then it is clear that number contain digit less than 2, i.e., either 1 or 0. Hence, number be

$$1\boxed{1}0000000$$

We can rotate the blocked digit in 8 different places. Hence, numbers are 8.

Correct answer is (a)

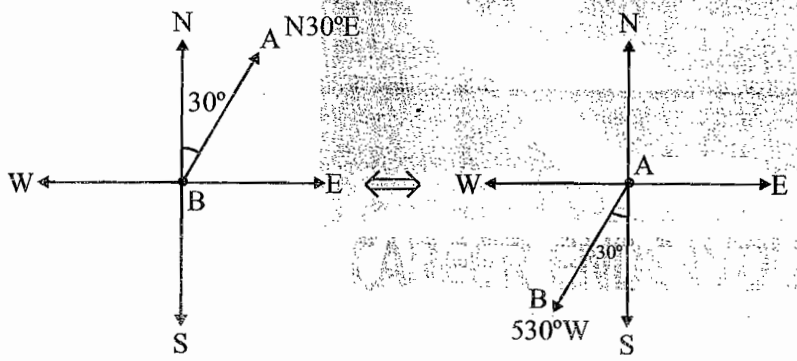
18. 366 players participate in a knock-out tournament. In each round all competing players pair together and play a match, the winner of each match moving to the next round. If at the end of a round there is an odd number of winners, the unpaired one moves to the next round without playing a match. What is the total number of matches played?
(a) 366 (b) 282 (c) 365 (d) 418

Soln. Whenever game is based on knockout system the number of matches for deciding the winner is less than one of number of players. Hence, 366-1 = 365

Correct answer is (c)

19. There are 2 hills, A and B, in a region. If hill A is located N30°E of hill B, what will be the direction of hill B when observed from hill A? (N 30°E means 30° from month towards east).
(a) S 30°W (b) S 60°W (c) S 30°E (d) S 60°E

Soln.



Correct answer is (a)

20. For real numbers x and y, $x^2 + (y-4)^2 = 0$. Then the vlaue of x + y is

(a) 0 (b) 2 (c) $\sqrt{2}$ (d) 4

Soln. We know that if $a^2 + b^2 = 0$ then both term should equal to zero.

$$\Rightarrow \quad x^2 + (y-4)^2 = 0 \quad \Rightarrow \quad x^2 = 0 \text{ and } (y-4)^2 = 0$$

$$\Rightarrow \quad x = 0 \text{ and } y - 4 = 0 \quad \Rightarrow y = 4$$

then $x + y = 0 + 4 = 4$

Correct answer is (d)

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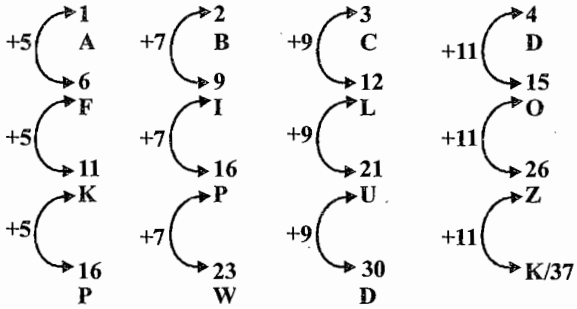
1. Find the missing letter

A	B	C	D
F	I	L	O
K	P	U	Z
P	W	D	?

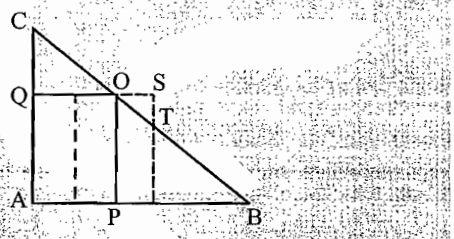
(a) P (b) K (c) J (d) L

Soln.



Put the place value of alphabates – in first column difference is 5, in 2nd–7, in 3rd–9, in 4th–11.

2. Consider a right-angled triangle ABC where AB = AC = 3. A rectangle APOQ is dranw inside it, as shown, such that the height of the rectangle is twice its width. The rectangle is moved horizontally by a distance 0.2 as shown schematically in the diagram (not to scale)



What is the value of the ratio $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta OST}$?

(a) 625 (b) 400 (c) 225 (d) 125

Soln. In this question, firstly we proof that ΔABC and ΔOST are similar. Therefore, in ΔABC and ΔOST

$\angle ABC = \angle OST = 90^\circ$

$\angle ACB = \angle OST = (\text{Alternate interior})$

Hence, $\angle ABC = \angle OST$

$\therefore \quad \Delta ABC \sim \Delta OST$

$\Rightarrow \quad \frac{AC}{ST} = \frac{AB}{OS} = \frac{BC}{OT}$

Now, $OS = 0.2$

$\frac{AC}{ST} = \frac{AB}{OS} \quad \Rightarrow \frac{3}{ST} = \frac{3}{0.2}$

$\therefore \quad ST = 0.2$

Now, $\frac{ar(\Delta ABC)}{ar(\Delta OST)} = \frac{\frac{1}{2} \times AB \times AC}{\frac{1}{2} \times OS \times ST} = \frac{\frac{1}{2} \times 3 \times 3}{\frac{1}{2} \times 0.2 \times 0.2} = \frac{9}{0.04} = 225$

Correct answer is (c)

3. 80 gsm paper is cut into sheets of 200 mm×300mm size and assembled in packets of 500 sheets. What will be the weight of a packet? (gsm = g/m²)

(a) 1.2 kg (b) 2.4kg (c) 3.6 kg (d) 4.8 kg

Soln. Weight of 1 m² sheet = 80 gram

Now, area of all 500 sheets = $200\times300\times500$ mm² $= \dfrac{200\times300\times500}{1000\times1000} = 30$ m²

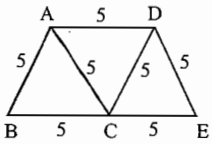
Hence, total weight = $80\times30 = 2400$ gram $= \dfrac{2400}{1000} = 2.4$ kg

Correct answer is (b)

4. Three identical flat equivalent-triangular plates of side 5 cm each are placed together such that they form a trapezium. The length of the longer of the two parallel sides of this trapezium is

(a) $5\sqrt{\frac{3}{4}}\ cm$ (b) $5\sqrt{2}\ cm$ (c) 10 cm (d) $10\sqrt{3}\ cm$

Soln.



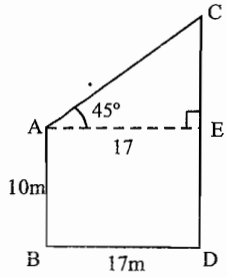
Hence, length of BE = 5 + 5 = 10 cm

Correct answer is (c)

5. An archer climbs to the top of a 10 m high building and aims at a bird on top of a tree 17m away. The line of sight from the archer to the bird makes an angle of 45° to the horizontal. What is the height of the tree?

(a) 17 m (b) 27 m (c) 37 m (d) 47 m

Soln.



Let AB is a building and CD is tree. Bird is located at point C and archer in point A. Draw a line parallel to BD i.e. AE.

$\Rightarrow$ AE = BD = 17

Now, in ΔAEC,

$$\frac{CE}{AE} = \tan 45° \Rightarrow \frac{CE}{17} = 1 \Rightarrow CE = 17$$

Hence, length of tree = 17 + 10 = 27 m.

Correct answer is (b)

6. Consider the set of numbers $\{17^1, 17^2, ..., 17^{300}\}$. How many of these numbers end with the digit 3?

(a) 60 (b) 75 (c) 100 (d) 150

Soln. Unit of $17^1 = 7$; $17^2 = 9$; $17^3 = 3$, $17^4 = 1$; $17^5 = 7$; $17^6 = 9$; $17^7 = 3$; $17^8 = 1$

Hence, $17^3, 17^7, 17^{11}$, ... will ends with 3 as unit.

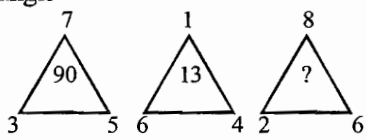
$$4\overline{)\,300\,(\,75}$$

Therefore, $\begin{array}{r} 28 \\ \times 20 \\ \hline 20 \\ \times \end{array}$

So, 75 is our answer.

Correct answer is (b)

7. Find the missing number in the triangle



(a) 16 (b) 96 (c) 50 (d) 80

Soln.
$7\times5\times3-(7+5+3)=90$
$6\times1\times4-(6+1+4)=13$
$8\times2\times6-(8+2+6)=80$

Correct answer is (d)

8. The time gap between the two instants, one before and one after 12.00 noon, when the angle between the hour hand and the minute hand is 66°, is
(a) 12 min (b) 16 min (c) 18 min (d) 24 min

Soln. Solve by hit and trial

Apply between hands of watch $=\left|\text{Hour}\times30-\text{min}\times\frac{1}{2}\right|$

in a gap of 24 min, there is angle between hands of watch is 66°.

Correct answer is (d)

9. A merchant buys equal numbers of shirts and trousers and pays Rs. 38000. If the cost of 3 shirts is Rs. 800 and that of a trouser is Rs. 1000, then how many shirts were bought?
(a) 60 (b) 30 (c) 15 (d) 10

Soln. Let he purchase y shirts and y trousers

then cost of 1 shirt $=\frac{800}{3}$ rs

and cost of 1 trousers = 1000 rs

Hence, $\frac{800}{3}\times y+1000\times y=38000$

$800y+3000y=3800\times3$

$3800y=38000\times3$

$y=30$

Correct answer is (b)

10. In the growing years of a child, the height increases as the square root of the age while the weight increases in direct proportion to the age. The ratio of the weight to the square of the height in this phase of growth.
(a) is constant
(b) reduces with age
(c) increases with age
(d) is constant only if the weight and height at birth are both zero

Soln.
$H\propto\sqrt{A}$
$W\propto A$
$\Rightarrow \quad H=k.\sqrt{A}$
$W=k_1.A$

Now, $H^2=\left(k.\sqrt{A}\right)^2=k^2.A=k'.A$ (where k' is other constant)

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and $\frac{H^2}{w}=\frac{k'A}{k_1.A}=\frac{k'}{k_1}=k_2$ (other constant)

Hence, correct answer is (d)

11. In 450 g of pure coffee powder 50g of chicory is added. A person buys 100g of this mixture and adds 5 g of chicory to that. What would be the rounded-off percentage of chicory in this final mixture?
(a) 10 (b) 5 (c) 14 (d) 15

Soln. After mixing chicory new mixture = 450 + 50 = 500 gram
Since, in 500 gram chicory is 50 gram

Therefore, in 1g chicory is $=\frac{50}{500}$

Therefore, in 100 gram chicory is $\frac{50}{500}\times 100=10$ gram

After adding 5g chicory
New mixture = 100 + 5 = 105 gram
and quantity of chicory = 10 + 5 = 15 gram

Hence, $\%=\frac{15}{105}\times 100=\frac{100}{7}=14.28=14.28\%$

Correct answer is (c)

12. Suppose in a box there are 20 red, 30 black, 40 blue and 50 white balls. What is the minimum number of balls to be drawn, without replacement, so that you are certain about getting 4 red, 5 black, 6 blue and 7 white balls?
(a) 140 (b) 97 (c) 104 (d) 124

Soln. So, minimum number of balls = 50 + 40 + 30 + 4 = 124
Correct answer is (d)

13. Suppose

$x\Delta y=(x-y)^2$

$x\circ y=(x+y)^2$

$x\bullet y=(x\times y)^{-1}$

$x\cdot y=x\times y$

+, – and × have their usual meanings. What is the value of

$\{(197\circ 315)-(197\Delta 315)\}\cdot(197*315)$?

(a) 118 (b) 512 (c) 2 (d) 4

Soln. $\{(197+315)^2-(197-315)^2\}.(197\times 315)^{-1}$

$\frac{\{512^2-(-118)^2\}}{197\times 315}=\frac{630\times 394}{197\times 315}=4$

Correct answer is (d)

14. Students in group A obtained the following marks : 40, 80, 70, 50, 60, 90, 30. Students in group B obtained 40, 80, 35, 70, 85, 45, 50, 75, 60 marks. Define

dispersion (D) = (maximum marks – minimum marks), and relative dispersion $(RD)=\frac{\text{dispersion}}{\text{mean}}$

Then,
(a) RD of group A = RD of group B (b) RD of group A > RD of group B
(c) RD of group A < RD of group B (d) D of group A < D of group B

Soln. **Group A :** 40, 80, 70, 50, 60, 90, 30

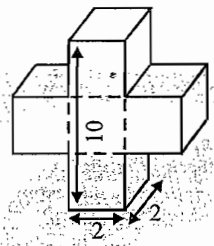
Group B : 40, 80, 35, 70, 85, 45, 50, 75, 60
Dispersion of Group A = 90–30 = 60
Group B = 85–35 = 50

$$\text{RD of Group A} = \frac{60}{\frac{40+80+70+50+60+90+30}{7}} = \frac{60}{60} = 1$$

$$\text{RD of Group B} = \frac{50}{\frac{40+80+35+70+85+45+50+75+60}{9}} = \frac{50}{60} < 1$$

RD of Group A > RD of Gropu B
Correct anwer is (b)

15. The following diagram shows 2 perpendicularly inter-grown prismatic crystals (twins) of identical shape and size. What is the volume of the object shown (units are arbitrary)?



(a) 60 (b) 65 (c) 72 (d) 80

Soln. Volume of both cuboid = (10×2×2) + (10×2×2) – (2×2×2) = 40 + 40 –8 = 72 unit³.
Correct answer is (c)

16. If A×B = 24, B×C = 32, C×D = 48, then A×D
(a) cannot be found (b) is a perfect square
(c) is a perfect cube (d) is odd

Soln. A×B = 24 ... (1)
B × C = 32 ... (2)
C × D = 48 ... (3)
(1) × (2) × (3)
A×B×B×C×C×D = 24×32×48
A×B²×C²×D = 24×32×48

Divide it by $(B\times C)^2$ – $\frac{A\times(B\times C)^2\times D}{(B\times C)^2} = \frac{24\times32\times48}{32\times32} = 36$

Which is perfect square. Hence, correct answer is **(b)**

17. Suppose n is a positive integer. Then $\left(n^2+n\right)(2n+1)$
(a) may not be divisible by 2
(b) is always divisible by 2 but may not be divisible by 3
(c) is always divisible by 3 but may not be divisible by 6.
(d) is always divisible by 6.

Soln. These type of question can be solved by hit and trial method.
Put n = 2, 3, 4, ... in (n² + n) (2n + 1)
For n = 2 $\Rightarrow$ (2² + 2) (2×3 +1) = 6×5 = 30
n = 3 $\Rightarrow$ (3² + 3) (2×3 + 1) = 12×7 = 84
n = 4 $\Rightarrow$ (4² + 4) (2×4 + 1) = 20×9 = 180
and we know that n (n + 1) (2n + 1) is always divisible by 6
Correct answer is (d)

18. There is a train of length 500 m, in which a man is standing at the rear end. At the instant the rear end crosses a stationary observer on a platform, the man starts walking from the rear to the front and the front to the rear of the train at a constant speed of 3 km/hr. The speed of the train is 80 km/hr. The distance of the man from the observer at the end of 30 minutes is

(a) 41.5 km (b) 40.5 km (c) 40.0 km (d) 41.0 km

Soln. In 30 minute train will cover $= 80 \times \frac{1}{2} = 40$ k.m.

Now, speed of man = 3 km/h

In $\frac{1}{2}$ hour man will cover $= 3 \times \frac{1}{2} = 1.5$ km $= 1500$ m

Length of train = 500 m.

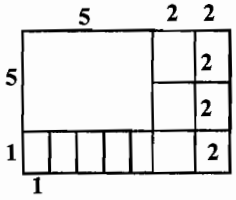
It means man will go ends to start, start to end and again ends to start. So, distance between observer and man will be 40 + 500 m = 40.5 km.

Correct answer is (b)

19. A rectangular area of sides 9 and 6 units is to be covered by square tiles of sides 1, 2 and 5 units. The minimum number of tiles needed for this is

(a) 3 (b) 11 (c) 12 (d) 15

Soln. Area of rectangle $= 9 \times 6 = 54$ unit^2.



$1(5\times5)+6(2\times2)+5(1\times1)=12$

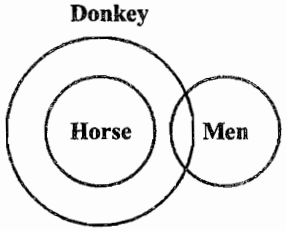
Correct answer is (c)

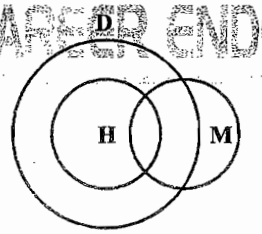
20. If all horses are donkeys, some donkeys are monkeys, and some monkeys are men, then which statement must be true?

(a) All donkeys are men (b) Some horses may be men

(c) Some horses are men (d) All horses are also monkeys.

Soln.





According to both figure,

only (ii) is true

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CSIR-NET/JRF - Dec. 2014 Paper

1. Lunch-dinner pattern of a person for m days is given below. He has a choice of a VEG or a NON-VEG meal for his lunch/dinner

(a) If he takes a NON-VEG lunch, he will have only VEG for dinner

(b) He takes NON-VEG dinner for exactly 9 days

(c) He takes VEG lunch for exactly 15 days

(d) He takes a total of 14 NON-VEG meals

What is m?

(a) 18 (b) 24 (c) 20 (d) 38

Soln.

	Non-veg	Veg
Lunch	5	15
Dinner	9	–

According to b and d

Total non-veg meals = 14 day

non veg dinnger = 9 day

Hence non-veg lunch = 14–9 = 5 day.

According to information (c), he takes 15 veg lunch

$$m = 15(\text{veg lunch}) + (5 \text{ non-veg lunch}) = 20 \text{ day}$$

Correct answer is (c)

2. Two locomotives are running towards each other with speeds of 60 and 40 km/h. An object keeps on flying to and fro from the front tip of one locomotive to the front tip of the other with a speed of 70 km/h. After 30 minutes, the two locomotives collide and the object is crushed. What distance did the object is crushed. What distance did the object cover before being crushed?

(a) 50 km (b) 45 km (c) 35 km (d) 10 km

Soln. Speed of object = 70 km/h

Flying time = 30 min = ½ hrs

hence, distance = 70×½ = 35 km

Correct option is (c)

3. A sphere is made up of very thin concentric shells of increasing radii (leaving no gaps). The mass of an arbitrarily chosen shell is

(a) equal to the mass of the preceding shell (b) proportional to its volume

(c) proportional to its radius (d) proportional to its surface area

Soln. Proportional to its surface area.

Correct answer is (d)

4. Find the missing letter:

A	?	Q	E
C	M	S	C
E	K	U	A
G	I	W	Y

(a) L (b) Q (c) N (d) O

Soln.

1 A	15 ?	17 Q	31 E
3 C	13 M	19 S	29 C
5 E	11 K	21 U	27 A
7 G	9 I	23 W	25 Y

Correct answer is (d)

5. A person sells two objects at Rs. 1035/- each. On the first object he suffers a loss of 10% while on the second he gains 15%. What is his net loss/gain percentage?

(a) 5% gain (b) <1% gain (c) <1% loss (d) no loss, no gain

Soln. Let CP = x

$x \times \frac{115}{100} = 1035$ ... (i) $\Rightarrow x = 900$

$x \times \frac{90}{100} = 1035$ ... (ii) $\Rightarrow x = 1150$

Total CP = 900 + 1150 = 2050

Total SP = 1035 + 1035 = 2070

Gain = 2070 – 2050 = 20

gain % = $\frac{20}{2050} \times 100 < 1\%$ *gain*

Correct answer is (b)

6. A bank offers a scheme wherein deposits made for 1600 days are doubled in value, the interest being compounded daily. The interest accrued on a deposit of Rs. 1000/- over the first 400 days would be Rs.

(a) 250 (b) 183 (c) 148 (d) 190

Soln. This is the question related with compound interest put in formula

$A = P\left(1 + \frac{r}{100}\right)^n$ and C.I. = A – P

Correct answer is (d)

7. The least significant bit of an 8-bit binary number is zero. A binary number whose value is 8 times the previous number has

(a) 12 bits ending with three zeros (b) 11 bits ending with four zeros

(c) 11 bits ending with three zeros (d) 12 bits ending with four zeros

Soln. Since the least significant digit of the binary number is zero, it means the number is even. If the decimal form of the number is X, then

X = 2Y, where Y is an integer.

Now, 8X = 16Y.

The binary form of 8X can be found as

2	16Y	→ 0
2	8Y	→ 0
2	4Y	→ 0
2	2Y=X	→ 0
	Y	

Hence, 8X has 11(8+3)

bits ending with four zeros.

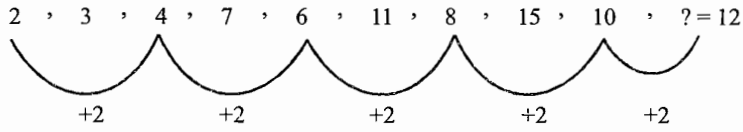
Correct option is (b)

8. What is the next number of the following sequence?

2, 3, 4, 7, 6, 11, 8, 15, 10 .....

(a) 12 (b) 13 (c) 17 (d) 19

Soln.



Correct answer is (d)

9. 18. 20% of students of a particular course get jobs within one year of passing. 20% of the remaining students get jobs by the end of second year of passing. If 16 students are still jobless, how many students had passed the course?

(a) 32 (b) 64 (c) 25 (d) 100

Soln. Let number of student is x.

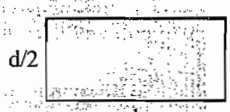
hence, $x \times \frac{80}{100} \times \frac{80}{100} = 16 \quad \Rightarrow x = 25$

Correct answer is (c)

10. A rectangle of length d and breadth $d/2$ is revolved once completely around its length and once around its breadth. The ratio of volumes swept in the two cases is

(a) 1:1 (b) 1:2 (c) 1:3 (d) 1:4

Soln. When folded according to length then $2\pi R = \frac{d}{2}$



$$R = \frac{d}{4\pi} \text{ and } h = d$$

Then volume $= \pi\left(\frac{d}{4\pi}\right)^2 \cdot d = \frac{\pi d^3}{16\pi^2}$

When folded according to width, $2\pi R = d$

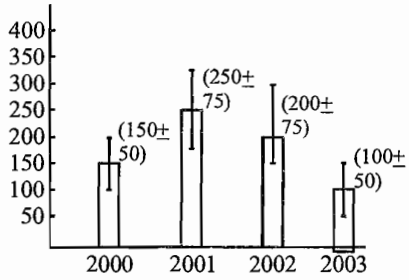
$$R = \frac{d}{2\pi} \text{ and } h = \frac{d}{2}$$

Then volume $= \pi\left(\frac{d}{2\pi}\right)^2 \cdot \frac{d}{2} = \frac{\pi d^3}{8\pi^2}$

Hence, ratio $= \frac{\pi d^3}{16\pi^2} : \frac{\pi d^3}{8\pi} = 1:2$

Correct answer is (b)

11. Average yield of a product in different years is shown in the histogram. If the vertical bars indicate variability during the year, then during which year was the percent variability over the average of that year the least?



(a) 2000 (b) 2001 (c) 2002 (d) 2003

Soln. For year 2000, $\frac{50}{100} \times 100 = 33.3\%$

For year 2001, $\frac{75}{250}\times 100 = 30\%$

For year 2002, $\frac{75}{200}\times 100 = 37.5\%$

For year 2003, $\frac{50}{100}\times 100 = 50\%$

Correct answer is (b)

12. A long ribbon is wound around a spool up to a radius R. Holding the tip of the ribbon, a boy runs away from the spool with a constant speed maintaining the unwound portion of the ribbon horizontal. In 4 minutes, the radius of the wound portion becomes $\frac{R}{\sqrt{2}}$. In what further time, it will become R/2?

(a) $\sqrt{2}$ min (b) 2 min

(c) $2\sqrt{2}$ min (d) 4 min

Soln. If radius is R, then area $= \pi R^2$

When radius is $\frac{R}{\sqrt{2}}$ then area $= \pi\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{\pi R^2}{2}$

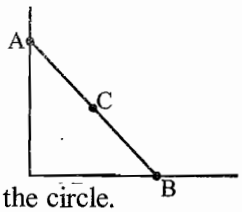
When radius is $\frac{R}{2}$, then area $= \pi\left(\frac{R}{2}\right)^2 = \frac{\pi R^2}{4}$

Since, area becomes $\frac{\pi R^2}{2} = 4$ min. Therefore, 1 area becomes $\frac{\pi R^2}{2} = \frac{4}{\pi R^2/2}$

Therefore, $\frac{\pi R^2}{4}$ becomes $\frac{\pi R^2}{2} = \frac{4\times\frac{\pi R^2}{4}}{\frac{\pi R^2}{2}} = 2$ min

Correct answer is (b)

13. A ladder rests against a wall as shown. The top and the bottom ends of the ladder are marked A and B. The base B slips. The central point C of the ladder falls along



(a) a parabola (b) the arc of a circle

(c) a straight line (d) a hyperbola

Soln. Since C is mid point of AB. So, its path make arc of the circle.

Correct answer is (b)

14. Binomial theorem in algebra gives $(1+x)^n = a_0 + a_1x + a_2x^2 + ... + a_nx^n$, where $a_0, a_1, ..., a_n$ are constants depending on n. What is the sum $a_0 + a_1 + a_2 + ... + a_n$?

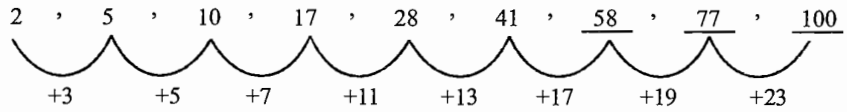
(a) 2^n (b) n (c) n^2 (d) $n^2 + n$

Soln. $a_0 + a_1 + a_2 + ... + a_n = 2^n$

Correct answer is (a)

15. Continue the sequence
2, 5, 10, 17, 28, 41, __, __, __
(a) 58, 77, 100 (b) 64, 81, 100 (c) 43, 47, 53 (d) 55, 89, 113

Soln.



Correct answer is (a)

16. A code consists of at most two identical letters followed by at most four identical digits. The code must have at least one letter and one digit. How many distinct codes can be generated using letters A to Z and digits 1 to 9?
(a) 936 (b) 1148 (c) 1872 (d) 2574

Soln. Correct answer is (c)

17. Two solid iron spheres are heated to 100°C and then allowed to cool. One has the size of a football; the other has the size of a pea. Which sphere will attain the room temperature (constant) first?
(a) The bigger sphere (b) The smaller sphere
(c) Both spheres will take the same time (d) It will depend on the room temperature

Soln. The heat contained in the sphere is proportional to the volume $\left(\frac{4\pi}{3}r^3\right)$ because the temperature of both spheres is same. The heat loss is proportional to surface area $\left(4\pi r^2\right)$. Hence, smaller sphere will reach the room temperature first.

Correct option is (b)

18. Weights (in kg) of 13 persons are given below:
70, 72, 74, 76, 78, 80, 82, 84, 86, 88, 90, 92, 94
Two new persons having weights 100 kg and 79 kg join the group. The average weight of the group increases by
(a) 0 kg (b) 1 kg (c) 1.6 kg (d) 1.8 kg

Soln. Average weight of 13 person $= 82$ kg

And average weight of 15 person $= \frac{82+100+79}{15} = 83\ kg$

Hence, increased average weight $= 83-82 = 1$ kg

Correct answer is (b)

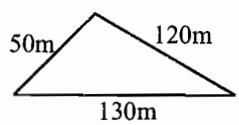
19. If n is a positive integer, then $n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)$ is divisible by
(a) 3 but not 7 (b) 3 and 7 (c) 7 but not 3 (d) neither 3 nor 7

Soln. Use hit and trials method, put $n = 1$
Then we see that given number is divisible by 3 and 7 both

Correct answer is (b)

20. The area (in m^2) of a triangular park of dimensions 50m, 120m and 130m is
(a) 3000 (b) 3250 (c) 5550 (d) 7800

Soln. $S = \frac{a+b+c}{2} = \frac{50+120+130}{2} = \frac{300}{2} = 150$



Hence, area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150(150-50)(150-120)(150-130)}$

$= \sqrt{150\times100\times30\times20} = 3000$

Correct answer is (a)

CSIR-NET/JRF - June 2015 Paper

1. Each of the following pairs of words hides a number, based on which you can arrange them in ascending order. Pick the correct answer:

I. Cloth reel
J. Silent wonder
K. Good tone
L. Bronze rod

(a) L, K, J, I (b) I, J, K, L (c) K, L, J, I (d) K, J, I, L

Soln. I. Clo[th re e]l J. Silen[t wo]nder K. Good t[one] L. Bron[ze ro]d

Hence, L, K, J, I

Correct answer is (a)

2. Which of the following values is same as $2^{2^{2^2}}$?

(a) 2^6 (b) 2^8 (c) 2^{16} (d) 2^{222}

Soln. $a^{m^{n^p}}$, at first we will solve $n^p = q$ (let)

$\Rightarrow a^{m^q}$

now solve $m^q = r$

$\Rightarrow a^r \quad \therefore \; 2^{(2^2)} = 2^{(2^4)} = 2^{16}$

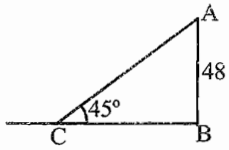
Correct answer is (c)

3. A 12 m × 4 m rectangular roof is resting on four 4 m tall thin poles. Sunlight falls on the roof at an angle of 45° from the east, creating a shadow on the ground. What will be the area of the shadow ?

(a) 24 m² (b) 36 m² (c) 48 m² (d) 60 m²

Soln. Area of rectangular sheet = 12 × 4 = 48m²

Let AB is rectangular sheet and BC is its shadow.



$\tan 45° = \frac{AB}{BC}, \quad 1 = \frac{48}{BC}, \quad BC = 48$

Correct answer is (c)

4. If

$$\begin{array}{r} 2\ a \\ \times\ b\ 2 \\ \hline c\ 6 \\ 8\ 4 \\ \hline 8\ d\ 6 \end{array}$$

Here a, b, c and d are digits. Then a + b =

(a) 4 (b) 9 (c) 11 (d) 16

CAREER ENDEAVOUR

Soln.

$$\begin{array}{r} 2\ a \\ b\ 2 \\ \hline c\ 6 \\ 8\ 4 \\ \hline 8\ d\ 6 \end{array} \qquad \begin{array}{r} 2\ 3 \\ b\ 2 \\ \hline 4\ 6 \\ 8\ 4 \\ \hline \end{array} \Rightarrow \qquad \begin{array}{r} 2\ 3 \\ 8\ 2 \\ \hline 4\ 6 \\ 1\ 8\ 4 \\ \hline 1\ 8\ 8\ 6 \end{array}$$

$a+b=?$

$3+8=11 \Rightarrow b=8$ which is contradiction

$$\begin{array}{r} 2\ 8 \\ b\ 2 \\ \hline 5\ 6 \\ 8\ 4 \\ \hline \text{Now b is 3} \end{array} \Rightarrow \qquad \begin{array}{r} 2\ 8 \\ 3\ 2 \\ \hline 5\ 6 \\ 8\ 4 \\ \hline 8\ 9\ 6 \end{array}$$

Correct answer is (c)

5. The maximum number of points formed by intersection of all pairs of diagonals of convex octagon is
(a) 70 (b) 400 (c) 120 (d) 190

Soln. **Correct option is (a)**

6. Find the height of a box of base area 24 cm × 48 cm, in which the longest stick that can be kept is 56 cm long.
(a) 8 cm (b) 32 cm (c) 37.5 cm (d) 16 cm

Soln. Box is a cuboid

Diagonal of a cuboid = $\sqrt{l^2+b^2+h^2}$

$d^2=l^2+b^2+h^2$, $56^2 = 24^2 + 48^2 + h^2$

$3136 = 576 + 2304 + h^2$

$h^2=256$

$h=16$ cm

Correct answer is (d)

7. The product of the perimeter of a triangle, the radius of its in-circle, and a number gives the area of the triangle. The number is
(a) 1/4 (b) 1/3 (c) 1/2 (d) 1

Soln. **Correct option is (c)**

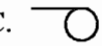
8. An infinite row of boxes is arranged. Each box has half the volume of the previous box. If the largest box has a volume of 20 cc, what is the total volume of all the boxes ?
(a) Infinite (b) 400 cc (c) 40 cc (d) 80 cc

Soln. This is a G.P. whose $a = 20$, $r = \frac{1}{2}$

$$\therefore \quad S\infty = \frac{a}{1-r} = \frac{20}{1-\frac{1}{2}} = 20\times 2 = 40\text{cc}$$

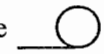
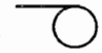
Correct answer is (c)

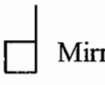
9. Find the missing element based on the given pattern

A. B. C. 

A. B. C. ?

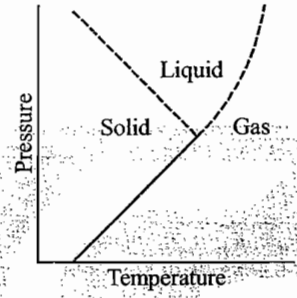
(a) (b) (c) (d)

Soln. A. Water image B. Mirror Image  Water image C. 

A. Water Image B.  Mirror Image Water Image C.

Correct answer is (b)

10. By reading the accompanying graph, determine the incorrect statement out of the following.



(a) Melting point increases with pressure
(b) Melting point decreases with pressure
(c) Boiling point increases with pressure
(d) Solid, liquid and gas can co-exist at the same pressure and temperature

Soln. By given graph, option (a) is wrong

Correct option is (a)

11. If you change only one observation from a set of 10 observations, which of the following will definitely change?

(a) Mean (b) Mediun (c) Mode (d) Standard deviation

Soln. If we change only one observation then mean will be change.

Correct option is (a)

12. A man starts his journey at 0100 Hrs local time to reach another country at 0900 Hrs local time on the same date. He starts a return journey on the same night at 2100 Hrs local time to his original place, taking the same time to travel back. If the time zone of his country of visit lags by 10 Hrs, the duration for which the man was away from his place is

(a) 48 hours (b) 20 hours (c) 25 hours (d) 36 hours

Soln. **Correct option is (a)**

13. Let r be a positive number satisfying $r^{(1/1234)} + r^{(-1/1234)} = 2$. Then $r^{4321} + r^{-4321} = ?$

(a) 2 (b) $2^{(4321/1234)}$ (c) 2^{3087} (d) 2^{1234}

Soln. $r^{1/1234} + r^{-1/1234} = 2$ ... (1)

then $r^{4321} + r^{-4321} = ?$ ... (2)

(i) and (ii) are alike but power in (i) is written in denominator form and in (ii) in numerator form.

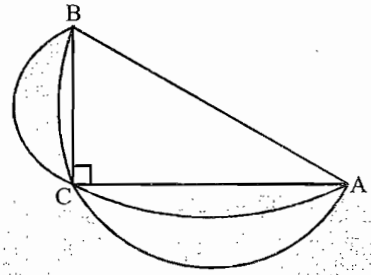
Correct option is (a)

14. A float is drifting in a river, 10 m downstream of a boat that can be rowed at a speed of 10 m/minute in still water. If the boat is rowed downstream, the time taken to catch up with the float
(a) will be 1 minute
(b) will be more than 1 minute
(c) will be less than 1 minute
(d) can be determined only if the speed of the river is known

Soln. Speed of current for stream and boat is same and float is 10 m ahead and speed of boat = 10m/min hence required time = 1 min.

Correct option is (a)

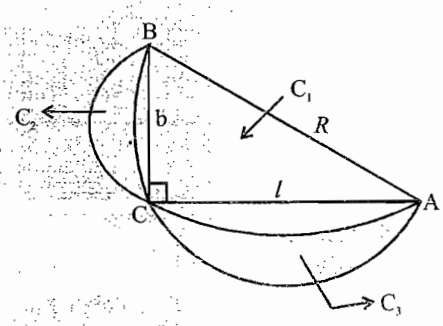
15. ABC is a right angled triangle inscribed in a semicircle. Smaller semicircles are drawn on sides BC and AC. If the area of the triangle is a, what is the total area of the shaded lumes ?



(a) a (b) πa (c) a/π (d) $a/2\pi$

Soln. $\Delta ABC = \frac{1}{2} \times b \cdot \ell$

$R^2 = b^2 + \ell^2$



Area of $C_1 = \frac{1}{2}\pi\left(\frac{R}{2}\right)^2 = \frac{1}{2}\pi\frac{R^2}{4}$

Area of $C_2 = \frac{1}{2}\pi\left(\frac{b}{2}\right)^2 = \frac{1}{2}\pi\frac{b^2}{4}$

Area of $C_3 = \frac{1}{2}\pi\left(\frac{\ell}{2}\right)^2 = \frac{1}{2}\pi\frac{\ell^2}{4}$

Hence, required area $= C_2 + C_3 - C_1 + \Delta$

$$= \frac{1}{2}\pi\frac{b^2}{4} + \frac{1}{2}\pi\frac{\ell^2}{4} - \frac{1}{2}\pi\frac{R^2}{4} + a = \frac{1}{2}\times\frac{\pi}{4}\left[\left(b^2+\ell^2\right) - R^2\right] + a$$

$$\Rightarrow \quad \frac{\pi}{8}\left[R^2 - R^2\right] + a = a$$

Correct optino is (a)

16. An ant can lift another ant of its size whereas an elephant cannot lift another elephant of its size, because
(a) ant muscle fibres are stronger than elephant muscle fibres
(b) ant has proportionately thicker legs than elephant
(c) strength scales as the square of the size while weight scales as cube of the size
(d) ants work cooperatively, whereas elephants work as individuals

Soln. Strength and weight both depends on size

Strength $\propto$ size^2

Weight $\propto$ size^3

Now size of Ant is very less with compare to an elephant. So it is possible for ant but not possible for elephant.

Correct answer is (b)

17. Consider a series of letters placed in the following way:

U__G__C__C__S__I__R

Each letter moves one step to its right and the extreme right letter takes the first position, completing one operation. After which of the following numbers of operations do the Cs not sit side by side ?

(a) 3 (b) 10 (c) 19 (d) 25

Soln. U_G_C_C_S_I_R

now each letter moves one step to his right

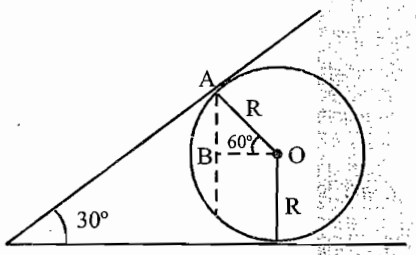
by sequence, 25 operations are required.

Correct answer is (d)

18. An inclined plane rests against a horizontal cylinder of radius R. If the plane makes an angle of 30° with the ground, the point of contact of the plane with the cylinder is at a height of

(a) 1.500 R (b) 1.866 R (c) 1.414 R (d) 1.000 R

Soln.



$$\Delta ABC \Rightarrow \frac{AB}{AC} = \sin 60^\circ \Rightarrow AB = AC \sin 60^\circ = R \times \frac{\sqrt{3}}{2}$$

Hence, height $= R + \frac{\sqrt{3}}{2} R = 1.866R$

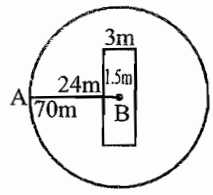
19. What is the maximum number of parallel, non-overlapping cricket pitches (length 24 m, width 3 m) that can be laid in a field of diameter 140 m, if the boundary is required to be at least 60 m from the centre of any pitch?

(a) 6 (b) 7 (c) 12 (d) 4

Soln. AB = 70m

BC = 1.5 m

AC = 68.5



This length form 6 more pitches

So number of pitches = 6 + 1 = 7

Correct answer is (b)

20. In a fast moving car with open windows, the driver feels a continuous incoming breeze. The pressure inside the car, however, does not keep increasing because,

(a) air coming in from the front windows goes out from the rear.

(b) air comes in as well as goes out through every window but the driver only feels the incoming one.

(c) no air actually comes in and the feeling of breeze is an illusion.

(d) cool air reduces the temperature therefore the pressure does not increase.

Soln. **Correct answer is (b)**

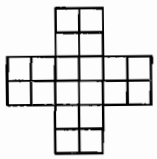
CSIR-NET/JRF - Dec. 2015 Paper (Physics)

1. In each of the following groups of words is a hidden number, based on which you should arrange them in descending order. Pick the correct answer:

E. Papers I Xeroxed
F. Wi-Fi veteran
G. Yourself ourselves
H. Breaks even

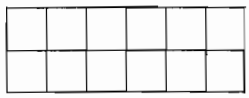
(a) H, F, G, H (b) E, G, F, H (c) H, F, G, E (d) H, E, F, G

Soln. E. Paper SIX eroxed
F. Wi- Five teran
G. Yoursel four selves
H. Break seven
HEFG
Correct option is (d)

2. 

The number of squares in the above figure is

(a) 30 (b) 29 (c) 25 (d) 20

Soln. 

Above figure have two condition vertically as well as horizontally.
Hence total number of squares $= (6 \times 2 + 5 \times 1) + (6 \times 2 + 5 \times 1)$
$= 17 + 17 = 34$
But this condition is common for both figures.

Hence we subtract 5 squares from total
Hence total number of squares = 34 – 5 = 29
Correct option is (b).

3. A shopkeeper purchases a product for Rs. 100 and sells it making a profit of 10%. The customer resells it to the same shopkeeper incurring a loss of 10%. In these dealing the shopkeeper makes

(a) no profit, no loss (b) Rs. 11 (c) Re. 1 (d) Rs. 20

Soln. Cost price for shopkeeper = 100 rs

Selling price for shopkeeper $= 100 \times \frac{(100+10)}{100}$

Profit of shopkeeper in this step = 110 – 100 = 10 rs
Now cost price for customer = 110 rs

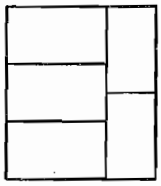
and selling price for customer = $110 \times \frac{(100-10)}{100} = 99$ rs

in this step shopkeeper's profit = 1 rs

hence total profit = 10 + 1 = 11 rs

Correct option is (b)

4. Five congruent rectangles are drawn inside a big rectangle of perimeter 165 as shown. What is the perimeter of one of the five rectangles?



(a) 37 (b) 75 (c) 15 (d) 165

Soln. From given figure $3b = 2a$

$a+a+b+a+b+b+b+a+b=165$

$4a+5b=165$ $\quad 2a=3\times15$

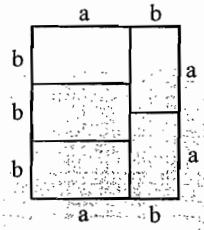
$2.2a+5b=165$ $\quad 2a=45$

$2.3b+5b=165$ $\quad a=22.5$

$6b+5b=165$

$11b=165$

$b=15$



Perimeter of one box = 2(22.5 + 15)

= 2 × 37.5 = 75

Correct option is (b)

5. A person walks downhill at 10 km/h, uphill at 6 km/h and on the plane at 7.5 km/h. If the person takes 3 hours to go from a place A to another place B, and 1 hour on the way back, the distance between A and B is

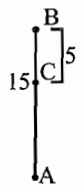
(a) 15 km (b) 23.5 km

(c) 16 km (d) Given data is insufficient to calculate the distance

Soln. This question is solved by hit and trail method - in every possible case.

One case is suppose a person walk for $2\frac{1}{2}$ hours up hill then came back $1h$ down hill.

So he will cover distance = $6 \times 2.5 - \frac{1}{2} \times 10$



distance from A to B = 15 and BC = 5

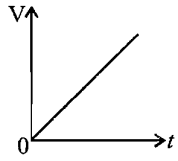
But person is at point C. From point C h[illegible]

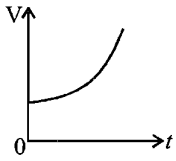
come at point A down hill in one hour.

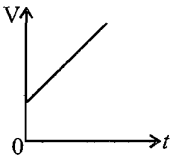
Distance between A to B = 15 K.M.

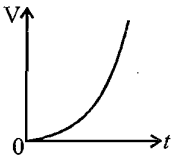
Correct option is (a).

6. A vessel is partially filled with water. More water is added to it at a rate directly proportional to time $[i.e., \frac{dV}{dt} \propto t]$. Which of the following graphs depicts correctly the variation of total volume V of water with time t?

(a) 

(b) 

(c) 

(d) 

Soln. Here initially vessel has some water. So option (a) and (d) wrong, since they represent no water in vessel. Since filling rate is directly proportional of time. So option (c) is also wrong then option (b) is correct.

7. At one instant, the hour hand and the minute hand of a clock are one over the other in between the markings for 5 and 6 on the dial. At this instant, the tip of the minute hand
   (a) is closer to the marking for 6
   (b) is equidistant from the markings for 5 and 6
   (c) is closer to making for 5
   (d) is equidistant from the markings for 11 and 12

Soln. Overlapping between 5:00 and 6:00.

$$0^\circ = \frac{1}{2}|60\times 5 - 11M|$$

$$0 = 300 - 11M$$

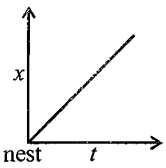
$$11M = 300$$

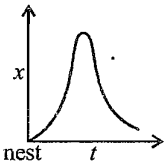
$$M = \frac{300}{11} = 27\frac{3}{11} \text{ min}$$

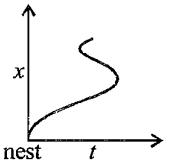
$$\frac{3}{11}\text{min} = \frac{3}{11}\times 60 = \frac{180}{11} = 16.3 = 16 \text{ sec. (approx.)}$$

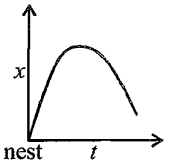
So time is 5 : 27 : 16 sec. If time is 5 : 27 : 30 then minute hand will exactly between 5:00 and 6:00. Since time is 5 : 27 : 16 hence minute hand closer to 5.

8. A bird leaves its nest and flies away. Its distance x from the nest is plotted as a function of time t. Which of the following plots cannot be right?

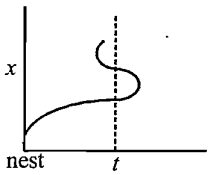
(a) 

(b) 

(c) 

(d) 

Soln. We draw a vertical line in a graph. It is intersect graph in 2 or more than 2 points, then it is wrong.



Since at a point of time, the distance from nest can not have two positions.

So option (c) is wrong.

9. A cubical cardboard box made of 1 cm thick card board has outer side of 29 cm. A tight-fitting cubical box of the same thickness is placed inside it, then another one inside it and so on. How many cubical boxes will be there in the entire set?

(a) 29 (b) 28 (c) 15 (d) 14

Soln. Each box cover 1 cm + 1 cm = 2 cm width

Total width of larger box = 29 cm

hence number of box = $\frac{29}{2}$ = 14.5 ≃ 14 small box

10. Secondary colours are made by a mixture of three primary colours, Red, Green and Blue, in different proportions; each of the primary colours comes in 8 possible levels. Grey corresponds to equal proportions of Red, Green and Blue. How many shades of grey exist in this scheme?

(a) 8^3 (b) 8 (c) 3^8 (d) 8 × 3

Soln. Each colour can come in 8 levels

So number of different combination = $\boxed{8}\boxed{8}\boxed{8}$ = 8^9 ways

but for grey colour equal ratio are required

hence possible shade of grey = $\boxed{8}\boxed{1}\boxed{1}$ = 8 × 1 × 1 = 8

11. The triangle formed by the lines $y = x$, $y = 1 - x$ and $x = 0$ in a two dimensional plane is (x and y axes have the same scale)

(a) isosceles and right-angled (b) isosceles but not right-angled

(c) right-angled but not isosceles (d) neither isosceles nor right-angled

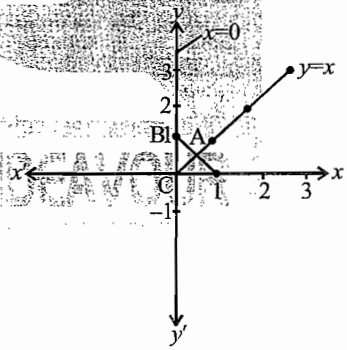
Soln. Graph $y = x$

x	1	2	3
y	1	2	3

for $x = 0$ it graph is y axis

for $y = 1 - x$

x	0	1	2
y	1	0	−1



Δ ABC is our required triangle

which is isosceles and right angle

Correct option is (a)

12. There are two buckets A and B. Initially A has 2 litres of water and B is empty. At every hour 1 litre of water is transferred from A to B followed by returning $\frac{1}{2}$ litre back to A from B half an hour later. The earliest A will get empty is in

(a) 5 h (b) 4 h (c) 3 h (d) 2 h

Soln.

	A	B
	2	0
After 1 hour	1	1
1/2 hour later	$1\frac{1}{2}$	$\frac{1}{2}$

After 2 hour	$\frac{1}{2}$	$1\frac{1}{2}$
1/2 hour later	1	1
After 3 hour	0	1

Correct option is (c)

13. Statement A: The following statement is true
Statement B: The preceding statement is false
Choose the correct inference from the following:
(a) Statements A and B are always true
(b) Statements A and B can be true if there is at least one statement between A and B
(c) Statements A and B can be true if there are at least two statements between A and B
(d) Statements A and B can never be true, independently

Soln. Statement A can be true of and only if at least one statement is given below statement A.
Statement B can be true if and only if at least one statement is given above it
Since both statement A and B are true hence they have at least two statement between them.
Correct option is (c)

14. A car is moving at 60 km/h. The instantaneous velocity of the upper most points of its wheels is
(a) 60 km/h forward (b) 120 km/h forward
(c) 60 km/h backward (d) 120 km/h backward

Soln. In a moving article, the instantaneous velocity of the lower most point is always equal to its speed and at upper most point the velocity is twice of its speed.
Correct option is (b)

15. If D + I + M = 1501
C + I + V + I + L = 157
L + I + V + I + D = 557
C + I + V + I + C = 207
What is V + I + M = ?
(a) Cannot be found (b) 1009
(c) 1006 (d) 509

Soln. ****

16. A living cell has a protoplasm which is water based and demarcated by a lipid bilayer membrane. If a cell is pierced up to $\frac{1}{5}$th of its diameter with a very sharp needle, after taking the needle out
(a) no effect will be observed
(b) protoplasm will leak out from the hole made by the needle for a few minutes until the cell heals the wound
(c) protoplasm will keep on leaking out till the cell is dead
(d) the cell will burst like a balloon

Soln. (a) No effect will be observed.

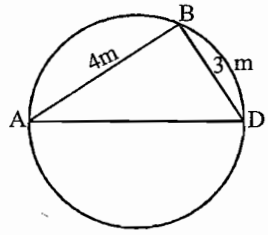
17. Density of a rice grain is 1.5 g/cc and bulk density of rice heap is 0.80 g/cc. If a 1 litre container is completely filled with rice, what will be the approximate volume of pore space in the container?
(a) 350 cc (b) 465 cc (c) 550 cc (d) 665 cc

Soln. **Correct option is (b)**

18. A turtle starts swimming from a point A located on the circumference of a circular pond. After swimming for 4 meters in a straight line it hits point B on the circumference of the pond. From there it changes direction and swims for 3 meters in a straight line and arrives at point D diametrically opposite to point A. How far is point D from A?

(a) 3 m (b) 4 m (c) 7 m (d) 5 m

Soln. Clearly, the given path is right angle triangle we have to find AD.



Hence, $AD^2 = AB^2 + BD^2$

$= 4^2 + 3^2$

$= 16 + 9$

$= 25$

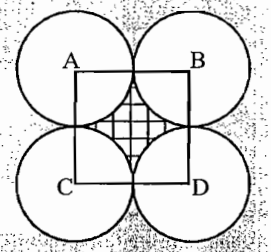
$\therefore$ AD = 5 m

Correct option is (d)

19. Four circles of unit radius each are drawn such that each one touches two others and their centres lie on the vertices of a square. The area of the region enclosed between the circles is

(a) $\pi - 1$ (b) $\pi - 2$ (c) $3 - \pi$ (d) $4 - \pi$

Soln. If we substract area of $\frac{1}{4}th$ part of all circle from square ABCD then we get the required area



Now side of a square = 1 + 1 = 2 units

Hence its area = $2^2 = 4$ unit^2

Radius of circle = 1 unit

Hence area of part of circle $= \frac{1}{4}\pi.1^2 = \frac{\pi}{4}$

Area of all 4 part $= 4 \times \frac{\pi}{4} = \pi$ unit^2

Therefore, required area = $4 - \pi$

CAREER ENDEAVOUR

Correct option is (d)

20. A film projector and microscope give equal magnification. But a film projector is not used to see living cells because

(a) a living cell cannot be placed in a film projector

(b) the viewer's eye is close to a microscope whereas it is far away from the projector's screen

(c) a microscope produces a virtual image whereas a projector produces a real image

(d) a microscope has greater resolving power than a projector.

Soln. **Correct option is (d)**

CSIR-NET/JRF - Dec. 2015 Paper (Chemistry)

1. "The clue is hidden in this statement", read the note handed to Sherlock by Moriarty, who hid the stolen treasure in one of the ten pillars. Which pillar is it?
(a) X (b) II (c) III (d) IX

Soln. THE CULE IS HIDD ENIN THIS STATEMENT.
Correct option is (d)

2. Suppose three meetings of a group of professors were arranged in Mumbai, Delhi and Chennai. Each professor of the group attended exactly two meetings. 21 professors attended Mumbai meeting, 27 attended Delhi meeting and 30 attended Chennai meeting. How many of them attended both the Chennai and Delhi meetings?

(a) 18 (b) 24
(c) 26 (d) cannot be found

Soln. Mumbai = 21, Delhi = 27, Chennai = 30

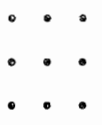
This question will solve hit and trait method
let Delhi and Chennai = 18
Remaining Chennai = 12
Delhi = 9
Out of Mumbai 21, 12 person attend with remain 12 Chennai
Now remaining 9 Mumbai can attend meeting with 9 Delhi.
Correct option is (a)

3. The probability that a ticketless traveler is caught during a trip is 0.1. If the traveler makes 4 tripes, the probability that he/she will be caught during at least one of the trips is

(a) $1-(0.9)^4$ (b) $(1-0.9)^4$ (c) $1-(1-0.9)^4$ (d) $(0.9)^4$

Soln. Probability of caught = 0.1
not caught = 1 – 0.1 = 0.9
So required probability = 1 – not caught in all 4 trips
$= 1 - 4_{Co}\,(0.1)\,(0.9)^4 = 1 - (0.9)^4$
Correct option is (a)

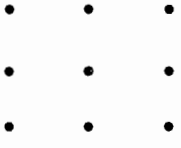
4.



The minimum number of straight lines required to connect the nine points above without lifting the pen or retracing is

(a) 3 (b) 4 (c) 5 (d) 6

Soln.

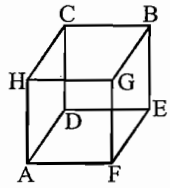


minimum number of lines required is 5
Correct option is (c)

5. Let A, B be the ends of the longest diagonal of the unit cube. The length of the shortest path from A to B along the surface is

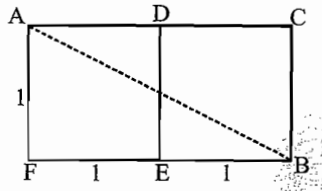
(a) $\sqrt{3}$ (b) $1+\sqrt{2}$ (c) $\sqrt{5}$ (d) 3

Soln.



Let side of cube is 1 unit

If we expand the surface ADEF and BCDE



FB = 2 units and AF = 1

then $AB^2 = AF^2 + BF^2 = 1^2 + 2^2 = 1 + 4$

$AB = \sqrt{5}$

Correct option is (c)

6. How many digits are there in 3^{16} when it is expressed in the decimal form?

(a) three (b) six (c) seven (d) eight

Soln. There are eight digit in 3^{16}.

when a 'n' digit number is multiplied to another 'n' digit number then result contain either '$2n$' digit or '$2n-1$' digit.

Correct option is (d)

CAREER ENDEAVOUR

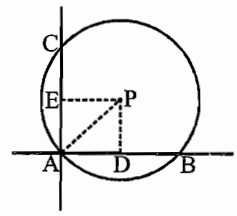
7. A circle drawn in the x-y coordinate plane passes through the origin and has chords of lengths 8 units and 7 units on the x and y axes, respectively. The coordinates of its centre are

(a) (8, 7) (b) (–8, 7) (c) (–4, 3.5) (d) (4, 3.5)

Soln. Let P is centre and its co-ordinates are (x, y)

AB = 8 unit and PD $\perp$ AB

hence $AD = \frac{8}{2} = 4$ units



AD = PE = Co-ordinate of x

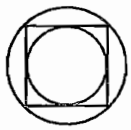
Now, AC = 7 units PE $\perp$ AC

$\Rightarrow$ AE = PD = co-ordinates of y = 3.5 unit

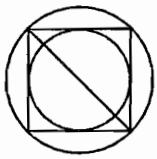
hence co-ordinate of centre = (4, 3.5)

Correct option is (d)

8. There is an inner circle and an outer circle around a square. What is the ratio of the area of the outer circle to that of the inner circle?



(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) $\sqrt{3/2}$

Soln. 

Let side of square is a, then diameter of incircle = a $\Rightarrow r_{inc} = \frac{a}{2}$

$\text{Area}_{inc} = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$

now diameter of outcircle = diagonal of square = $\sqrt{2}a$. Hence $r_{cc} = \frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

$\text{Area}_{cc} = \pi\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{\pi a^2}{2}$ $\therefore \frac{\text{Area}_{cc}}{\text{Area}_{inc}} = \frac{\pi a^2 \times 4}{2 \times \pi a^2} = 2$

Correct option is (b)

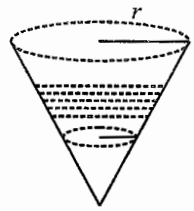
9. The base diameter of a glass is 20% smaller than the diameter at the rim. The glass is filled to half the height. The ratio of empty to filled volume of the glass is

(a) $\frac{\sqrt{10}-\sqrt{9}}{\sqrt{9}-\sqrt{8}}$ (b) $\frac{10-9}{9-8}$ (c) $\frac{10^2-9^2}{9-8}$ (d) $\frac{10^3-9^3}{9^3-8^3}$

Soln. Let glass is a part of given cone

Let at top radius is r and bottom of

glass $r = r \times \frac{(100-20)}{100} = r \times \frac{80}{100} = \frac{4r}{5}$



in cone $r \propto h$

for top let height is h

and for bottom of class = $\frac{4h}{5}$

Since glass is filled half so at filling level of glass $r = r \times \frac{90}{100} = \frac{9r}{10}$ and $h = \frac{9h}{10}$

Volume of empty part of glass $= \frac{1}{3}\pi r^2 h - \frac{1}{3} r\pi \left(\frac{9r}{10}\right)^2 \times \left(\frac{9h}{10}\right)$

$$= \frac{1}{3}\pi\left[r^2h - \left(\frac{9}{10}\right)^3 r^2h\right] = \frac{1}{3}\pi r^2 h\frac{(10^3 - 9^3)}{10^3}$$

Volume of filled part $= \frac{1}{3}\pi\left(\frac{9r}{10}\right)^2\left(\frac{9h}{10}\right) - \frac{1}{3}x\pi\times\left(\frac{8r}{10}\right)^2\left(\frac{8h}{10}\right)$

$$= \frac{1}{3}\pi r^2 h\left[\left(\frac{9}{10}\right)^3 - \left(\frac{8}{10}\right)^3\right]$$

$$= \frac{1}{3}\pi r^2 h\frac{(9^3 - 8^3)}{10^3}$$

hence ratio $= \dfrac{\frac{1}{3}\pi r^2 h\frac{(10^3 - 9^3)}{10^3}}{\frac{1}{3}\pi r^2 h\frac{(9^3 - 8^3)}{10^3}} = \dfrac{10^3 - 9^3}{9^3 - 8^3}$

Correct option (d)

10. A wheel barrow with unit spacing between its wheels is pushed along a semi-circular path of mean radius 10. The difference between distances covered by the inner and outer wheels is

(a) 0 (b) 10 (c) π (d) 2π

Soln. Correct option is (d)

11. Write d = 1 degree, r = 1 radian and g = 1 grad. Then which of the following is true? (100 grad = a right angle)

(a) $\cos d < \cos r < \cos g$ (b) $\cos r < \cos g < \cos d$

(c) $\cos r < \cos d < \cos g$ (d) $\cos g < \cos d < \cos r$

Soln. $\because$ 100 grad = 90° $\therefore$ 180° = π^C

$\therefore$ 1 grad $= \frac{90}{100} = 0.9°$ $\therefore$ $1° = \frac{\pi}{180} = \frac{22}{7\times180}$ (which is less than 1^C)

$\therefore$ 1 grad < 1 degree $\therefore$ 1 degree < 1 radian

Now combining these two

1 grad $\leq$ 1 degree < 1 radian

Hence, $\cos g > \cos d > \cos r$

Correct option is (c)

12. A vendor sells articles having a cost price of Rs. 100 each. He sells these articles at a premium price during first eight months, and at a sale price, which is half of the premium price, during next four months. He makes a net profit of 20% at the end of the year. Assuming that equal numbers of articles are sold each month, what is the premium price of the article?

(a) 122 (b) 144 (c) 150 (d) 160

Soln. Let CP of an article = 100 rs

Let premium price = x rs and sale price = $\frac{x}{2}$ rs

Let each month he sold 1 article then in 8 month he sold 8 article and next 4 month, 4 articles

$$\Rightarrow 8x+4\cdot\frac{x}{2}=12\times100\frac{(100+20)}{100}$$

$10x=12\times120$

$x=12\times12 = 144$ rs

Correct option is (b)

13. The statement: "The father of my son is the only child of your parents"
(a) can never be true
(b) is true in only one type of relation
(c) can be true for more than one type of relations
(d) can be true only in a polygamous family

Soln. It is true in one type of relation if the statement is said by the wife of person

Father of wife's son – wife's husband

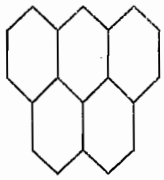
Only child of your parents - 'you'

Here wife tells the statement to her husband.

Correct option is (b)

14. One is required to tile a plane with congruent regular polygons. With which of the following polygons is this possible?
(a) 6-gon (b) 8-gon (c) 10-gon (d) 12-gon

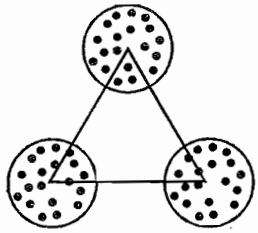
Soln. 6-gon



from easy looking it is clear that only in case of Hexagon it is possible.

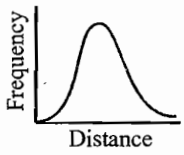
Correct option is (a)

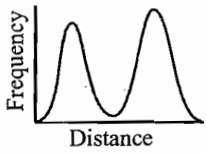
15. Three circles of equal diameters are placed such that their centres make an equilateral triangle as in the figure

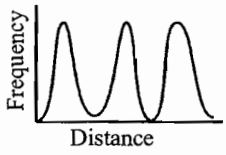


Within each circle, 50 points are randomly scattered. The frequency distribution of distances between all possible pairs of points will look as

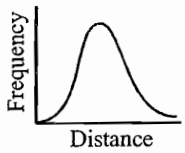
(a)

(b) 

(c) 

(d) 

Soln. When number of points are large and it randomly scattered then it will follow gaussian distribution.



Correct option is (b)

16. Most Indian tropical fruit trees produce fruits in April-May. The best possible explanation for this is
(a) optimum water availability for fruit production
(b) the heat allows quicker ripening of fruit.
(c) animals have no other source of food in summer.
(d) the impending monsoon provides optimum conditions for propagation.

Soln. The impending monsoon provides optimum conditions for propagation.

Correct option is (d)

17. The number of diagonals of a convex deodecagon (12-gon) is
(a) 66 (b) 54 (c) 55 (d) 60

Soln. No. of diagonals in any polygon $= n_{c_2} - n$

for diagonals $n = 12$

$$= 12_{c_2} - 12$$

$$= \frac{12 \times 11}{4 \times 1} = 12$$

$$= 46 - 12 = 54$$

Correct option is (b)

18. Three boxes are coloured red, blue and green and so are three balls. In how many ways can one put the balls one in each box such that no ball goes into the box of its own colour?
(a) 1 (b) 2 (c) 3 (d) 4

Soln. Let 3 boxes are R, B, G and 3 balls are r, b and g

Now 3 balls can be put in 3 boxes. | 3 | 2 | 1 |

$= 3 \times 2 \times 1 = 6$ ways

R	B	G
r	b	g
r	g	b
b	r	g
b	g	r
g	r	b
g	b	r

So, there are only 2 way.

Correct option (b)

19. Decode

G	E	N	T	S	T	U
I	S	S	O	L	V	D
L	I	I	S	P	A	E
L	M	H	T	R	B	N
E	E	L	B	O	L	T
T	N	I	Y	B	E	S

(a) GENT STUDENT CAUSE LITTLE HEART BURNS
(b) STUDENTS ARE INTELLIGENT BUT PROBLEM IS NOT SOLABLE
(c) THIS PROBLEM IS UNSOLVABLE BY ANY STUDENT
(d) THIS PROBLEM IS SOLVABLE BY INTELLIGENT STUDENTS.

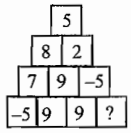
Soln. From given alphabets only this sentence can be formed

THIS PROBLEM IS SOLVABLE BY INTELLIGENT STUDENTS.

Correct option is (d)

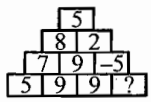
CAREER ENDEAVOUR

20. The missing number is



(a) –19 (b) –5 (c) 9 (d) –9

Soln.



Every above number is average of its two below number as

$$\frac{8+2}{2}=5,\ \frac{7+9}{2}=8,\ \frac{9+(-5)}{2}=2,\ \frac{5+9}{2}=7,\ \frac{9+9}{2}=9$$

hence, $\frac{9+x}{2}=-5,\ 9+x=-10,\ x=-19$

Correct option is (a)

CSIR-NET/JRF - June 2016 Paper (Chemistry)

1. It takes 2 hours for Tiwari and Deo to do a job. Tiwari and Hari take 3 hours to do the same job. Deo and Hari take 6 hours to do the same job. Which of the following statements is incorrect?
(a) Tiwari alone can do the job in 3 hours
(b) Deo alone can do the job in 6 hours
(c) Hari does not work at all
(d) Hari is the fastest worker

Soln. 1 hour work of T and D

$$\frac{1}{T}+\frac{1}{D}=\frac{1}{2} \quad \ldots (1)$$

1 hour work of T and H

$$\frac{1}{T}+\frac{1}{H}=\frac{1}{3} \quad \ldots (2)$$

1 hour work of D and H

$$\frac{1}{D}+\frac{1}{H}=\frac{1}{6} \quad \ldots (3)$$

By adding (1), (2) and (3)

$$2\left(\frac{1}{T}+\frac{1}{D}+\frac{1}{H}\right)=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$$

$$\therefore \quad \frac{1}{T}+\frac{1}{D}+\frac{1}{H}=\frac{1}{2}$$

Subtracting (1), (2) and (3) by above equation $H=0,\ D=6,\ T=3$

Hence, option (d) is incorrect.

2. Abdul travels thrice the distance Catherine travels, which is also twice the distance that Binoy travels. Catherine's speed is 1/3 of Abdul's speed, which is also 1/2 of Binoy's speed. If they start at the same time then who reaches first?
(a) Both Abdul and catherine (b) Binoy
(c) Catherine (d) All three together

Soln. Let C covers = 10 km, let speed of C = 10 km/h, then A covers = 30 km, then speed of A = 30 km/h, and B covers = 15 km and that of B = 20 km/h.

So time taken by $C=\frac{10}{10}=1$ hour

$$\text{By } A=\frac{30}{30}=1 \text{ hour}$$

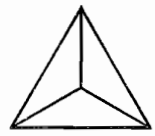
$$\text{By } B=\frac{15}{20}=\frac{3}{4} \text{ hour}$$

Clearly, B will take least time and reach his destination first.

Hence, option (b) is correct.

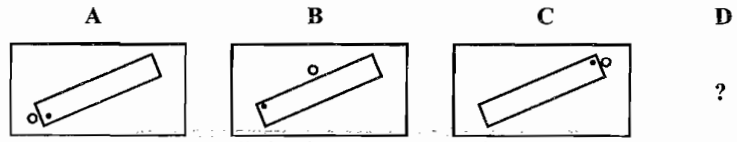
3. For a certain regular solid; number of faces + number of vertics = number of edges + 2. For three such distinct (not touching each other) objects, what is the total value of faces + vertices – edges?
(a) two (b) four (c) six (d) zero

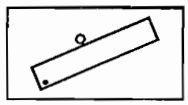
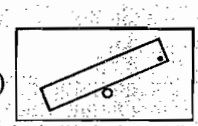
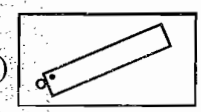
Soln. Number of faces + number of vertices = number of edges + 2
The above condition is true for tetrahedron.



Face + vertices – edges = 4 + 4 – 2 = 6.
Hence, option (c) is correct.

4. What will be the next figure in the following sequence?



(a)  (b) (c)  (d) 

Soln. By observation of given figures, the (3)rd figure is correct.
Hence, option (c) is correct.

5. A, B, C and D are points on a circle with AB = 5 cm, BC = 12 cm, AC = 13 and AD = 7 cm. Then, the closest approximation of CD is
(a) 9 cm (b) 10 cm (c) 11 cm (d) 14 cm

Soln. In ΔABC

$\because \quad AB^2 + BC^2 = AC^2$

$5^2 + 12^2 = 13^2$

Hence, AC is diameter of given circle, then ΔADC must be right angle triangle then,

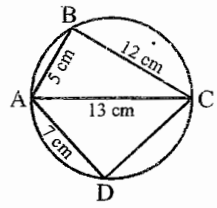
$AD^2 + CD^2 = AC^2$

$7^2 + CD^2 = 13^2$

$49 + CD^2 = 169$

$CD = 120$

$CD = \sqrt{120} \approx 11 \text{ cm}$

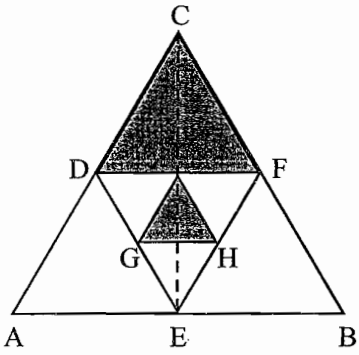


Hence, option (c) is correct.

6. Choose the four digit number, in which the product of the first and fourth digits is 40 and the product of the middle digits is 28. The thousands digit is as much less than the unit digit as the hundreds digit is less than the tens digit
(a) 5478 (b) 5748 (c) 8745 (d) 8475

Soln. By observation of given numbers, only 5478 satisfies given conditions.
Hence, option (a) is correct.

7. Equilateral triangles are drawn one inside the other as shown. What is the ratio of the two shaded areas?



(a) 2 : 1 (b) $\sqrt{3}:4$ (c) 4 : 1 (d) 8 : 1

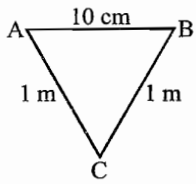
Soln. In given figure, the largest triangle firstly divided in 4 equilateral triangle having same area. Thereafter one of them is again divided in 4 smaller equilateral triangle having same area. Hence, ratio will be 4 : 1.

Hence, option (c) is correct.

8. A frogs hops and lands exactly 1 meter away at a time. What is the least number of hops required to reach a point 10 cm away?

(a) 1
(b) 2
(c) 3
(d) it cannot travel such a distance

Soln.



Frog firstly jump from point A to C then it jump C to B. So 2 least jump are required.

Hence, option (b) is correct.

CAREER ENDEAVOUR

9. A train running at 36 km/h crosses a mark on the platform in 8 sec and takes 20 sec to cross the platform. What is the length of the platform?

(a) 120 m (b) 280 m (c) 40 m (d) 160 m

Soln. Let length of train is d m and speed of train = 36 km/h = $36\times\frac{5}{18}=10$ m/sec.

when it cross the pole, it will cover the distance equal to its own length, then $\frac{d}{10}=8$. Since d = 80 m and when it crosses the platform, it covers the distance equals to sum of its own length and length of platform, hence

$\frac{80+x}{10}=20$; $80+x=200$; $x=120$ m.

Hence, option (a) is correct.

10. When a polynomial $f(x)$ is divided by $x-5$ or $x-3$ or $x-2$ it leaves a remainder of 1. Which of the following would be the polynomial?

(a) $x^3-10x^2+31x+31$ (b) $x^3-10x^2+31x-29$

(c) $x^3-10x^2+31x-31$ (d) $x^3-10x^2+31x+29$

Soln. By remainder theorem, we will put $x=5$, 3 and 2 in each polynomial. Only polynomial $x^3-10x^2+31x-29$ gives remainder 1 for each x.

Hence, option (b) is correct.

11. Water is slowly dripping out of a tiny hole at the bottom of a hollow metallic sphere initially full of water. Ignoring the water that has flowed away, the centre of mass of the system

(a) remains fixed at the centre of the sphere

(b) moves down steadily as the amount of water decreases

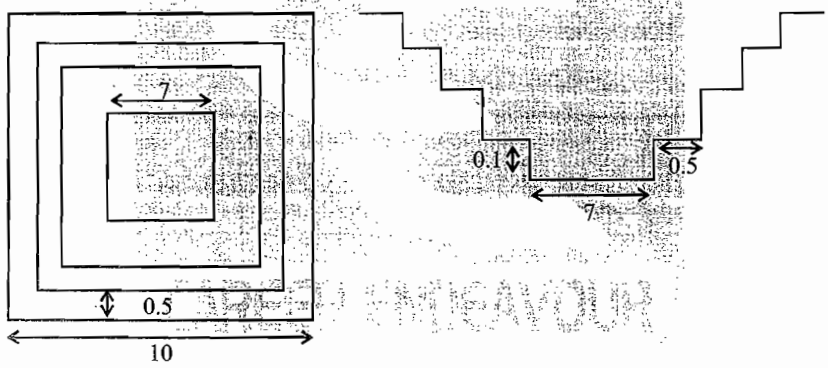
(c) moves down for some time but eventually returns to the centre of the sphere

(d) moves down until half of the water is lost and then moves up

Soln. If water is dripping out slowly, its center of mass moves down slowly. But finally we have to ignoring water that has flowed away then again it will come on the center of sphere.

Hence, option (c) is correct.

12. The diagram (not to scale) shows the top view and cross-section of a pond having a square outline and equal sized steps of 0.5 m width and 0.1m height. What will be the volume of water (in m^3) in the pond when it is completely filled?



(a) 40.0 (b) 29.4 (c) 19.4 (d) 11.3

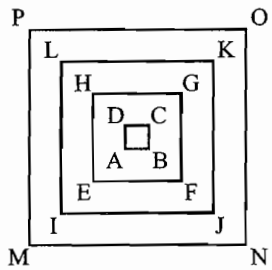
Soln. Volume of ABCD = $7 \times 7 \times 0.1 = 4.9$

Volume of EFGH = $(7 + 0.5 - 0.5)\,(7 + 0.5 + 0.5) \times 0.1 = 8 \times 8 \times 0.1 = 6.4$

Volume IJKL = $(8 + 0.5 + 0.5)\,(8 + 0.5 + 0.5) \times 0.1 = 9 \times 9 \times 0.1 = 8.1$

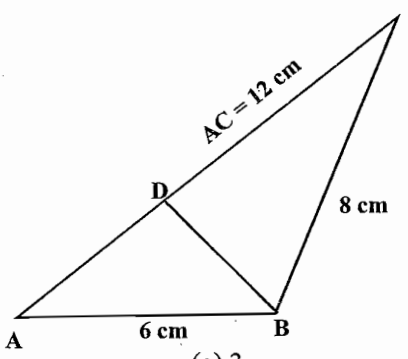
Volume of MNOP = $(9 + 0.5 + 0.5) \times (9 + 0.5 + 0.5) \times 0.1 = 10 \times 10 \times 0.1 = 10$

Adding all $4.9 + 6.4 + 8.1 + 10.0 = 29.4$



Hence, option (b) is correct.

13. D is a point on AC in the following triangle such that $\angle ADB = \angle ABC$. Then BD (in cm) is



(a) 8 (b) 6 (c) 3 (d) 4

Soln. In ΔADB and ΔABC

$\angle ADB = \angle ABC$ (given)

and $\angle A = \angle A$ (common)

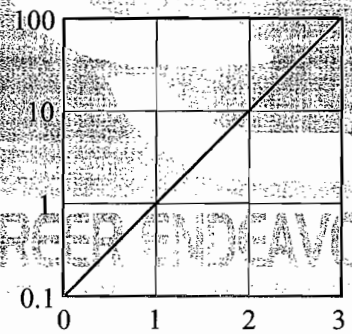
hence $\Delta ADB \sim \Delta ABC$

thereafter $\frac{AB}{AC} = \frac{BD}{BC}$, $\frac{6}{12} = \frac{BD}{8}$

$\Rightarrow BD = 4.$

Hence, option (d) is correct.

14. The function $f(x)$ is plotted against x as shown. Extrapolate and find the value of the function at $x = -1$.



(a) –0.01 (b) –0.1 (c) 0.01 (d) 0.1

Soln. By observation of given graph, the function of graph is $f(x) = 10^{x-1}$

Since, $f(0) = 10^{0-1} = 10^{-1} = \frac{1}{10} = 0.1$

$f(1) = 10^{1-1} = 10^0 = 1$

$f(2) = 10^{2-1} = 10^1 = 10$

$f(3) = 10^{3-1} = 10^2 = 100$

Hence, $f(-1) = 10^{-1-1} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$

Hence, option (c) is correct.

15. A notebook contains only hundred statements as under :
(1) This notebook contains 1 false statement
(2) This notebook contains 2 false statements
.
.
.
(99) This notebook contains 99 false statements
(100) This notebook contains 100 false statements
Which of the statements is correct?
(a) 100^{th} (b) 1^{st} (c) 99^{th} (d) 2^{nd}

Soln. By analysis of all option only statement 99 is correct.
Hence, option (c) is correct.

16. A chocolate bar having m×n unit square tiles is given. Calculate the number of cuts needed to break it completely, without stacking, into individual tiles
(a) $(m\times n)$ (b) $(m-1)\times(n-1)$ (c) $(m\times n)-1$ (d) $(m\times n)+1$

Soln. This type question is solved hit and trial method.
Let $m=2$ and $n=3$

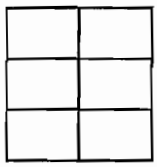


Figure-1

Let $m=3$ and $n=4$

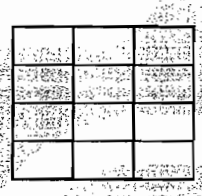


Figure-2

For figure (1) for, m we will give only 1 cut. For each m we will give 2 cuts for finding n. So number of cuts $2\times3-1=5$ for figure (2) for m we will give 2 cuts. For every m, we will give 4 cuts for n. So number of cuts $=3\times4-1=11$
Generalization of it, clearly $m\times n-1$ satisfies.
Hence, option (c) is correct.

17. A person paid income tax at the rate of R% for the first Rs 2 lakhs, and at the rate of (R + 10)% for income exceeding Rs 2 lakhs. If the total tax paid is (R + 5)% of the annual income, then what is the annual income?
(a) Rs 2.5 lakhs (b) Rs. 3.0 lakhs (c) Rs. 4.0 lakhs (d) Rs 5.0 lakhs

Soln. Let total income of person is $=x$ rs then according to condition

$$\Rightarrow \quad 200000\times\frac{R}{100}+(x-200000)\times\frac{(R+10)}{100}=x\times\frac{(R+5)}{100}$$

$$\Rightarrow \quad 200000\times\frac{R}{100}+\frac{x\cdot R}{100}+\frac{10x}{100}-200000\times\frac{R}{100}-\frac{200000\times10}{100}=\frac{x\cdot R}{100}+\frac{5x}{100}$$

$$\Rightarrow \quad \frac{10x}{100}-\frac{5x}{100}=200000\times\frac{10}{100}$$

$$\Rightarrow \quad \frac{5x}{100}=\frac{200000\times10}{100}$$

$$\Rightarrow \quad x=400000 \text{ rs.}$$

Hence, option (c) is correct.

18. An experiment leads to the following set of observations of the variable 'v' at different time 't'

t	0	1	2	3	4	5	6
v	5	6.1	9.1	13.7	20.6	30.8	41.1

Allowing for experimental errors, which of the following expressions best describes the relationship between t and v?

(a) $v \propto t^2$ (b) $(v-5) \propto t^2$ (c) $v = 5t + t^2$ (d) $(v-5) = (t+5)^2$

Soln. Putting value of t in all equations with value of v only $(v-5) \propto t^2$ equation is satisfies.

Hence, option (b) is correct.

19. The difference between the squares of the ages (in complete years) of a father and his son is 899. The age of the father when his son was born

(a) cannot be ascertained due to inadequate data

(b) is 27 years

(c) is 29 years

(d) is 31 years

Soln. This type question can be solved hit and trial method.

Suppose at the time of birth of son, father is 29 years, after 1 year father will 30 years and son = 1 year.

Therefore, $30^2 - 1^2 = 899$.

Hence, option (c) is correct.

20. A bicycle tube has a mean circumference of 200 cm and a circular cross-section of diameter 6 cm. What is the approximate volume of water (in cc) required to completely fill the tube, assuming that it does not expand?

(a) $600\,\pi$ (b) $1200\,\pi$ (c) $3600\,\pi$ (d) $1800\,\pi$

Soln. If we cut the bicycle tube, it will take form of a cylinder

whose height = 200 cm

$r = \frac{6}{2} = 3$ cm

Hence volume $\pi \cdot 3^2 \cdot 200 = 1800\pi$ cm^3.

Hence, option (d) is correct.

CAREER ENDEAVOUR

CSIR-NET/JRF - June 2016 Paper (Physics)

1. An infinite number of identical circular discs each of radius $\frac{1}{2}$ are tightly packed such that the centers of the discs are at integer values of coordinates x and y. The ratio of the area of the uncovered patches to the total area is

(a) $1-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $1-\pi$ (d) π

Soln. **Correct option is (a)**

2. It takes 5 days for a steamboat to travel from A to B along a river. It takes 7 days to return from B to A. How many days will it take for a raft to drift from A to B (all speeds stay constant)?

(a) 13 (b) 35 (c) 6 (d) 12

Soln. Let distance of A and B is 1 and speed of steam boat $= x$
that of current $= y$

then, $\frac{1}{x+y}=5$ ... (i)

and $\frac{1}{x-y}=7$ ... (ii)

$\Rightarrow \quad 5x+5y=1\times 7$

$7x-7y=1\times 5$

$\Rightarrow \quad 35x+35y=7$

$35x-35y=5$

$70y=2$

$\Rightarrow \quad y=\frac{2}{70}$ then $5x+5\cdot\frac{2}{70}=1$

$\Rightarrow \quad 5z=7\Rightarrow x=\frac{7}{5}$

Then for a raft $=\frac{1}{2/70}=35$ *days*

Correct option is (b)

3. "My friend Raju has more than 1000 books", said Ram. "Oh no, he has less than 1000 books", said Shyam. "Well, Raju certainly has at least one book", said Geeta. If only one of these statements is true, how many books does Raju have ?

(a) 1 (b) 1000 (c) 999 (d) 1001

Soln. Since one of them is telling true. Hence, if we take 1000 books then Geeta's statement becomes true
Correct option is (b)

4. Of the following, which is the odd one out ?

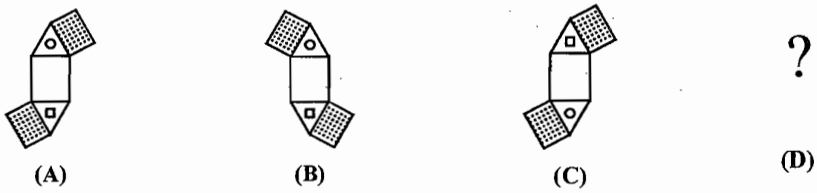
(a) Cone (b) Torus (c) Sphere (d) Ellipsoid

Soln. Cone is a figure containing straight line, but remaining three do not contain straight line.
Correct option is (b)

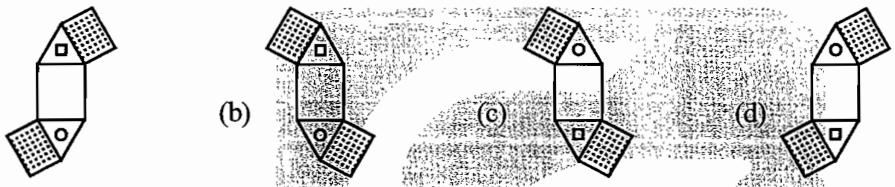
5. A student appearing for an exam is declared to have failed the exam if his/her score is less than half the median score. This implies
(a) 1/4 of the students appearing for the exam always fail.
(b) If a student scores less than 1/4 of the maximum score, he/she always fails.
(c) If a student scores more than 1/2 of the maximum score, he/she always passes.
(d) It is possible that no one fails.

Soln. By definition of median, it is a number which divides the given group in two equal parts. So, option(a), (b) and (c) can not satisfies in each case, while option (d) can be possible.
Correct option is (d)

6. Find the next figure 'D"



(a) (b) (c) (d)



Soln. By observation of given figure, second figure satisfies.
Correct option is (b)

7. N is a four digit number. If the leftmost digit is removed, the resulting three digit number is 1/9th of N. How many such N are possible ?
(a) 10 (b) 9 (c) 8 (d) 7

Soln. Let N = abcd. Now, $\frac{1}{9}\times N = bcd$

put according to place value

$\frac{1}{9}\times(1000a+100b+10c+d)=100b+10c+d$

After solving it, we get

$125a = 100b+10c+d$

$125a = bcd$

a	bcd	$N = abcd$
1	125	1125
2	250	2250
3	375	3375
4	500	4500
5	625	5625
6	750	6750
7	875	6750
8	1000	not acceptable

So, there is total 7 numbers.
Correct option is (d)

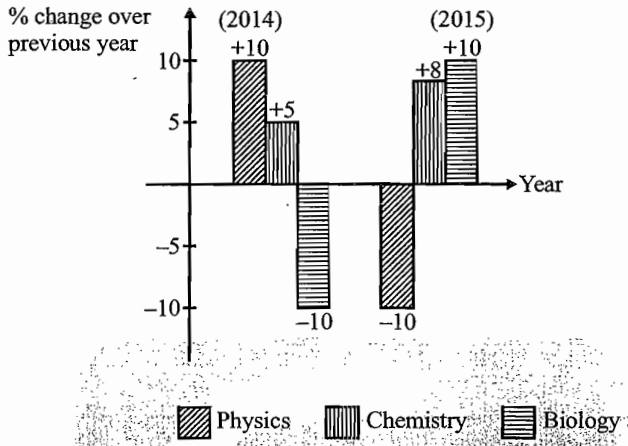
CAREER ENDEAVOUR

8. AB and CD are two chords of a circle subtending 60° and 120° respectively at the same point on the circumference of the circle. Then AB : CD is

(a) $\sqrt{3}:1$ (b) $\sqrt{2}:1$ (c) 1 : 1 (d) $\sqrt{3}:\sqrt{2}$

Soln. We know that angle made by any arc on the centre of the circle is twice of the angle made by same arc on any point on the remaining surface. So, by given condition both chords AB and CD must be equal length.
Correct option is (c)

9.



Which of the following inferences can be drawn from the above graph ?

(a) The total number of students qualifying in Physics in 2015 and 2014 is the same
(b) The number of students qualifying in Biology in 2015 is less than that in 2013
(c) The number of Chemistry students qualifying in 2015 must be more than the number of students who qualified in Biology in 2014
(d) The number of students qualifying in Physics in 2015 is equal to the number of students in Biology that qualified in 2014

Soln. Let in 2013, student in each subject = 100

Subject	2013	2014	2015
Physics	100	110	99
Chemistry	100	105	113.4
Biology	100	90	99

From above table it is clear that (b) is correct
Correct option is (b)

10. What is the minimum number of moves required to transform figure 1 to figure 2 ? A move is defined as removing a coin and placing it such that it touches two other coins in its new position.

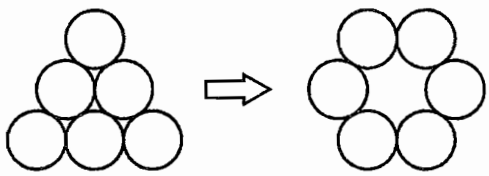
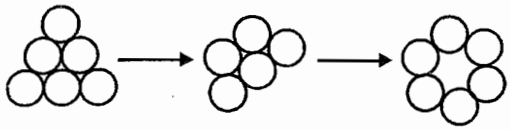


Fig. 1 **Fig. 2**

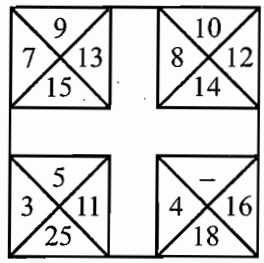
(a) 1 (b) 2 (c) 3 (d) 4

Soln. Minimum number of moves required = 2



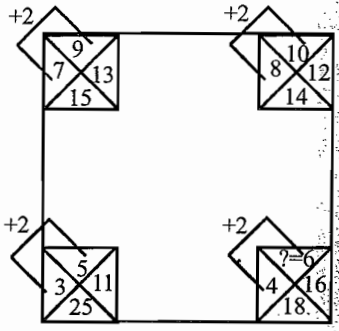
Correct option is (b)

11. The relationship among the numbers in each corner square is the same as that in the other corner squares. Find the missing number.



(a) 10 (b) 8 (c) 6 (d) 12

Soln.



Correct option (c)

12. Which of the following best approximates sin (0.5°) ?

(a) 0.5 (b) $0.5\times\frac{\pi}{90}$ (c) $0.5\times\frac{\pi}{180}$ (d) $0.5\times\frac{\pi}{360}$

Soln. $\because \quad 180^\circ = \pi^c$

$\therefore \quad 1^0 = \frac{\pi}{180}$

$\therefore \quad 0.5^0 = 0.5\times\frac{\pi}{180}$

Correct option is (c)

13. What comes next in the sequence ?

A B C D

(a) (b) (c) (d)

Soln. By observation of given year, the half of each letter starting from A is shown here. So, next term of sequence is half of 'E'.

Correct option is (c)

14. Which of the following statements is logically **incorrect**?
(a) I always speak the truth (b) I occasionally lie
(c) I occasionally speak the truth (d) I always lie

Soln. According to logic, statement "I always lie" is incorrect.
Correct option is (d)

15. How many times starting at 1 : 00 pm would the minute and hour hands of a clock make an angle of 40° with each other in the next 6 hours?
(a) 6 (b) 7 (c) 11 (d) 12

Soln. Put $\theta = 40°$ for each hour starting from 1 : 00

$$\theta = \frac{1}{2}|60 \times H - 11M|$$

So, between 1 : 00 to 2:00, only one time and for each next 5 hours 2 times.
So, total 11 times
Correct option is (c)

16. Brothers Santa and Chris walk to school from their house. The former takes 40 minutes while the latter, 30 minutes. One day Santa started 5 minutes earlier than Chris. In how many minutes would Chris overtake Santa?
(a) 5 (b) 15 (c) 20 (d) 25

Soln. Let distance between school and house = d km.

then speed of Santa $= \frac{d}{40}$ km/min

and speed of chris $\frac{d}{30}$ km/min

Now, Santa started 5 min earlier. So, he will cover the distance when Chris will start $= \frac{d}{40} \times 5 = \frac{d}{8}$ k.m.

Now relative speed of Chris $= \frac{d}{30} - \frac{d}{40} = \frac{4d - 3d}{120} = \frac{d}{120}$

So, Chris overtakes Santa $= \frac{d}{8} / \frac{d}{120} = 15$ min

Correct option is (b)

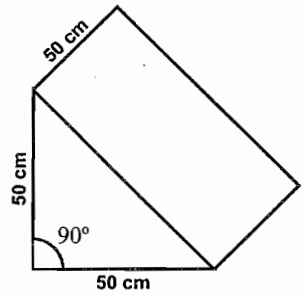
17. The set of numbers (5, 6, 7, m, 6, 7, 8, n) has an arithmetic mean of 6 and mode (most frequently occuring number) of 7. Then $m \times n =$
(a) 18 (b) 35 (c) 28 (d) 14

Soln. Mean, $M = \frac{5+6+7+m+6+7+8+n}{8} = 6$

$$39 + m + n = 48$$
$$m + n = 9$$

Since, 7 is mode. Hence, at least either m or n must be 7
Let m = 7, then n = 2
Hence, m×n = 7×2 = 14
Correct option is (d)

18. The diagram shows a block of marble having the shape of a triangular prism. What is the maximum number of slabs of $10 \times 10 \times 5$ cm^3 size that can be cut parallel to the face on which the block is resting ?



(a) 50 (b) 100 (c) 125 (d) 250

Soln. **Correct option is (b)**

19. A solid contains a spherical cavity. The cavity is filled with a liquid and includes a spherical bubble of gas. The radii of cavity and gas bubble are 2 mm and 1 mm, respectively. What proportion of the cavity is filled with liquid ?

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{7}{8}$

Soln. Volume of cavity $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 2^3 = \frac{32\pi}{3}$

and volume of bubble $= \frac{4}{3}\pi 1^3 = \frac{4}{3}\pi$

So, volume of liquid $= \frac{32\pi}{3} - \frac{4\pi}{3} = \frac{28\pi}{3}$

Hence, ratio $= \frac{\frac{28\pi}{3}}{32\pi/3} = \frac{7}{8}$

Correct option is (d)

CAREER ENDEAVOUR

20. Fill in the blank: F2,_____, D8, C16, B32, A64.

(a) C4 (b) E4 (c) C2 (d) G16

Soln.

2^1		2^2	2^3	2^4	2^5	2^6
F2	E	4	D8	C16	B32	A64
	−1	−1	−1	−1	−1	

Correct option is (b)

CHAPTER

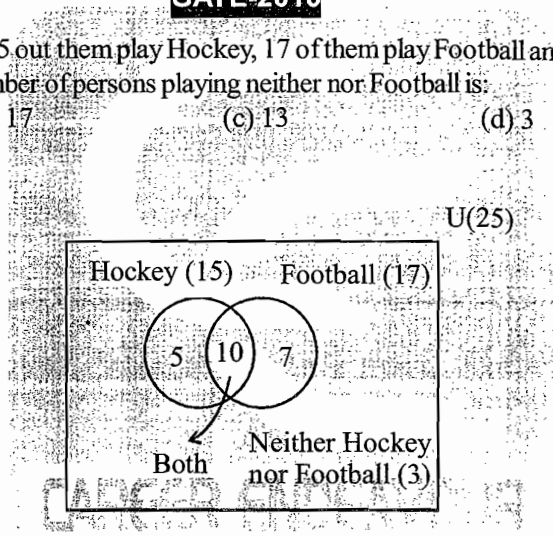
21

Previous Year GATE Questions with Solutions

GATE-2010

1. 25 persons are in a room. 15 out them play Hockey, 17 of them play Football and 10 of them play both Hocky and Football. Then the number of persons playing neither nor Football is:
(a) 2 (b) 17 (c) 13 (d) 3

Soln. Total persons = 25



Thus the number of persons playing neither Hockey nor Football is 3.
Answer is (d).

2. If 137 + 276 = 435 how much is 731 + 672?
(a) 534 (b) 1403 (c) 1623 (d) 1531

Soln. The numbers are written in reverse order. When they coded.
137 + 276 = 435
731 + 672 = 534
Answer is (a).

3. 5 skilled workers can build a wall in 20 days; 8 semiskilled can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semiskilled and 5 unskilled workers, how long will it take to build the wall?
(a) 20 days (b) 18 days (c) 16 days (d) 15 days.

Soln. 5 skilled worker can build a wall in 20 days

Thus, a skilled worker efficiency per day $= \frac{1}{20\times 5} = \frac{1}{100}$

8 semiskilled worker can build a wall in 25 days.

Thus, semiskilled worker efficiency per day $= \frac{1}{8\times 25} = \frac{1}{200}$

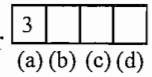
10 unskilled worker can build a wall in 30 days.

Thus, a unskilled worker efficiency per day $= \frac{1}{10\times 30} = \frac{1}{300}$

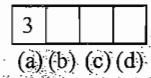
Per day worked done by 2 skilled, 6 semiskilled and 5 unskilled worker $= \frac{2}{100} + \frac{6}{200} + \frac{5}{300} = \frac{40}{600} = \frac{1}{15}$

Thus time to complete the work is equal to 15 days.
Answer is (d).

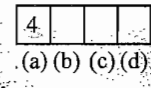
4. Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed?
(a) 50 (b) 51 (c) 52 (d) 54

Soln. Four digit number 

Number is greater than 3000 thus first digit number must be either 3 or 4.

Case-I: When thousands digit place (a) is 3. 

If there is no restriction on number of two's threes and fours. Then each of b, c, d be filled with 2 or 3 or 4 each in three ways.
So, $3\times3\times3 = 27$, numbers are there. Out of which 3222, 3333 are invalied as 2 can be used twice and three thrice.
Thus valid numbers are $27-2 = 25$

Case-II: When thousands digit place (a) is 4. 

Without restriction on number of 2's, 3's and 4's.
a, b, c can filled is $3\times3\times3 = 27$ ways.
Out of which 4222 is only invalied as 2 is used thrice. Thus valid numbers are $27-1 = 26$
Total numbers from both case $= 25 + 26 = 51$
Answer is (b)

5. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:
(1) Hari's age + Gita's age > Irfan's age + Saira's age.
(2) The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
(3) There are no twins.
In what order were they born (oldest first)?
(a) HSIG (b) SGHI (c) IGSH (d) IHSG

Soln. Let us Hari's age is 4, Gita's age is G, Saira's age is S, Irfan age is I.
From statement (1) $H + G > I + S$
From statement (2) Either $S - G = 1$ or $G - S = 1$
From statement (3) No twins.
Option (a), HSIG not possible because G and S must be lie together cause there age difference is exact one year.
Option (b), SGHI is a possible order

Option (c), IGSH not possible because H + G > I + S. From IGSH option we get I + S > G + H
So, both are contradict each other.
Option (d), IHSG from here I > H, S > G, thus I + S > H + G
which contradict statement (1)
Answer is (b)

6. If log (P) = (1/2), log (Q) = (1/3), log (R), then which of the following options is TRUE? **[GATE-2011, Set-I]**
(a) $P^2 = Q^3R^2$ (b) $Q^2 = PR$ (c) $Q^2 = R^3P$ (d) $R = P^2Q^2$.

Soln. Let $\log(P)=\left(\frac{1}{2}\right)\ \log(Q)=\left(\frac{1}{3}\right)\ \log(R)=k$

Thus, $\log P = k,\ P = 10^k$; $\frac{1}{2}\log Q = k;\ \log Q = 2k,\ Q = 10^{2k}$ and $\frac{1}{3}\log R = k,\ \log R = 3k,\ R = 10^{3k}$

(a) $P^2 = 10^{2k}$; $Q^3R^2 = \left(10^{2k}\right)^3\left(10^{3k}\right)^2 = 10^{6k}\ 10^{6k} = 10^{12k}$

(b) $Q^2 = \left(10^{2k}\right)^2 = 10^{4k}$; $PR = \left(10^k\right)\left(10^{3k}\right) = 10^{4k}$

(c) $Q^2 = \left(10^{2k}\right)^2 = 10^{4k}$; $R^3P = \left(10^{3k}\right)^3 10^k = \left(10^{9k}\right)\left(10^k\right) = 10^{10k}$

(d) $R = 10^{3k}$; $P^2Q^2 = \left(10^k\right)^2\left(10^{2k}\right)^2 = 10^{2k+4k} = 10^{6k}$

Answer is (b)

GATE-2011

7. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this processes is repeated one more time. How much spirit is now left in the container?
(a) 7.58 litres (b) 7.84 litres (c) 7 litres (d) 7.29 litres

Soln. Every time if we take 1 litre of mixture out and replace with water, content of pure spirit . We keep on reducing by 10%.
So, finally the spirit after 3 operation = 10×0.9×0.9×0.9 = 7.29 litre.
Answer is (d)

8. The variable cost (V) of manufacturing a product varies according to the equation V = 4q, where 'q' is the quantity produced. The fixed cost (F) of production of same product reduces with 'q' according to the equation F= 100/q. How many units should be produced to minimize the total cost (V + F)?
(a) 5 (b) 4 (c) 7 (d) 6

Soln. Total cost = Variable cost + Fixed cost = V + F

Total cost = $4q+\frac{100}{q}$

(a) For q = 5, T.C. = $(4)(5)+\frac{100}{5}=20+20=40$

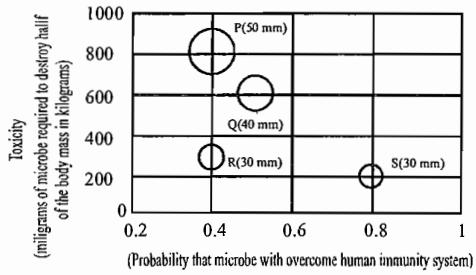
(b) For q = 4, T.C. = $(4)(4)+\frac{100}{4}=16+25=41$

(c) For q = 7, T.C. = $(4)(7)+\frac{100}{7}=28+14.29=42.29$

(d) For q = 6, T.C. $= (4)(6) + \frac{100}{6} = 24 + 16.61 = 40.67$

Answer is (a).

9. P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below.



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its attempt first ?

(a) P (b) Q (c) R (d) S

Soln. Danger to human beings varies proportionately with the toxicity and potency, from the graph.

	P	Q	R	S
Requirement	800	600	300	200
Potency	0.4	0.5	0.4	0.8

From table 'S' is the most dangerous microbe thus it should be first attempt.

Answer is (d)

10. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of hte 4th day he can clear all the orders. Alternately, if he uses only 3 trucks, then all orders are cleared at the end of hte 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day?

(a) 4 (b) 5 (c) 6 (d) 7

Soln. Let each truck carry 100 units.

'n' is the number of order each day.

'b' is the pending backlogs.

Thus, $4n + b = 28$ ... (i)

and in normal day $10n + b = 30$ ... (ii)

Solving (i) and (ii), $n = \frac{1}{3}$ and $b = \frac{80}{3}$

Since, we need to find out number of trucks so that no pending order will be there at the end of 5th day.

$5n + b = 5d$

Where 'd' is number of truck required.

$$d = \frac{5n + b}{5} = \frac{5\left(\frac{1}{3}\right) + \frac{80}{3}}{5} = \frac{\frac{85}{3}}{5} = 5\frac{2}{3}$$

As number of trucks must be natural number. Hence minimum 6 trucks will be required.

11. There are two candidates P and Q in an election. During the campaign 40% of the voters promised to vote for P, and rest for Q. However, on the day of electron 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters?
(a) 100 (b) 110 (c) 90 (d) 95

Soln. Let total number of voters = x
40% of voter promised to give vote to P thus number of votes to P = 0.4x
60% of voter promised to give vote to P thus number of votes to Q = 0.6 x
On voting day,

Total votes to P = $(0.4\,x)(0.85)+(0.6x)(0.25)=0.49x$

Total votes to Q = $(0.6\,x)(0.75)+(0.4\,x)(0.15)=0.51\,x$

Given that P lost by 2 votes thus, $0.51\,x-0.49\,x=2$; $x=100$
Answer is (a).

12. Three friends R, S and T shared toffee from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?
(a) 38 (b) 31 (c) 48 (d) 41

Soln. Let total number of toffee in the bowl = x

Number of toffee in after R took her share = $\frac{2}{3}x+4$

Number of toffee in bowl after 'S' took his share = $\frac{3}{4}\left(\frac{2}{3}x+4\right)+3$

Number of toffee in bowl after T took his share = $\frac{1}{2}\left\{\frac{3}{4}\left(\frac{2}{3}x+4\right)+3\right\}+2$

At lost in bowl 17 toffees one left thus:

$$\frac{1}{2}\left\{\frac{3}{4}\left(\frac{2}{3}x+4\right)+3\right\}+2=17;\quad x=48$$

Answer is (c).

13. The sum of 'n' terms of the series $4+44+444+.....$is

(a) $(4/81)\left[10^{n+1}-9n-1\right]$ (b) $(4/81)\left[10^{n-1}-9n-1\right]$

(c) $(4/81)\left[10^{n+1}-9n-10\right]$ (d) $(4/81)\left[10^{n}-9n-10\right]$

Soln. Let $S=4+44+444+.....$n terms

$=4[1+11+111+.....$n terms$]$

$=\frac{4}{9}[9+99+999+.....$n terms$]$

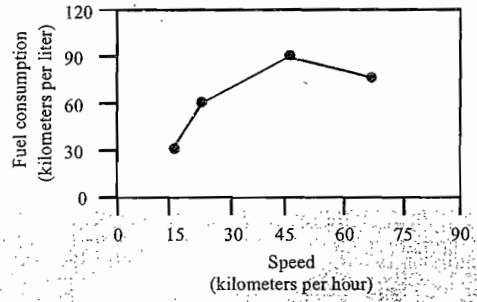
$=\frac{4}{9}[(10-1)+(100-1)+(1000-1)+.....$n terms$]$

$$=\frac{4}{9}\left[(10+100+1000.....\text{n term})-(1+1+1.....\text{n term})\right]$$

$$=\frac{4}{9}\left[\frac{10\left(10^{n}-1\right)}{10-1}-n\right]=\frac{4}{9}\left[\frac{10^{n+1}-10}{9}-n\right]=\frac{4}{81}\left[10^{n+1}-9n-10\right]$$

Answer is (c).

12. The fuel consumed by a motorcycle during a journey while travelling at various speeds is indicated in the graph below.



The distance covered during four laps of the journey are listed in the table below.

Lap	Distance (kilometers)	Average speed (kilometers per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometer was least during the lap.

(a) P (b) Q (c) R (d) S

Soln. From the graph, the highest point is speed of 45 and fuel consumption is 90.

Speed = 45 km/hour

Fuel consumption = 90 km/liter

Means for one liter is motorcycle travel 90 km which is maximum for the graph.

Answer is (b)

15. Given that $f(y)=|y|/y$, and q is an y non-zero real number, the value of $|f(q)-f(-q)|$ is:

(a) 0 (b) –1 (c) 1 (d) 2

Soln. Case-I: When q is positive number then $|f(q)-f(-q)|=\left|\frac{|q|}{q}-\frac{|q|}{-q}\right|=|1+1|=|2|=2$

Case-II: When q is negative number then $|f(q)-f(-q)|=\left|\frac{|q|}{q}-\frac{|q|}{-q}\right|=\left|\frac{-q}{q}-\frac{(-q)}{q}\right|=|-1-1|=|-2|=2$

Answer is (d)

16. The cost function for a product in a firm is given by $5q^2$., where 'q' is the amount of production. The firm can shell the product at a market price of Rs. 50 per unit. The number of units to be produced by the firm such that the profit is maximized is:
(a) 5 (b) 10 (c) 15 (d) 25

Soln. Total cost of production = $5q^2$.
Earning through market sharing = (per unit cost) (no of units) = (50) (q) = 50q
Profit = $50q - 5q^2$.
(a) q = 5, profit = $50(5) - 5(5)^2 = 250 - 125 = 125$
(b) q = 10, profit = $50(10) - 5(10)^2 = 500 - 500 = 0$
(c) q = 15, profit = $50(15) - 5(15)^2 = 750 - 1125 = -375$
(d) q = 20, profit = $50(20) - 5(20)^2 = 1000 - 2000 = -1000$
Answer is (a)

17. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where 'y' is the height of the arch in meters. The maximum possible height of the arch is:
(a) 8 meters (b) 10 meters (c) 12 meters (d) 14 meters.

Soln. Height of arch, $y = 2x - 0.1x^2$ meters.

(a) arch = 8 meters, $y = 2(8) - 0.1(8)^2 = 16 - 6.4 = 9.6$ meters

(b) arch = 10 meters, $y = 2(10) - 0.1(10)^2 = 20 - 10 = 10$ meters

(c) arch = 12 meters, $y = 2(12) - 0.1(12)^2 = 24 - 14.4 = 9.6$ meters

(d) arch = 14 meters, $y = 2(14) - 0.1(14)^2 = 28 - 19.6 = 8.4$ meters

Answer is (b).

18. Given the sequnce of terms, AD CG FK JK the next term is:
(a) OV (b) OW (c) PV (d) PQ

Soln. AD, CG, FK, JP are the terms of sequence
A is 1st and D is 4th letter of english alphabet.
C is 3rd and G is the 7th letter of english alphabet.
F is 6th and K is the 11th letter of english alphabet.
J is 10th and P is 16th letter of english alphabet.
First letter value varies as 1, 3, 6, 10 difference between the alphabets are gradually increasing by 1 starting from value 2, so the next term is 10 + 5 = 15, it means english alphabet O.
Seond letter value varies as 4, 7, 11, 16 difference between the alphabets are gradually increasing by 1 starting value 3, so the next term is 16 + 6 = 22 it means english alphabet V.
Hence, answer is OV.
Answer is (a).

19. Which of the followng assertions are CORRECT?
P: Adding 7 to each entry in a list adds 7 to the mean of the list.
Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list.
R: Doubling each entry 'n' a list doubles the mean of the list
S: Doubling each entry in a list leaves the standard deviation of the list unchanged.
(a) P, Q (b) Q, R (c) P, R (d) R, S

Soln. (P) Mean of n number $= \frac{x_1 + x_2 + x_3 + \ldots\ldots x_n}{n} = \bar{x}$

If we add 7 in each number then,

$$\frac{(x_1+7)(x_2+7).....+(x_n+7)}{n}=\frac{x_1+x_2+.....x_n+7n}{n}=\frac{x_1+x_2+.....x_n}{n}+7$$

Thus it is correct sentence

(R) Mean of n numbers $=\frac{x_1+x_2+x_3+.....x_n}{n}=\overline{x}$

Mean of after doubling each number $=\frac{7x_1+7x_2+....7x_n}{n}==\frac{7(x_1+x_2+....x_n)}{n}=7\overline{x}$

Thus this statement is also correct.
'P' and 'R' are correct statements.
Answer is (c)

20. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbes. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbes, 96% are reliable. Of Y's shock absorbs, 72% are reliable.
The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is
(a) 0.288 (b) 0.334 (c) 0.667 (d) 0.720

Soln. Let total 100 shock absorbes are supplied
Number of shock absorbers from supplier 'X' = 60
Number of shock absorbes from suppliers 'Y' = 40
Chance of X's shock absorbes is reliable = 60×0.96 = 57.6
Chance of Y's shock absorbes is reliable = 40×0.72 = 28.8
Shock absorber is found is reliable thus it must be belongs to (57.6 + 28.8) group

Probability that reliable shock absorber belongs to Y $=\frac{28.8}{57.6+28.8}=0.333$

Answer is (b).

21. If $(1.001)^{1259}=3.52$ and $(1.001)^{2062}=7.85$, then $(1.001)^{3321}=$
(a) 2.23 (b) 4.33 (c) 11.37 (d) 27.64

Soln. $(1.001)^{1259}=3.52$ ... (i)

$(1.001)^{2062}=7.85$ ... (ii)

(i) × (ii)

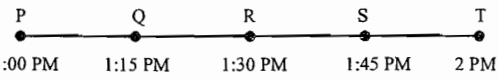
$(1.001)^{1259+2062}=3.52\times7.85$; $(1.001)^{3321}=27.632$

Answer is (d)

22. A and B are friends. They decide to meet between 1 PM and 2PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is:

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{7}{16}$ (d) $\frac{9}{16}$

Soln. 1 PM to 2PM time can be divided in four block of 15 minute each



Lets blocks are named as (1) PQ, (2) QR, (3) RS, (4) ST

Point of time when first friend arrivd	Block in which if second friend come then they will meat.	Block in which if second friend come then won't meet.	Favourable condition ratio
P	(1)	(2), (3), (4)	1/4
Q	(1), (2)	(3), (4)	2/4
R	(2), (3)	(1), (4)	2/4
S	(3), (4)	(1), (2)	2/4
T	(4)	(1), (2), (3)	1/4

From the above table probability that they will meet lie must be in between $\frac{1}{4}$ and $\frac{2}{4}$.

$$\frac{1}{4} < \text{probability} < \frac{2}{4};\ \frac{1}{4} < \text{probability} < \frac{1}{2}$$

Only value $\frac{7}{16}$ lie in the desire range.

Answer is (c).

23. Raju has 14 currency notes in his pocket consisting of only Rs 20 notes and Rs 10 notes. The total money value of the notes is Rs 230. The number of Rs 10 notes the Raju has is:
(a) 5 (b) 6 (c) 9 (d) 10

Soln. Lets number of notes of Rs 10 are x.

Then, $20(14-x)+10x=230;\ 280-20x+10x=230;\ 280-10x=230;\ 10x=50;\ x=5$

Answer is (d).

24. There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighting balance , the minimum number of weighing required to identify the heavier bag is:
(a) 2 (b) 3 (c) 4 (d) 8

Soln. **Step-I:** Put the three bags in each sides of balance from which we get the heavier side. If both sides are equal then the heavier bag must be in remaining two bags and if havier bag in any one side then that side will be bent.
Step-II: If heavier bag in any one side then take all three bags and put two of them. Again, in balance sheet. If balance sheet are equal then heavier is remaining third one.
Answer is (a)

25. The data given the following table summarizes the monthly budget of an average household.

Category	Amount
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Others	1800

The approximate percentage of the monthly budget NOT spent on savings is
(a) 10% (b) 14% (c) 81% (d) 86%

Soln. Approximate percentage of the monthly budget

$$\text{not spent on saving} = \frac{4000+1200+2000+1800}{4000+1200+2000+1500+1800}\times 100 = \frac{9000}{10500}\times 100 = 85.71$$

Answer is (d).

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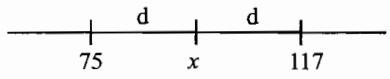
26. A number is as much greater than 75 as it is smaller than 117. The number is:
(a) 91 (b) 93 (c) 89 (d) 96

Soln.



$2d = 117 - 75 = 42$

$d = 21$

Hence, $x = 75 + d = 96$

Answer is (d)

27. X and Y are two positive real numbers such that $2X + Y \le 6$ and $X + 2Y \le 8$. For which of the following values of (X, Y) the function $f(X, Y) = 3X + 6Y$ will give maximum value?
(a) (4/3, 10/3) (b) (8/3, 20/3) (c) (8/3, 10/3) (d) (4/3, 20/3)

Soln. Put all the optional value of (in) all equation.
Olnly option (a) satisfied all equation.
Answer is (a)

28. If $|4X - 7| = 5$ then the values of $2|X| - |-X|$ is:
(a) 2, 1/3 (b) 1/2, 3 (c) 3/2, 9 (d) 2/3, 9

Soln. $|4x - 7| = 5$

$4x - 7 = 5$

$4x = 12$

$x = 3$

$-(4x - 7) = 5$

$-4x + 7 = 5$

$-4x = -2$

$x = \frac{1}{2}$

$\Rightarrow \quad 2|3| - |-3|$

$2 \times 3 - 3 = 3$

$2\left(\frac{1}{2}\right) - \left(-\frac{1}{2}\right)$

$1 - \frac{1}{2} = \frac{1}{2}$

Answer is (b)

29. Following table provides figures (in rupees) on annual expenditure of a firm for two years-2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power & Fuel	7000	9450
Salary & Wages	9000	12600
Plant & Machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?
(a) Raw material and Salary & wages (b) Salary & wages and Advertising
(c) Power & fuel and Advertising (d) Raw material and Research & Development

Soln. We see that raw material $= \frac{6240-5200}{5200} \times 100 = 20\%$

Research and development $= \frac{26400-22000}{22000} \times 100 = 20\%$

Answer is (d)

30. A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

(a) 5 (b) 10 (c) 15 (d) 30

Soln. Let the quantity produced = x unit

Total C.P. = 100 rs

Total selling price = $60 \times x = 60x$

Total profit = 500 rs

Hence, $60x = 500 + 100$

$60x = 600; \quad x = 10$

Let new cost = $100 \times \left(\frac{100+30}{100}\right) = 130$ rs

Total unit is 10 and profit = 500

Hence, total S.P. = 500 + 130 = 630 rs

So, we have to increase the total price = 630 – 600 = 30rs $\Rightarrow \frac{3}{60} \times 100 = 5\%$

There are 10 articles.

Hence, increasing price for each article $= \frac{30}{10} = 3$ rs

Increasing % = 5%

Answer is (a)

31. Abhishek is elder to Savar.

Savar is younger to Anshul

Which of the given conclusion is logically valid and is inferred from the above statements?

(a) Abhishek is elder to Anshul (b) Anshul is elder to Abhishek

(c) Abhishek and Anshul are of the same age (d) No conclusion follows

Soln. Savar < Abhishek

Savar < Anshul

No conclusion because there are no direct relation between Abhishek and Anshul.

Answer is (d)

32. If $3x \le X \le 5$ and $B \le Y \le 11$ then which of the following options is TRUE?

(a) $\frac{3}{5} \le \frac{X}{Y} \le \frac{8}{5}$ (b) $\frac{3}{11} \le \frac{X}{Y} \le \frac{5}{8}$ (c) $\frac{3}{11} \le \frac{X}{Y} \le \frac{5}{8}$ (d) $\frac{3}{5} \le \frac{X}{Y} \le \frac{8}{11}$

Soln. $3 \le x \le 5 \qquad 8 \le y \le 11$

$\frac{3}{5} \le x \le 1 \qquad \frac{8}{11} \le y \le 1$

Answer is ( )

33. Velocity of an object fired directly in upward direction is given by V = 80–32t, where t (time) is in seconds. When will the velocity be between 32 m/sec and 64 m/sec?

(a) (1, 3/2) (b) (1/2, 1) (c) (1/2, 3/2) (d) (1, 3)

Soln. The question will be solved by hit and trail method.

$V = 80-32t$

Put the value of 't' from option (c)

$t = ½,\ v = 80-32\times\frac{1}{2} = 64$

$t = \frac{3}{2},\ v = 80-32\times\frac{3}{2} = 32$

Answer is (c)

34. In a factory, two machines M1 and M2 manufacturer 60% and 40% of the autocomponents respectively. Out of the total production, 2% of M1 and 3% of M2 are found to be defective. If a randomly drawn autocomponent from the combined lot is found defective, what is the probability that it was manufactured by M2?

(a) 0.35 (b) 0.45 (c) 0.5 (d) 0.4

Soln. Let total production is 100

Then production of $M_1 = 100\times\frac{60}{100} = 60$ and that of $M_2 = 100\times\frac{40}{100} = 40$

Production of defective article from $M_1 = 60\times\frac{2}{100} = \frac{12}{10} = 1.2$ that of $M_2 = 40\times\frac{30}{100} = \frac{12}{10} = 1.2$

Hence, probability given by Baye's theorem

$$P(E) = \frac{\frac{1}{2}\times 1.2}{\frac{1}{2}(1.2+1.2)} = \frac{0.6}{1.2} = \frac{1}{2} = 0.5$$

Answer is (c)

35. Following table gives data on tourists from different countries visiting india in the year 2011.

Country	Number of Tourists
USA	2000
England	3500
Germany	1200
Italy	1100
Japan	2400
Australia	2300
France	1000

Which two countries contributed to the one third of the total number of tourists who visited India in 2011?

(a) USA and Japan (b) USA and Australia

(c) England and France (d) Japan and Australia

Soln. Total number of tourist = 13500

Then $\frac{1}{3}rd$ of total $= \frac{1}{3}\times 13500 = 4500$

England + France = 3500 + 1000 = 4500

Answer is (c)

36. If $|-2X+9|=3$ then the possible value of $|-X|-X^2$ will be

(a) 30 (b) –30 (c) –42 (d) 42

Soln. By the definition of Mod

$|-2x+9|=(-2x+9)$ ... (i)

and $-(-2x+9)$ ... (ii)

then for equation (i) then $|-x|-x^2$

$-2x+9=3$ $=|-3|-3^2$

$-2x=3-9$ $=3-9=-6$

$2x=-6$

$x=3$

Now for equation (ii) $|-6|-6^2$

$-(-2x+9)=3$ $\Rightarrow$ $6-36$

$2x=3+9$ $\Rightarrow$ -30

$x=\frac{12}{2}=6$

Answer is (b)

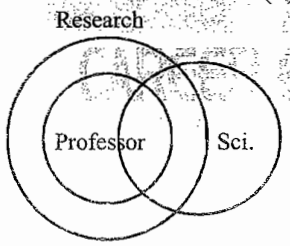
37. All professors are researchers

Some scientists are professors.

Which of the given conclusions is logically valid and is inferred from the above arguments:

(a) All scientists are researchers (b) All professors are scientists

(c) Some researchers are scientists (d) No conclusion follows



Then it is clear that some researchers are scientist.

Answer is (c)

38. Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.
Which one of the statements below is logically valid and can be inferred from the above sentences?
(a) Rajan has decided to work only in a group.
(b) Rajan and Sajan were formed into a group against their wishes
(c) Sajan had decided to give in to Rajan's request to work with him.
(d) Rajan had believed that Sajan and he would be working together.

Soln. Answer is (d)

39. If $y = 5x^2 + 3$, then the tangent at $x = 0, y = 3$
(a) passes through $x = 0, y = 0$ (b) has a slope of +1
(c) is parallel to the x-axis (d) has a slope of −1

Soln. $y = 5x^2 + 3 \Rightarrow \frac{dy}{dx} = 10x$

For tangent $= -\frac{dy}{dx} = 10x$

Put $x = 0$ and $y = 3$, then $-\frac{dy}{dx} = 0$

Hence, tangent is parallel to the x-axis.
Answer is (c)

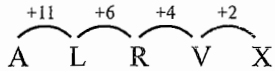
40. A foundry has a fixed daily cost of Rs 50, 000 whenever it operates and a variable cost of Rs. 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs. per tonne for a daily production of 100 tonnes?

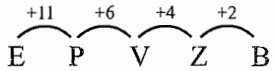
Soln. Total cost for 100 tonnes = fixed cost + variable cost
$= 50000 + 800Q$
Here, Q = 100 tonnes = 50000 + 800×100 = 50000 + 80000 = 130000 Rs.

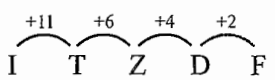
Hence per tonne $= \frac{130000}{100} = 1300\ Rs$

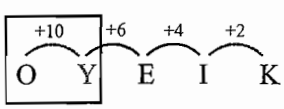
41. Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK
(a) ALRVX (b) EPVZB (c) ITZDF (d) OYEIK

Soln.









Answer is (d).

42. Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(a)	6	2	5	1	3	4
(b)	2	6	5	1	3	4
(c)	4	2	6	3	1	5
(d)	2	4	6	1	3	5

From (i) and (ii) from (iii) (v)

Anuj/Bhola — Chandan — Bhola ×

Eshwar×

Bhola ×

Anuj/Bhola — Faisal — from (vi) Faisal

from (iv) Dileep ×

Anuj/Bhola — Dileep

Comparing these, we get
Bhola
Chandan
Faisal
Eshwar
Anuj
Dileep
Answer is (b)

43. The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3 : 4 : 5 : 6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

Soln. Value of angle of quadrilateral = $3x + 4x + 5x + 6x = 360^\circ$

$18x = 360^\circ$

$x = 20^\circ$

Hence angles are 60, 80, 100, 120

$$\text{Smallest angle of triangle} = \frac{2}{3} \times 60 = 40^\circ$$

Smallest angle of triangle = $2 \times 40 = 80^\circ$

Hence remaining third is $180 - (40 + 80) = 60^\circ$

Hence, $60 + 120 = 180^\circ$

44. One percent of the people of country X are taller than 6 fit. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

(a) 3.0 (b) 2.5 (c) 1.5 (d) 1.25

Soln. Let number of poeple in country Y = 100

then 2% of $Y = \frac{2}{100} \times 300 = 3$

Number of people in country $X = 3Y = 3 \times 100 = 300$

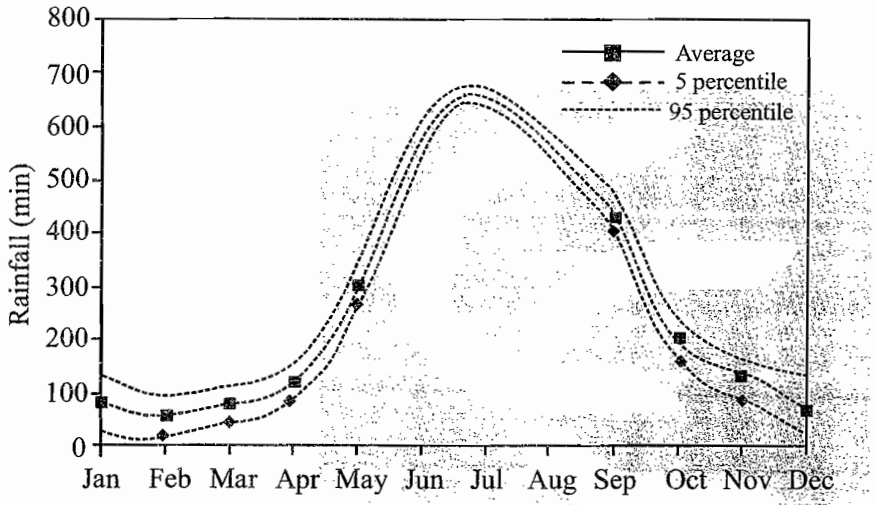
then 1% of $X = \frac{1}{100} \times 300 = 3$

Sum of person taller than 6ft. $= 2 + 3 = 5$

then $\% = \frac{5}{(100+300)} \times 100 = \frac{1.25}{400} \times 100 = 1.25\%$

Answer is (d)

45. The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (k percentile is the value such that 'k' percent of the data fall below that value)



(i) On average, it rains more in july than in December
(ii) Every year, the amount of rainfall in August is more than that in January
(iii) July rainfall can be estimated with better confidence than February rainfall.
(iv) In August, there is at least 500 mm of rainfall.

(a) i, ii (b) i and iii (c) ii and iii (d) iii and iv.

Soln. By given graph we can't say result (2) because graph show average rainfall. So, (ii) is wrong.
Same way we can't say there is at least 500 mm ranifall in August because 95 percentile data fall below that value and rainfall is between 500 to 600.
So, (i) and (iii) is true.
Answer is (b)

GATE (CS)-2014 [SET-7]

46. If $(z+1/z)^2 = 98$, compute $\left(z^2+1/z^2\right)$

Soln. $\left(z+\frac{1}{z}\right)^2 = 98 \quad \Rightarrow z^2+\left(\frac{1}{z}\right)^2+2.z.\frac{1}{z}=98$

$z^2+\frac{1}{z^2}+2=98 \quad \Rightarrow z^2+\frac{1}{z^2}=96$

47. The roots of $ax^2+bx+c=0$ are real and positive. a, b and c are real. Then $ax^2+b|x|+c=0$ has

(a) no roots (b) 2 real roots (c) 3 real roots (d) 4 real roots

Soln. $ax^2+b|x|+c=0$

Now, $|x|$ can hold two values –(i) –x (ii) x

Hence, $ax^2-bx+c=0$ gives us two roots.

And, $ax^2+bx+c=0$ gives us two roots.
So, there are 4 real roots.

48. The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terian to its north and south. The exact reasons for the formation ofthis gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures.
What can be inferred from this passage?
(a) The Palghat gap is caused by high rainfall and high temperatures in southern Tamil Nadu and Kerala.
(b) The regions in Tamil Nadu and Kerala that are near the Palghat gap are low-lying
(c) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.
(d) Higher summer temperatures result in higher rainfall near the Palghat Gap area.

Soln. The given passage described how Palghat controls the weathering condition around its neighbouring areas. Other option tells about results due to Palghat which do not contain conclusion of given passage.

49. Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and schiozophrenia, and consequently, that doctors will be able to eradicate these diseases through easily identification and gene therepy.
On which of the following assumptions does the statement above rely?
(a) Strategies are now available for eliminating psychiatric illnesseses
(b) Certain psychiatric illnesses have a genetic and how they are expressed.
(c) All human diseases can be traced back to genes and how they are expressed.
(d) In the future, genetics will become the only relevant field for identifying psychiatric illness.

Soln. Assumption (c) and (d) contain all and only words. Hence assumption is wrong, and strategies was also available for it in previous time. Hence, only (b) is true.

49. Round-trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is Rs. 100, a group of 5 tourists purchasing round-trip tickets will be charged Rs _____

Soln. One way single person fare is 100 Rs.
So round trip ticket for one person = 100/2 = 200 Rs.
But there are 10% discounts for round trip ticket and for more than 5 person another discount of 5% also given.

So, actual value for a round trip ticket for one person $= 200 \times \frac{(100-15)}{100} = 170\ Rs$

Hence, for 5 person = 170×5 = 850 Rs.

50. In a servey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

Soln. Number of male who do not own scooter = 40 + 20 = 60
Number of female who do not own scooter = 34 + 50 = 84
Total number of person who do not scooter = 84 + 60 = 144

Hence, percentage $= \frac{144}{300} \times 100 = 48\%$

51. When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines?

Soln. A point inside tetrahedron when touches it all corner by straight line will make six new planes inside it.

[SET-8]

52. What is the average of all multiples of 10 from 2 to 198?
(a) 90 (b) 100 (c) 110 (d) 120

Soln. $\frac{10+20+30+40+.....+180+190}{19} = \frac{\frac{19}{2}(10+190)}{19} = \frac{\frac{19}{2}\times 200}{19} = 100$

Answer is (b)

CAREER ENDEAVOUR

53. The value of $\sqrt{12+\sqrt{12+\sqrt{12+.....}}}$ is
(a) 3.464 (b) 3.932 (c) 4.000 (d) 4.444

Soln. Let $\sqrt{12+\sqrt{12+\sqrt{12+.....}}}$

squaring both sides,

$x^2 = 12+\sqrt{12+\sqrt{12+\sqrt{12+...}}}$

$x^2 = 12+x$

$\Rightarrow \quad x^2 - x - 12 = 0$

$\Rightarrow \quad x^2 - 4x + 3x - 12 = 0$

$\Rightarrow \quad x(x-4)+3(x-4) = 0$

$(x-4)(x+3) = 0 \qquad \Rightarrow x = 4, x = -3$

Since, 'x' can't hold negative value.

54. The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of poland to the south and west and Lithuania to the east respectively. Which of the statements below can be inferred from this passage?
(a) Kaliningrad was historically Russian in its ethnic make up
(b) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia
(c) Koenigsberg was renamed Kaliningard, as that was its original Russian name.
(d) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

Soln. The purpose of this passage is about the location of Kaliningsberg with respect to maintand of Russia. So, option (b) represents it correctly.

55. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have conclueded that measures to control the mosquito population have failed in this region.
Which one of the following statements, if true, does not contradict this conclusion?
(a) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent.
(b) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency.
(c) Many more cases of dengue are being diagonsed this year since the introduction of a new and effective diagnostic test.
(d) The number of people with material fever (also contracted from mosquito bites) has increased this year.

Soln. The given passage tells about only the present situation. It donot gives any reason. Also A is a assumptions (b) is wrong and (c) is also wrong. Passage do not tells about any new diagnotic test.
Hence, (d) is correct.

56. If x is real and $\left|x^2-2x+3\right|=11$, then possible values of $\left|-x^3+x^2-x\right|$ include
(a) 2, 4 (b) 2, 14 (c) 4, 52 (d) 14, 52

Soln. $\left|x^3-2x+3\right|=11$

$\Rightarrow \quad x^2-2x+3=11 \quad \Rightarrow \quad x^2-2x+3-11=0$

$\Rightarrow \quad x^2-2x-8=0 \quad \Rightarrow \quad x^2-4x+2x-8=0$

$\Rightarrow \quad x(x-4)+2(x-4)=0 \quad \Rightarrow \quad (x-4)(x+2)=0$

$\Rightarrow \quad x=4,-2 \quad \Rightarrow \quad \left|-x^2+x^3-x\right|$

put $x=4$

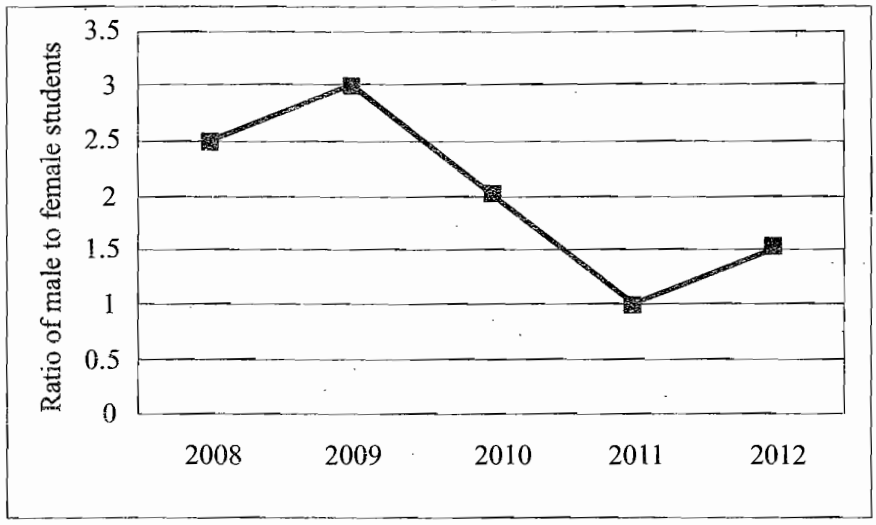
$\Rightarrow \quad \left|-4^3+4^2-4\right|=\left|-64+16-4\right|=\left|-52\right|=52$

put $x=-2$

$\Rightarrow \quad \left|-(-2)^3+(-2)^2-(-2)\right|=\left|8+4+2\right|=\left|14\right|=14$

Answer is (d)

57. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



Soln. Ratio of male to female in 2008 = 2.5 : 1
Let female = 100 and M = 250
Now in 2009, M : F = 3 : 1
Hence, F = 200
Then M = 3×200 = 600
Number of male increasing = 600–250 = 350

Hence, increasing $\% = \frac{350}{250} \times 100 = 140\%$

48. At what time between 6 am and 7 am will the minute hand and hour hand of a clock make an angle closest to 60°?
(a) 6 : 22 am (b) 6 : 27 am (c) 6 : 38am (d) 6 : 45 am

Soln. 6 : 22 $\left|6\times 30 - 11\times\frac{11}{2}\right| = |180-121| = 59°$

6 : 27 $\left|6\times 30 - 27\times\frac{11}{2}\right| = |180-148.5| = 31.5°$

6 : 38 $\left|6\times 30 - 38\times\frac{11}{2}\right| = |180-209| = |-29| = 29°$

6 : 45 $\left|6\times 30 - 45\times\frac{11}{2}\right| = |180-247.5| = |-67.5| = 67.5°$

Formula: $\boxed{\left|30\times \text{Hour part} - \frac{11}{2}\times \text{Minute part}\right|}$

SET-9

49. Which number does not belong in the series below?
2, 5, 10, 17, 26, 37,50, 64
(a) 17 (b) 37 (c) 64 (d) 26

Soln. $4 - 2^{+3}, 5^{+5}, 10^{+7}, 17^{+9}, 26^{+11}, 37^{+13}, 50^{+15}, \boxed{64}$

Answer is (c)

50. The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

Q.No.	Marks	Answered correctly	Answered wrongly	Not attempted
1	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination?

Soln. Total students = 21 + 17 + 6 = 44

Total marks = 7×44 = 308

Total marks scored by student = 2×21+3×15+2×23 = 42+45+46 = 133

Hence, average marks $= \frac{133}{44} = 3.02(\text{approx})$

51. A dance programme is scheduled for 10, 00 a.m. Some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be acompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is

(a) students should come at 9.00 a.m. and parents should come at 10.00 am.

(b) Participating students should come at 9.00 a.m. accompanied by a parent, and other parents and students should come by 10.00 a.m.

(c) Students who are not participating should come by 10.00 a.m. and they should not bring their parents. Participating students should come at 9.00 am.

(d) Participating students should come before 9.00 a.m. Parents who accompany them should come at 9.00 a.m. All others should come at 10.00 a.m.

Soln. From given passage it is very clear that the student, who participating in dance program, come before 1 hour with accompanied by a parent.

So, (b) is true.

52. By the beginning of the 20th century, several hypothesses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.

Which of the following inference(s) may be drawn from the above passage?

(i) Our understanding of the universe changes based on the positions of stars

(ii) Paradigm shifts usually occur at the beginning of centuries

(iii) Stars are important objects in the universe

(iv) Experimental evidence was important in confirming this paradigm shift.

(a) i, ii and iv (b) iii only (c) i and iv (d) iv only

Soln. Answer is (d)

53. The Gross Domestic Product (GDP) in Rupees grew at 7% during 2012-2013. For international comparison, the GDP is compared in US Dollars (USD) after conversion based on the market exchange rate. During the period 2012-2013 the exchange rate for the USD increased from Rs. 50/ USD to Rs. 60 USD. India's GDP in USD during the period 2012-2013.

(a) increased by 5% (b) decreased by 13% (c) decreased by 20% (d) decreased by 11%

Soln. Supposed GDP in Rs. for 2011-12 = 100 Rs.

Rate = 50 rs/D

$$\text{GDP in D} = \frac{100}{50} = 2\text{D}$$

Now GDP in Rs. for 2012–13 = 107

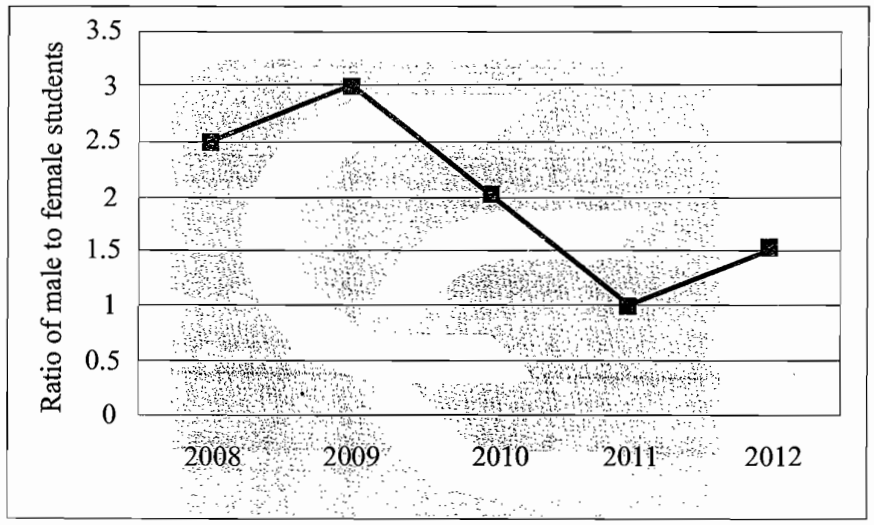
New rate = 60 rs/dollar

$$\text{GDP in D} = \frac{107}{60}$$

$$\text{Hence, percent decreasing} = \frac{\frac{107}{60} - 2}{2} \times 100 = \frac{107 - 120}{2 \times 60} \times 100 = \frac{-13}{2 \times 60} \times 100 = \frac{-13}{2 \times 60} \times 100$$

$$= \frac{-65}{6} = -10.83 = 11\%(\text{approx})$$

54. The ratio of male to female students in a college for five years in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?



(a) 1 : 1 (b) 2 : 1 (c) 1.5 : 1 (d) 2.5 : 1

Soln. Ratio of male to female in 2011 = 1 : 1

Hence, suppose M = 100, F = 100

In 2012, M : F = 1.5 : 1

Since female are equal for both year

Then number of male in 2012 = 1.5×100 = 150

Hence, ratio of male students for 2012 to 2011 is 150 : 100, 1.5 : 1

55. Consider the equation : $(7526)_8 - (Y)_8 = (4364)_8$, where $(X)_N$ stands for X to the base N. Find Y.

(a) 1634 (b) 1737 (c) 3142 (d) 3162

Soln. $(7526)_8 - (Y)_8 = (4364)_8$

$$\text{Hence, } (Y)_8 = (7526)_8 - (4364)_8 = \begin{array}{r} 7526 \\ -\,4364 \\ \hline 3142 \end{array}$$

$$(Y)_8 = (3142)_8$$

Answer is (c)

GATE – 2015 (PHYSICS - PH)

1. Tanya is older than Eric
Cliff is older than Tanya
Eric is older than Cliff.
If the first two statements are true, then the third statement is
(a) True (b) False (c) Uncertain (d) Data insufficient

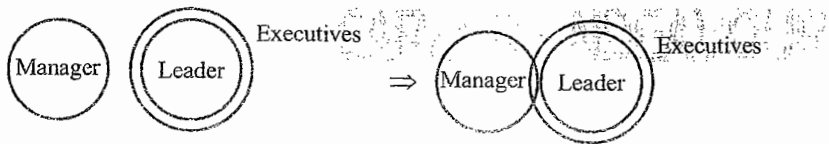
Soln. **(b)** Combining (i) and (ii) statements.
Cliff > Tanya > Eric

2. Five teams have to compete in a league, with every team playing every other team exactly once, before going to the next round. How many matches will have to be held to complete the league round of matches?
(a) 20 (b) 10 (c) 8 (d) 5

Soln. **(b)**
Since there are five teams
= No. of matches = $\Sigma(n-1)$
= $\Sigma 4 = 4 + 3 + 2 + 1 = 10$

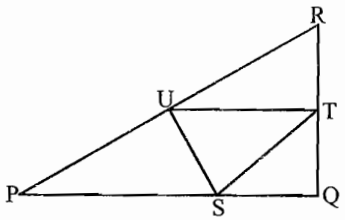
3. Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.
Statements:
I. No manager is a leader
II. All leaders are executives
Conclusions:
I. No manager is an executive
II. No executive is a manager
(a) Only conclusion I follows (b) Only conclusion II follows
(c) Neither conclusion I nor II follows (d) Both conclusion I and II follow

Soln. **(d)**



	N	M
(i)	✓	×
(ii)	✓	×

4. In the given figure angle Q is a right angle, PS:QS = 3:1. RT:QT = 5:2 and PU:UR = 1:1. If area of triangle QTS is 20cm^2, then the area of triangle PQR in cm^2 is .....



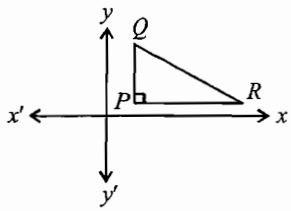
Soln. $\frac{QS}{PQ}=\frac{1}{4} \Rightarrow PQ=4QS \Rightarrow \frac{QT}{QR}=\frac{2}{7} \Rightarrow QR=\frac{7}{2}QT$

$= \frac{1}{2}PQ\times QR = \frac{1}{2}\times 4QS\times\frac{7}{2}QT = 14 \times 20 = 280 \text{ cm}^2$

5. Right triangle PQR is to be constructed in the xy-plane so that the right angle is at P and line PR is parallel to the x-axis. The x and y coordinates of P, Q and R are to be integers that satisfy the inequalities $-4 \le x \le 5$ and $6 \le y \le 16$. How many different triangle could be constructed with these properties?

(a) 110 (b) 1,100 (c) 9,900 (d) 10,000

Soln. **(a)**



x can hold 10 different values, y can hold 11 different value
hence (x, y) can hold $(10 \times 11 = 110)$ values.

GATE 2015 (CHEMISTRY-CY)

1. Operators, $\square$, $\Diamond$ and $\rightarrow$ are defined by: $a\square b=\frac{a-b}{a+b}; a\Diamond b=\frac{a+b}{a-b}; a\rightarrow b=ab$

Find the value of $(66\square 6)\rightarrow(66\Diamond 6)$

(a) –2. (b) –1 (c) 1 (d) 2

Soln. **(c)**

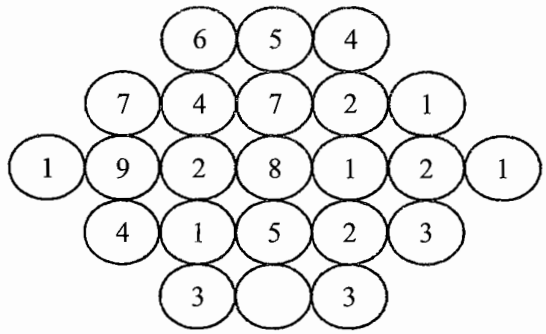
$$\frac{66-6}{66+6}\times\frac{66+6}{66-6}=\frac{60}{72}\times\frac{72}{60}=1$$

2. If $\log_x(5/7)=-1/3$, then the value of x is

(a) 343/125 (b) 125/343 (c) –25/49 (d) –49/25

Soln. **(a)** $\because \log_a b = x \Rightarrow b = a^x$; $\log_x(5/7)=-\frac{1}{3}$, $\frac{5}{7}=x^{-1/3}$; $x^{1/3}=\frac{7}{5}$, $x=\left(\frac{7}{5}\right)^3=\frac{343}{125}$

3. Fill in the missing value



Soln. $\frac{6+4}{2}=\frac{10}{2}=5$

$$\frac{7+4+2+1}{2}=\frac{14}{2}=7$$

:

:

Answer is 3.

Exact middle number is equal to half of sum of other circle in that row.

4. A cube of side 3 units is formed using a set of smaller cubes of side 1 unit. Find the proportion of the number of faces of the smaller cubes visible to those which are NOT visible.

(a) 1 : 4 (b) 1 : 3 (c) 1 : 2 (d) 2 : 3

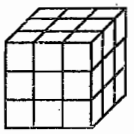
Soln. **(c)**

No. of cube's visible = 18

Total cubes = 54

not visible = 36

18 : 36 = 1 : 2



5. Humpty Dumpty sits on a wall every day while having lunch. The wall sometimes breaks. A person sitting on the wall falls if the wall breaks.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(a) Humpty Dumpty always falls while having lunch

(b) Humpty Dumpty does not fall sometimes while having lunch

(c) Humpty Dumpty never falls during dinner

(d) When Humpty Dumpty does not sit on the wall, the wall does not break

Soln. **(b)**

GATE 2015 (COMPUTER SCIENCE & IT)

1. Based on the given statements, select the most appropriate option to solve the given question.

If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements:

(I) Each step is 3/4 foot high

(II) Each step is 1 foot wide

(a) Statement I alone is sufficient, but statement II alone is not sufficient

(b) Statement II alone is sufficient, but statement I alone is not sufficient

(c) Both statements together are sufficient, but neither statement alone is sufficient

(d) Statement I and II together are not sufficient

Soln. **(a)**

Difference between two floors = 9 feet

1 step = 3/4 feet

hence required steps = $\frac{9}{3}\times 4=12$

2. Given Set A = {2, 3, 4, 5} and Set B = {11, 12, 13, 14, 15}, two numbers are randomly selected one from each set. What is the probability that the sum of the two numbers equals 16?

(a) 0.20 (b) 0.25 (c) 0.30 (d) 0.33

Soln. **(a)**

$n(s) = 4 \times 5 = 20$

$n(E)$ = sum of 16 = {(2, 14), (3, 13), (4, 12), (5, 11)} = 4

hence, $P(E) = \frac{4}{20} = \frac{1}{5} = 0.2$

3. The given statement is followed by some courses of action. Assuming the statement to be true, decide the correct option.

Statement: There has been a significant drop in the water level in the lakes supplying water to the city.

Course of action:

(I) The water supply authority should impose a partial cut in supply to tackle the situation

(II) The government should appeal to all the residents through mass media for minimal use of water

(III) The government should ban the water supply in lower areas

(a) Statements I and II follow (b) Statements I and III follow

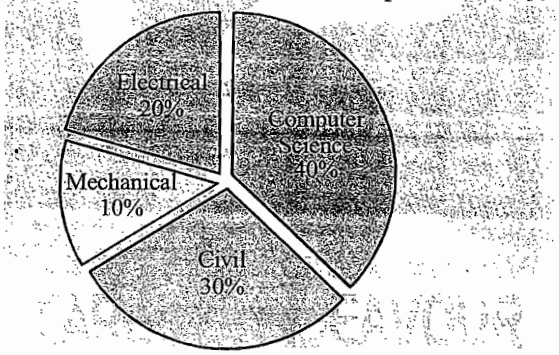
(c) Statements II and III follow (d) All statements follow

Soln. (a)

I and II both has positive sense.

Course of action is a immediate step taken against to solve a problem.

4. The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is 5:4. There are 40 males in Electrical Engineering. What is the difference between the numbers of female students in the Civil department and the female students in the Mechanical department?



Soln. M:F = 5:4

Let x = total students

$$x \times \frac{20}{100} \times \frac{3}{9} = 40, \quad x = 360$$

Female in civil : $360 \times \frac{30}{100} \times \frac{4}{9} = 48$

Female in mechanical : $360 \times \frac{10}{100} \times \frac{4}{9} = 16$

$= 48 - 16 = 32$

5. The probabilities that a student passes in Mathematics, Physics and Chemistry are m, p and c respectively. Of these subjects, the students has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in m, p, c.

(I) $p+m+c=27/20$ (II) $p+m+c=13/20$ (III) $(p)\times(m)\times(c)=1/10$

(a) Only relation I is true (b) Only relation II is true

(c) Relations II and III are true (d) Relations I and III are true

Soln. **(c)**

$p(m \cup p \cup c) = p(m) + p(p) + p(c) - (m \cap p) - p(m \cap c) - p(p \cap c) + p(m \cap p \cap c)$

Passing in exactly two = Pass in at least two – Pass in all three subject

Hence, probability $\frac{4}{10} = \frac{50}{100}$ – Pass in all three

Pass in all three $= \frac{1}{2} - \frac{2}{5} = \frac{1}{10}$

III is definitely true.

Statement (I) is wrong.

Pass in exactly one = Pass in at least one – Pass in exactly two – Pass in all three

$= \frac{75}{100} - \frac{40}{100} - \frac{10}{100} = \frac{25}{100} = 2.5\%$

6. If ROAD is written as URDG, then SWAN should be written as

(a) VXDQ (b) VZDQ (c) VZDP (d) UXDQ

Soln. **(b)**

18	15	1	4
R	O	A	D
+3	+3	+3	+3
U	R	D	G
21	18	4	7

S W A N

↓

V Z D Q

7. A function $f(x)$ is linear and has a value of 29 at $x = -2$ and 39 at $x = 3$. Find its value at $x = 5$.

(a) 59 (b) 45 (c) 43 (d) 35

Soln. **(c)**

$f(x) = a_0 + a_1 x$, $x = -2 \Rightarrow a_0 - 2a = 29$

$x = 3$, $a_0 + 3a_1 = 39$

$f(5) = 33 + 2 \times 5 = 43$

$a_0 - 2a_1 = 29$

$\underline{-a_0 + 3a_1 = 39}$

$-5a_1 = -10$

$a_1 = 2$, $a_0 - 2 \times 2 = 29$

$a_0 = 29 + 4 = 33$

8. Alexander turned his attention towards India, since he had conquered Persia.

Which one of the statements below is logically valid and can be inferred from the above sentence?

(a) Alexander would not have turned his attention towards India had he not conquered Persia.

(b) Alexander was not ready to rest on his laurels, and wanted to march to India.

(c) Alexander was completely in control of his army and could command it to move towards India

(d) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.

Soln. **(d)**

9. Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament. He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addressed this problem success at the highest level will continue to elude him.

Which of the statement (s) below is /are logically valid and can be inferred from the above passage?

(i) He was already a successful batsman at the highest level

(ii) He has to improve his temperament in order to become a great batsman

(iii) He failed to make many of his good starts count

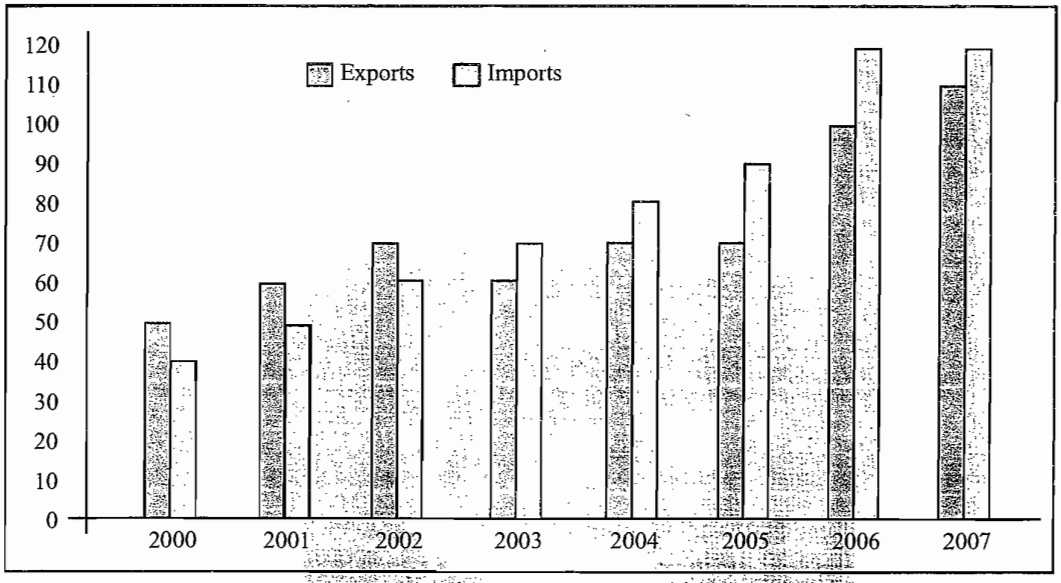
(iv) Improving his technical skills will guarantee success

(a) (iii) and (iv) (b) (ii) and (iii)

(c) (i), (ii) and (iii) (d) (ii) only

Soln. Correct option is (a)

10. The exports and imports (in crores of Rs.) of a country from the year 2000 to 2007 are given in the following bar chart. In which year is the combined percentage increase in imports and exports the highest?

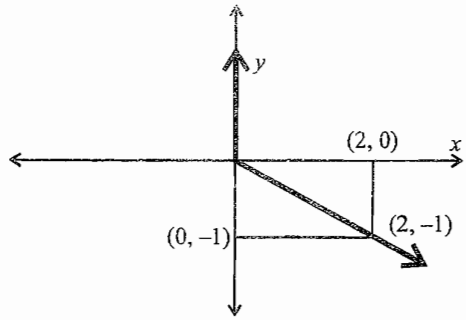


Soln. Import increasing in 2006, $\frac{120-90}{90}\times 100 = 33.3\%$

Export increasing in 2006, $\frac{100-70}{70}\times 100 = 42.8\%$

$42.8 + 33.3 = 76.1$

11. Choose the most appropriate equation for the function drawn as thick line, in the plot below.



(a) $x = y - |y|$ (b) $x = -(y - |y|)$ (c) $x = y + |y|$ (d) $x = -(y + |y|)$

Soln. (b)

A point is taken in line $(2, -1)$

if means $y = -1 \Rightarrow x = 2$

in option (b) $x = -[(-1) - |(-1)|] = -[-1-1] = (-2) = 2$

12. The head of a newly formed government desires to appoint five of the six selected members P, Q, R, S, T and U to portfolios of Home, Power, Defense, Telecom, and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets either Power or Telecom then she must get the other one. T insists on a portfolio if P gets one.

Which is the valid distribution of portfolios?

(a) P-Home, Q-Power, R-Defence, S-Telecom, T-Finance

(b) R-Home, S-Power, P-Defence, Q-Telecom, T-Finance

(c) P-Home, Q-Power, T-Defence, S-Telecom, U-Finance

(d) Q-Home, U-Power, T-Defence, R-Telecom, P-Finance

Soln. **(b)**

By condition

(i) U and S can not be together, hence option (c) is wrong

(ii) R can get either home or finance, hence (a) and (d) is wrong

$U \rightarrow S$, R-Home / Finance

$Q \rightarrow S$, (Power / Telecom)

$T \rightarrow P$

GATE 2016 (PHYSICS-PH)

1. **Fact:** If it rains, then the field is wet.

Read the following statements:

(i) It rains (ii) The field is not wet

(iii) The field is wet (iv) It did not rain

Which one of the options given below is NOT logically possible, based on the given fact?

(a) If (iii), then (iv) (b) If (i), then (iii)

(c) If (i), then (ii) (d) If (ii), then (iv)

Soln. If (i), then (ii)

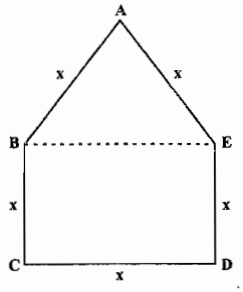
it is just opposite our fact.

Correct option is (c)

2. A window is made up of a square portion and an equilateral triangle portion above it. The base of the triangular portion coincides with the upper side of the square. If the perimeter of the window is 6m, the area of the window in m^2 is _____

(a) 1.43 (b) 2.06 (c) 2.68 (d) 2.88

Soln.



$\therefore \quad 5x = 6$

$$x = \frac{6}{5} = 1.2\,\text{m}$$

$$\text{area (ABCDE)} = \text{area}\,(\Delta ABCD) + \text{area}\,(BECD)$$

$$\frac{\sqrt{3}}{4} \times x^2 + x^2$$

$$= \frac{\sqrt{3}}{4} \times (1.2)^2 + (1.2)^2 = 0.6235 + 1.44 = 2.0635 = 2.06$$

Correct option is (b)

3. Students taking an exam are divided into two groups, **P** and **Q** such that each group has the same number of students. The performance of each of the students in a test was evaluated out of 200 marks. It was observed that the mean of group **P** was 105, while that of group **Q** was 85. The standard deviation of group **P** was 25, while that of group **Q** was 5. Assuming that the marks were distributed on a normal distribution, which of the following statements will have the highest probability of being **TRUE**?

(a) No student in group **Q** scored less marks than any student in group **P**

(b) No student in group **P** scored less marks than any student in group **Q**

(c) Most students of group **Q** scored marks in a narrower range than students in group **P**

(d) The median of the marks of group **P** is 100

Soln. Mean for P = 105 S.D. = 25 Mean for Q = 85 S.D = 5

$$\text{RSD} = \frac{25}{105} \times 100 = 23.809\% \qquad \text{RSD} = \frac{5}{85} \times 100 = 5.88$$

Hence we can say easily option (c)

4. A smart city integrates all modes of transport, uses clean energy and promotes sustainable use of resources. It also uses technology to ensure safety and security of the city, something which critics argue, will lead to a surveillance state.

Which of the following can be logically inferred from the above paragraph?

(i) All smart cities encourage the formation of surveillance states

(ii) Surveillance is an integral part of a smart city

(iii) Sustainability and surveillance go hand in hand in a smart city

(iv) There is a perception that smart cities promote surveillance

(a) (i) and (iv) only (b) (ii) and (iii) only

(c) (iv) only (d) (i) only

Soln. **Correct option is (b)**

8. Find the missing sequence in the letter series.

B, FH, LNP, ...

(a) SUWY (b) TUVW (c) TVXZ (d) TWXZ

Soln.

	+4		+4			+4				
2		6	8		12	14	16		20	22	24	26
B,		F	H,		L	N	P,		T	V	X	Z

Hence, correct option is (c)

9. The binary operation $\square$ is defined as $a \square b = ab + (a+b)$, where a and b are any two real numbers. The value of the identity element of this operation, defined as the number x such that $a \square x = a$, for any a, is

(a) 0 (b) 1 (c) 2 (d) 10

Soln. $a \square b = ab + (a+b)$

$a \square x = a \cdot x + (a+x) = a$

if $x = 0$

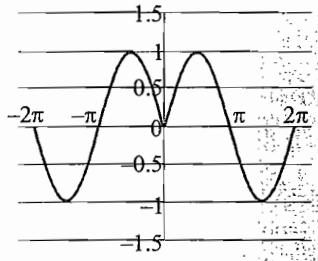
$a \square x = a \cdot 0 + (a+0) = 0 + a = a$

hence, $x = 0$

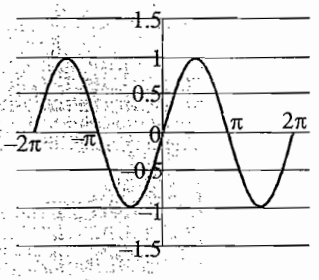
Correct option is (a)

10. Which of the following curves represents the function $y = \ln(|\, e^{[|\sin(|x|)|]}\,|)$ for $|x| < 2\pi$? Here, x represents the abscissa and y represents the ordinate.

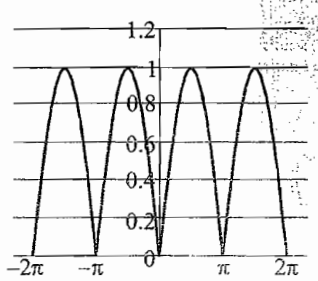
(a)



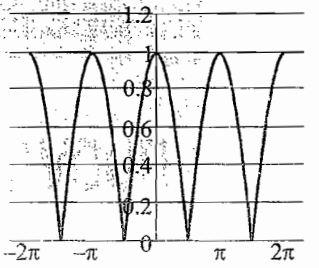
(b)



(c)



(d)



Soln. $y = \ln\left(e^{\left[|\sin(|x|)|\right]}\right)$

then $y = \left[|\sin(|x|)|\right] \ln e$

$y = |\sin(|x|)|$

So all negative values are positive also for $\sin|x|$.

Correct option is (c).

GATE 2016 (COMPUTER SCIENCE & IT)

1. Pick the odd one from the following options:

(a) CADBE (b) JHKIL (c) DVYWZ (d) ONPMQ

Soln.

-2 3 -2 3	-2 3 -2 3	-2 3 -2 3	-1 2 -3 4
C A D B E	J H K I L	X V Y W Z	O N P M Q

Correct option is (d)

2. In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$$

(a) n^4 (b) 4^n (c) 2^{2n-1} (d) 4^{n-1}

Soln.

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}} \Rightarrow \frac{\alpha^n + \beta^n}{\frac{1}{\alpha^n} + \frac{1}{\beta^n}} \Rightarrow \frac{\alpha^n + \beta^n}{\frac{\beta^n + \alpha^n}{\alpha^n \beta^n}} \Rightarrow \frac{\left(\alpha^n + \beta^n\right) \times \alpha^n \beta^n}{\alpha^n + \beta^n}$$

$$\Rightarrow (\alpha \times \alpha \times \ldots\ldots \text{n times}) \times (\beta \times \beta \times \ldots\ldots \text{n times})$$

$$\Rightarrow (\alpha \times \beta) \times (\alpha \times \beta) \times \ldots\ldots (\text{n times}) \Rightarrow (4 \times 4 \times 4 \ldots\ldots \text{n times}) = 4^n$$

Correct option is (b)

3. Among 150 faculty members in an institute, 55 are connected with each other through Facebook and 85 are connected through WhatsApp. 30 faculty members do not have Facebook or WhatsApp accounts. The number of faculty members connected only through Facebook accounts is _____

(a) 35 (b) 45 (c) 65 (d) 90

Soln. Number of person connected to either Facebook or WhatsApp = 150–30 = 120.

$$120 = n(F) + n(W) - n(F \cap W)$$ Where, F = Facebook and W = WhatsApp.

$$120 = 55 + 85 - n(F \cap W)$$

$$120 = 140 - n(F \cap W)$$

$$\therefore \; n(F \cap W) = 20$$

Then, number of member connected only through Facebook = 55-20 = 35

Correct option is (a)

4. Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) the author believes that computers are not good for us

(ii) Mobile computers and the internet are both intended inventions

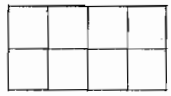
(a) (i) only (b) (ii) only (c) both (i) and (ii) (d) neither (i) nor (ii)

Soln. **Correct option is (b)**

5. All hill-stations have a lake. Ooty has two lakes
Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?
(i) Ooty is not a hill-station
(ii) No hill-station can have more than one lake
(a) (i) only (b) (ii) only (c) both (i) and (ii) (d) neither (i) nor (ii)

Soln. Correct option is (c)

6. In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?



(a) 21 (b) 27 (c) 30 (d) 36

Soln. Correct option is (c)

ENGLISH SECTION

CHAPTER

1

Sentence Correction

Sentence Correction:

1. Researchers at Cornell University have demonstrated that homing pigeons can sense changes in the earth's magnetic field, see light waves that people cannot see, detect low-frequency sounds from miles away, sense changes in air pressure, and can identify familiar odors.

   (a) sense changes in air pressure, and can identify familiar odors

   (b) can sense changes in air pressure, and can identify familiar odors

   (c) sense changes in air pressure, and identify familiar odors

   (d) air pressure changes can be sensed, and familiar odors identified

   (e) air pressure changes are sensed, and familiar odors identified

   Answer is (c).

2. In ancient times, Nubia was the principal corridor where there were cultural influences transmitted between Black Africa and the Mediterranean basin.

   (a) where there were cultural influences transmitted

   (b) through which cultural influences were transmitted between

   (c) where there was a transmission of cultural influences

   (d) for the transmitting of cultural influences

   (e) which was transmitting cultural influences

   Answer is (b).

3. It is a special feature of cell aggregation in the developing nervous system that in most regions of the brain the cells not only adhere to one another and also adopt some preferential orientation.

   (a) to one another and also adopt (b) one to the other, and also they adopt

   (c) one to the other, but also adopting (d) to one another but also adopt

   (e) to each other, also adopting

   Answer is (d)

4. Among the reasons for the decline of New England agriculture in the last three decades were the high cost of land, the pressure of housing and commercial development, and basing a marketing and distribution system on importing produce from Florida and California

   (a) basing a marketing and distribution system on importing produce from Florida and California

(b) basing a marketing and distribution system on the imported produce of Florida and California

(c) basing a system of marketing and distribution on the import of produce from Florida and California

(d) a marketing and distribution system based on importing produce from Florida and California

(e) a marketing and distribution system importing produce from Florida and California as its base

Answer is (d)

11

5. Like Byron at Missolonghi, Jack London was slowly killed by the mistakes of the medical men who treated him.

(a) Like Byron (b) Like Byron's death (c) Just as Byron died

(d) Similar to Byron (e) As did Byron

Answer is (a)

6. One of every two new businesses fail within two years.

(a) fail (b) fails (c) should fail (d) may have failed (e) has failed

12

Answer is (b)

7. Even today, a century after Pasteur developed the first vaccine, rabies almost always kills its victims unless inoculated in the earliest stages of the disease.

(a) its victims unless inoculated (b) its victims unless they are inoculated

(c) its victims unless inoculation is done (d) the victims unless there is an inoculation

(e) the victims unless inoculated

Answer is (b)

8. In a period of time when women typically have had a narrow range of choices, Mary Baker Eddy became a distinguished writer and the founder, architect, and builder of a growing church.

13

(a) In a period of time when women typically have

(b) During a time in which typically women have

(c) Typically, during a time when women

(d) At a time when women typically

(e) Typically in a time in which women

Answer is (d)

14.

9. The message conveyed by the conference speakers make me feel fortunate that I did not invest all of my money in stocks traded on the NASDAQ stock exchange.

(a) make me feel fortunate that I did not invest all of my money in stocks traded on the NASDAQ stock exchange.

(b) make me feel fortunate that I did not invest all of my money in NASDAQ stocks.

(c) make me feel fortunate that I did not invest my money in NASDAQ stocks.

15.

(d) makes me feel fortunate that I did not invest all of my money in NASDAQ stocks.

(e) makes me feel fortunate that I did not invest my money in NASDAQ stocks.

Answer is (d)

10. Cynics charge that Major League Baseball lobbied for stadiums with fewer seats to reduce supply and increase ticket prices.

(a)lobbied for stadiums with fewer seats to reduce supply and increase ticket prices.

(b) lobbied for stadiums with less seats to reduce supply and increase ticket prices.

(c) lobbied for stadiums that had fewer seats to reduce supply and increase ticket prices.

(d) lobbied for stadiums that had less seats to reduce supply and increase ticket prices.

(e) lobbied for stadiums that had less seats as a means of reducing supply so they could increase ticket prices.

Answer is (a)

11. The Navy used the nuclear submarine accident off the coast of Hawaii to show that everyone must be held responsible for their actions.

(a) to show that everyone must be held responsible for their actions.

(b) to demonstrate their belief that everyone must be held responsible for their actions.

(c) to demonstrate its belief that everyone must be held responsible for their actions.

(d) to signal how no one should be able to get away with committing acts of terrible negligence.

(e) to show that everyone must be held responsible for his actions.

Answer is (e)

12. I never fail to get frustrated on the golf course: nonetheless, I always resolve to continue my involving in the game.

(a) on the golf course: nonetheless, I always resolve to continue my involving in the game.

(b) on the golf course: nonetheless I always resolve to continue my involvement in the game.

(c) on the golf course; nonetheless, I always resolve to continue my involvement in the game.

(d) on the golf course; nonetheless I always resolve to continue my involvement in the game.

(e) on the golf course but I always still manage to find a way to resolve my self to continue my pursuit of the involvement of the game.

Answer is (c)

13. Each of the movies were great and the choice for the best one among all three was very difficult indeed.

(a) were great and the choice for the best one among all three was very difficult indeed.

(b) was great and the choice for the best one among all three was very difficult indeed.

(c) were great and the choice for the best one between all three was very difficult indeed.

(d) was great and the choice for the best one between all three was very difficult indeed.

(e) were great and the choices between all three were very difficult indeed.

Answer is (b)

14. The employees viewed the work-sharing plan with hostility, having feared that it will undermine the seniority system and negatively affect their retirement benefits.

(a) having feared that it will undermine
(b) fearing that it would undermine
(c) having the fear that it would undermine
(d) feared that it would undermine
(e) fearing that it will undermine

Answer is (b)

15. In many coastal New England towns, the fisherman still operates as they have for generations, displaying and selling their catch dockside at the end of each day.

(a) the fisherman still operates as they have

(b) the fisherman still operates as was done

(c) fishermen still operate as they have done

(d) the fisherman still operates as he has

(e) fishermen still operate as they had

Answer is (c)

CHAPTER

2

Sentence Correction & Grammar

Accuracy and Appropriateness

An important prerequisite for communication to take place is mutual intelligibility between sender and receiver(s). This mutual understanding comes from a shared understanding of the meaning of words as well as grammar. Two teams competing in a cricket match play according to a predetermined and mutually agreed upon set of rules; similarly, two people who are communicating, do so with the mutual understanding that they are following the same set of rules within the language being used.

Accuracy refers to the 'correct' use of spelling and grammar rules while encoding the message to be communicated. Appropriateness is trickier and refers to the choice of words and tone while drafting the message keeping the receiver and the occasion in mind. So while you are encoding your message not only is it important for you to be aware of the rules of English grammar and spelling, but also to keep all the guidelines of effective communication in mind so that your message is decoded in the manner you intended and you get the desired feedback.

Let us now look at some features of the English language that will help us to communicate better.

Spelling: When in doubt, use a dictionary as your helpline.

Since words are the building blocks of the language, we can begin with spelling, the most basic feature of accurate communication. While the spelling skill is one that comes with reading and practice, the prescribed rules refer to how the spelling of a word changes when a suffix is added to it.

Let us look at some of these rules.

- Adding suffixes to words ending in a consonant
- The final consonant is doubled in one syllable words ending in a single vowel + single consonant, when we add a suffix beginning with a vowel.
- For example: beg (a single syllable word with a single vowel and a single consonant at the end) willbecome begged when the suffix ed is added to it: beg+ed=begged.
- When we add the suffix *ful* to a word that ends in *ll*, the second l of the word is dropped.
- For example: when we add *ful* to *skill* the final word is not skillful, but skilful: skill+ful=skilful.
- When a word ends in two consonants, a consonant followed by a *y*, the *y* changes to *i*, when a suffix is added (except when the suffix is *ing*).
- For example: happy+ly=happily; carry+ed=carried; beauty+ful=beautiful; BUT, marry+ing=marrying; carry+ing=carrying.
- Adding suffixes to words ending in a vowel

- Words ending in *e* drop the final *e* when the suffix we add to it begins with a vowel. For example: live+ing = living, not living; move+ed=moved, not moveed; and drive+er=driver, not driveer. This rule also applies even if the final e is silent and not pronounced, so the word ends on a vowel, but with a consonant sound as in the words above, where we write the final e while spelling the word, but do not say it while pronouncing the words, which would be spoken as liv, mov and driv.
- When words ending in *ie* are extended with the suffix *ing*, the *ie* becomes *y*. For example: die + ing = dying; lie + ing = lying; tie + ing = tying.
- Exceptions to the rules and Special Cases.
- When we add a suffix that begins with a consonant to a word ending in *e*, the *e* is retained in the final word. For example: hope+ful=hopeful and engage+ment=engagement. The same is the case when the word ends in a double e: see+ing=seeing; agree+ment=agreement.
- This exception too has exceptions to it:
- true+ly=truly (the final *e* is retained); whole+ly=wholly (the final *e* has gone but the *l* of the suffix has doubled); due+ly=duly (the final *e* has gone); nine+th=ninth (the final *e* has gone); argue+ment=argument (the final *e* has gone); awe+ful=awful (the final *e* has gone).
- Words ending in *ce* and *ge*, retain the final *e* when we add the suffixes *able* and *ous*. For example: notice+able=noticeable; courage+ous=courageous.
- When y is the final letter of a word and it follows not a consonant, but a vowel, it does not change when a suffix is added to the word. For example: pray+ed=prayed; play+er=player.
- This exception too has exceptions to it:
  (i) pay+ed=paid; say+ed=said; lay+ed=laid (the y has gone and the e of the suffix has become an i)
  (ii) day+ly=daily; gay+ly=gaily (the y has gone and i has been added to the suffix).
- *ie* or *ei*: We have all heard the rule *i before e except after c*. Did you know that this rule refers to the sequence of the two vowels *i* and *e* as they come together in the word and that it is applicable only when the *ie* is pronounced like *ee* in the word jeep/keep, etc. Some examples of such words are: believe; relieve; grieve; yield; field; and, receive; deceive; receipt; conceive.
- An easy way to remember these exceptions is to see how the *ie* are pronounced. If they sound like the *ee* in jeep/keep then we know that the *i* must precede the *e*, but if they sound like *a* as in the words n*ei*ghbor/ w*ei*gh, then the e comes first.
- This rule, like all others in the English language has its exceptions and some words that do not follow the rule are: seize; protein; counterfeit; weird; and, surfeit. You could use this one liner to remember them: "Neither the weird financier nor the foreigner seizes leisure at its height".

However many rules we may learn, it is practice that makes perfect, that is the reason we have tried to balance theory and practice in the writing of this book.

When in doubt about the spelling of a word, the dictionary is your best guide to spelling right, so don't hesitate to look it up. If you are using the computer to encode your message, most software will underline a word in red to warn you that you may have misspelled the word. If you right click on the underlined/highlighted word, you will get spelling suggestions and you can choose the one you want. Do remember that the computer is not omniscient and you may be using a word, particularly a non-English word or a proper noun, that the computer does not have in its dictionary. In such a case, choose to ignore the computer's warning or if you are likely to be using the word frequently, add it to the computer's dictionary so it will not keep underling it and will recognise it as correct the next time.

Another interesting feature of spelling is the different conventions followed even within a single language. Perhaps one of the most frequently used words in this book is 'organisation'. When you type this word on your computer, the software is likely to underline it and give you 'organization' as the correct option. Why is this so? This happens because the word is spelt with a 's' in British English and with a 'z' in American English. There are many such words like theatre (theater in American English), colour (color in American English) and centre (center in American English) that are spelt differently on either side of the Atlantic Ocean. You are free to use either convention, just make sure you stick to the same convention throughout your message.

Denotation and Connotation:

Denotation is the dictionary meaning of a word and connotation is its sense in the context in which it is used. For example if you open the dictionary and look up the word 'Goodwill' you will find that its primary denotations are: "1) kindly feeling; 2) established reputation of business, etc as enhancing its value".

What then is the connotation of goodwill? That would depend on how you used the word in a sentence. If we say, "His act of charity generated a lot of goodwill in the community" it would obviously connote that he generated kindly feelings among the community by his act of charity. On the other hand if we say, "The goodwill of Tata Motors ensured the continued sales of its cars even during recession", we are implying that the value of the company remains undiminished due to its established reputation.

Vocabulary: Synonyms, Antonyms, Homophones, Homonyms, Idiomatic usage

After spelling, let's expand our vocabulary and look at various kinds of words and see how an idea can be expressed in multiple ways. This is where we need to pay attention not only to accuracy but also to appropriateness.

1. Synonyms: Words that are similar in meaning to each other are called synonyms. Synonyms are useful because they help us avoid the monotony of repeating the same word again and again, something that the reader/listener is likely to find very tedious. What we need to remember is that similar is not the same as identical so while there may be more than one way of expressing an idea; we need to choose the most appropriate word from among all the synonyms to encode our message. This is where our understanding of the difference between the denotation and the connotation will come in handy as it is the latter that will help us decide which word to use. For example hysterical and funny are synonyms but the former has a negative connotation as well so we might want to careful while using it to describe our teacher and her/his jokes.

Just as we suggest that you use the dictionary to help you find the spelling and denotation of a word, the thesaurus is very useful for finding synonyms.

Here is a sample list of words that are useful in descriptive writing, especially if we are describing a person. Read the list carefully and keep in mind the points about similar and identical made above. **(From** http://examples.yourdictionary.com/examples-of-synonyms.html)

- **Beautiful**: Attractive, Pretty, Lovely, Stunning
- **Bossy:** Controlling, Tyrannical
- **Fair**: Just, Objective, Impartial, Unbiased
- **Funny**: Humorous, Comical, Hilarious, Hysterical
- **Happy**: Content, Joyful, Mirthful, Upbeat
- **Hardworking**: Diligent, Determined, Industrious, Enterprising
- **Honest**: Honorable, Fair, Sincere, Trustworthy
- **Intelligent**: Smart, Bright, Brilliant, Sharp
- **Introverted**: Shy, Bashful, Quiet, Withdrawn
- **Kind**: Thoughtful, Considerate, Amiable, Gracious
- **Lazy**: Idle, Lackadaisical, Lethargic, Indolent
- **Lucky:** Auspicious, Fortunate
- **Mean**: Unfriendly, Unpleasant, Bad-tempered, Difficult
- **Old:** antiquated, ancient, obsolete, extinct, past, prehistoric, venerable, aged
- **Outgoing**: Friendly, Sociable, Warm, Extroverted
- **Positive:** Optimistic, Cheerful, Starry-eyed, Sanguine
- **Recalcitrant:** obstinate, stubborn
- **Rich**: Affluent, Wealthy, Well-off, Well-to-do
- **Strong**: Stable, Secure, Solid, Tough
- **Unhappy**: Sad, Depressed, Melancholy, Miserable

2. Antonyms: An antonym is the antonym of a synonym – that is, a word that is the exact opposite in meaning of a given word. "**Antonymy** is a key feature of everyday life. Should further evidence be required, try visiting a public bathroom without checking which the 'gents' is and which is the 'ladies.' On your way out, ignore the instructions which tell you whether to 'push' or 'pull' the door. And once outside, pay no attention to whether traffic lights are telling you to 'stop' or 'go.' At best, you will end up looking very foolish; at worst, you will end up dead." (http://grammar.about.com/od/ab/g/antonymsterms.htm)

"Linguists identify three types of **antonymy**: (1) *Gradable antonyms*, which operate on a continuum: (*very*) *big*, (*very*) *small*. Such pairs often occur in phrases with *and*: (*blow*) *hot and cold*, (*search*) *high and low*. (2) *Complementary antonyms*, which express an either/or relationship: *dead* or *alive*, *male* or *female*. (3) *Converse* or *relational antonyms*, expressing reciprocity: *borrow* or *lend*, *buy* or *sell*, *wife* or *husband*." (Tom McArthur, "Antonym." *The Oxford Companion to the English Language*. Oxford Univ. Press, 1992)
In the case of antonyms you will have to think of both denotation and connotation since the two antonyms must be the opposite of each other in both their denotative and connotative meanings. Here too a thesaurus will come in handy.
Here are some quotes from famous people that contain antonyms in them.

- "You always pass *failure* on the way to *success*." (Mickey Rooney)
- "Some have been thought to be *brave* because they were *afraid* to run away." (Thomas Fuller)
- "Every day I remind myself that my *inner* and *outer* lives are based on the labors of other men, *living* and *dead*, and that I must exert myself in order to *give* in the same measure as I have *received* and am still receiving." (Albert Einstein)
- "The opposite of love is not hate, its indifference. The opposite of art is not ugliness, its indifference. The opposite of faith is not heresy, its indifference. And the opposite of life is not death, its indifference." (Elie Wiesel)

3. Homophones and Homonyms: Homophones are words that sound the same (notice the suffix 'phone' that is associated with sound) but are spelt differently. Common examples of these are: See/Sea; Write/Right; There/Their; Which/Witch; Steel/Steal.
Homonyms are also words that sound the same, are spelt the same but have different meanings. Think of the words club, rock and fine. In all three there are two possible meanings of the word, depending on how it is used: club can be a place where you go for entertainment but it is also a heavy object that can be used to hit someone; rock is a hard stone but is also a kind of music; and, fine is what you pay if you break a rule but also what you feel when you are happy and healthy.

4. Idioms: An idiom relies completely on connotative value and each language has its own idioms that are very difficult to translate. An idiomatic sentence is a synergy in which the whole is greater than and different from the sum of its parts, i.e. the meaning of the sentence as a whole is different from the combined denotative meaning of each word in it.
English too has an extensive list of idioms, we give you only a few here as a sampler. Notice how the combination of verb and preposition used in a particular context is what gives the sentence its idiomatic connotation.

- His evidence did not *bear upon* (have any relevance to) the enquiry.
- They *cried out* (protested) against the injustice done to the fresher.
- The family was *cast down* (depressed) by the robbery in their house.
- This issue is *held over* (deferred/postponed) until the next meeting.
- Dr. Ashwin Baruah told all his employees that they have to *pull together* (cooperate) to sustain the success of RiceCalls.Org

Conjunctions

From individual words let's now move to phrases and sentences which we make by putting words together in a particular order so that we create a meaningful message. As you know a conjunction is a word that joins two words together to make a phrase or two or more phrases together to make a sentence. Conjunctions can be divided into two classes depending on the function they serve in a sentence.

1. Co-ordinating Conjunctions: These are conjunctions that join two independent statements or two statements that are of equal importance. The major co-ordinating conjunctions are: and, but, for, or, nor, also, either…or, neither…nor.
   For example: The sun rises in the east *and* sets in the west.
2. Subordinating Conjunctions: These are conjunctions join together two statements, one of which depends on the other to complete its meaning. The major subordinating conjunctions are: after, because, if, that, though, although, till, before, unless, as, when, where, while.
   For example: They ran away from the scene of the crime *because* they were afraid of being questioned by the police.

Prepositions:

A preposition is a word placed before a noun or a pronoun to show what relationship it has with something else in the sentence.

When we introduced the organisation RiceCalls.org to you this is what we said, "**RiceCalls.Org** was set up in Guwahati, Assam in 2010 by Dr. Ashwin Baruah who hails from Majuli, and did his PhD in Agriculture from the Assam Agricultural University, Jorhat."

In this opening sentence note the words we have underlined: in, by, from. These are prepositions that tell us what relationship between the noun that follows and something else in the sentence. The first preposition in the sentence in tells us the relationship between Guwahati which follows it and the name of the organisation that is located there.

The most important function performed by a preposition is to show relationship and you need to choose the right preposition to express the relationship that you intend to convey. In a sentence in which you want to show the relationship between RicecCalls.org and the Indian Rice Research Institute, notice what happens depending on the preposition that is used:

1. RiceCalls.Org was set up with a grant received *from* the Indian Rice Research Institute.
2. RiceCalls.Org was set up with a grant received *by* the Indian Rice Research Institute.
   In the first sentence it is clear that the grant from the IRRI helped in the setting up of RiceCalls.Org; in the second sentence it appears as if RiceCalls.org has been set up by the IRRI and the grant has come from elsewhere. See how the meaning of the sentence is transformed by a single preposition.

Here is a list of some prepositions that express certain kinds of relationship:

- Place: as, about, across, against, among, at, below, beside, between, by, from, in, into, near, round, under, upon, within.
- Time: after, at, before, behind, by, during, for, from, since, in, throughout, till, with.
- Agency/instrumentality: at, by, through, with.
- Manner: by, with.
- Cause/reason/purpose: for, of, from, through, with.
- Possession: on, of, with.
- Measure/standard/weight/value: at, by.
- Contrast: after, for, with.
- Inference/motive/source/origin: from.

Notice that a single preposition can be used to express more than one kind of relationship depending on how it is used. For example the preposition *with* can be used:

- for time in the sentence, "rise with the sun"
- for agency in the sentence, "cut it with a knife"
- for manner in the sentence, "he passed with ease"
- for cause in the sentence "she is down with a cold"
- for possession in the sentence, "the boy with the brown eyes".

The lesson we learn from this is of course that a preposition is a useful but tricky part of speech and must be used accurately and appropriately so that our message is not distorted.

Tenses:

The verb is the doing word in a sentence and one of the important indications we need to include is the time of the action being done and its completeness – whether it is being done at the time of speaking/writing or has been done or will be done, whether it is complete or in the process of being done, etc. The different tenses help us to indicate this by modifying the verb accordingly. A useful way to remember how to denote the verb to show time and completeness is by seeing how a verb like 'communicate' changes in each when the time is the present. In addition to time and completeness determining the nature of the verb that will be used, the subject/ doer of the action is also a determinant – is the action being done by one person or many; is the subject being referred to in the first/second or third person?

Time/ Person	Simple Present	Present Continuous	Present Perfect	Present Perfect Continuous
I (first person singular)	I communicate	I am communicating	I have communicated	I have been communicating
We (first person plural)	We communicate	We are communicating	We have communicated	We have been communicating
You (second person singular/ plural)	You communicate	You are communicating	You have communicated	I have been communicating
He/She/It (third person singular)	He/She/It communicates	He/She/It is communicating	He/She/It have communicated	He/She/It have been communicating
They (third person plural)	They communicate	They are communicating	They have communicated	They have been communicating

The verb will undergo a similar modification for both the past and the future tenses. Remember you will need to draft your message carefully to ensure that you communicate with accuracy who the action is being done by, when it is scheduled and how complete it is.

Tenses and their uses

There are three main tenses.

(1) The Present Tense (2) The Past Tense (3) The Future Tense

The present tense has four forms

(a) Simple Present

(b) Present Continuous

(c) Present Perfect

(d) Present Perfect Continuous

(a) Simple Present tense is used

(i) To express general truth such as scientific facts. e.g.: Squares have four sides.

The sun rises in the east and sets in the west.

The moon moves round the earth once every 28 days.

(ii) To indicate a habitual action, event or condition. e.g.: I go for a walk every morning.
We decorate our house every Diwali.
(iii) To indicate abilities. e.g.: He is a good tennis player.
She paints very well.

(b) Present Continuous tense is used
(i) To express what is happening at this very moment. e.g.:
He is doing his homework.
She is reading a book.
(ii) To indicate the future. e.g.:
My sister is arriving this evening.
I am not going to the movie tonight.
(iii) For temporary or new habits. e.g. :
He is smoking too much.
She is eating a lot these days.

(c) Present Perfect tense is used
(i) To indicate an action that began in the past but is continuing at the time of speaking. e.g.:
She has done only one sum till now.
Mother has lived here for ten years.
(ii) To indicate an action that has been completed in the immediate past. e.g.:
She has just finished her homework.

(d) Present perfect continuous tense
(i) To indicate an activity which started in the past and is continuing in the present. e.g.:
She has been learning singing for three years.
The baby has been sleeping since morning.
Similarly, **Past tense** has four forms:
(a) Simple Past
(b) Past Continuous
(c) Past Perfect
(d) Past Perfect Continuous

(a) Past simple tense is used to indicate an action that has taken place in the pa .g.:
They left for Delhi yesterday.
It rained in the morning.
(b) Past Continuous tense indicates an action going on in the past. e.g.:
The watchman was sleeping last night.
While Mina was doing homework, her brother was playing.
(c) Past Perfect tense is used for an action which happened before another action in the past. e.g.:
Before I reached the stadium, the match had already begun.
We had that television before it had broken down.
(d) Past Perfect Continuous tense indicates an action which happened before another action in the past and was continuing at a certain point in the past. e.g.:
She had been working in this office for two years when I joined.
He had been waiting for half an hour when the train arrived.

Future tense also has three forms:
(a) Simple Future
(b) Future Continuous
(c) Future Perfect
(d) Future Perfect Continuous

Simple Future tense is used to indicate an action which has not yet taken place but will take place in the future. Usually will or going to form is used. e.g.:

I will leave for Delhi tomorrow.
They are going to shift to their new house next week.

The Future Continuous tense is used to express an action which will take place in the future and will be continuing at some point of time in the future. e.g.:
I will be doing my homework in the evening tomorrow.
We will be going to Japan next year.

The Future Perfect tense is used to express an action which will be completed in the future. e.g.:
I will have been in this company for two years.
She will have had finished reading the book by next week.

The Future Perfect Continuous tense is used to express an action that started in the past but will end at some time in the future. e.g.:
By October next year, I will have been living here for five years.
She will have been studying in this school for seven years by next March.

Subject-Verb Agreement:

Both the subject and the verb must co-operate in the communication of meaning. Just as an organisation depends on the all the employees concurring about its goals and working towards their fulfillment, a sentence too depends on the agreement between subject and verb. This agreement between subject and verb must be on two counts: number and person.
For example in the sentence "The quality of the fruit were not good" we see that the sentence is awkward because the subject 'fruit' is singular but the verb being used is plural.
Let us look at some useful rules to keep in mind while writing and speaking which will help us avoid the clash between subject and verb:

(1) Two or more singular nouns or pronouns joined by the conjunction *and* will be accompanied by a plural verb. For example: Jacob Mahapatra and Varuni Choudhary *travel* to work using a car pool.
(2) Two or more singular nouns or pronouns joined by the conjunction *or/nor* will be accompanied by a singular verb. Neither Jacob Mahapatra and Varuni Choudhary nor *travels* to work using a car pool.
(3) Two nouns if qualified by *each/every* even when joined by the conjunction *and* will take a singular verb. For example: Each senior research scientist and every research scientist in RiceCalls.Org *was* given a new Smartphone o: Bihu, the Assamese New Year.
(4) Some nouns which are plural in form but singular in meaning, take a singular verb. For example: Mathematics *is* viewed as a very important subject in school.
(5) Some nouns which are singular in form but plural in meaning, take a plural verb. For example: According to the present market rate, a dozen eggs *cost* twenty four rupees.
(6) A collective noun takes a singular verb when the collection is thought of as one whole; a plural verb when the members of the collective are considered as individuals. For example: The Committee *has* submitted its report; but, The Committee *are* divided on the point of pricing.
(7) When the plural noun is a proper name for some single object or collective unit, it must be accompanied by a singular verb. For example: RiceCalls.Org *is* the name of Dr.Ashwin Baruah's flagship company.

Modal Verbs:

Modal Verbs are Auxiliary verbs that provide additional and specific meaning to the main verb of the sentence.
For example –

- Can
- Could
- May
- Might
- Must
- Shall
- Should

- Ought to
- Will
- Would

Thus it can be said that Modals are those helping verbs, which express the 'mode' or 'manner' of the actions indicated by the main verbs. They express modes such as ability, possibility, probability, permission, obligation, etc.

Modals are used to:

- Ask permission —may, can, could
  Examples:
  May I come in?
  Could I use your pen, please?
- Make a request —can, could
  Example:
  Could you please give me the doctor's telephone number?
- Express a possibility—may, might, could
  Example: It might rain during the night.
- Give advice or suggestion —should
  Example:
  You should wear a helmet while riding your motorbike.
- Express necessity or compulsion —must, have to
  Examples: We must slow down while driving in front of a school.
  I have to submit my project by tomorrow.
- Express prohibition
  Example: You must not talk loudly in the library.
- Express a promise or intention—will, shall
  Example: I will mail you my address.
- Express a wish—may
  Example: May you have a long life!

A modal does not change according to the number or person of the subject.

Examples: He can learn. I can learn. You can learn. They can learn. We can learn.

A modal is always used with a verb in its basic form. The modal takes the tense while the main verb remains in its dictionary form.

Modals can be used alone in response to a question.

Examples: Can you sing? I can.

Modals, when joined with 'not' to form a negative, can be contracted.

Examples: I cannot run. I can't run.

Modal verbs do not have all the tenses.

Modal verbs use other verbs to complete the tenses. For Example

- Can is completed with be able to
- Must is completed with have to

They can have more than one meaning depending on the situations.

- Single Concept Modal: they have one meaning
- Double Concept Modal: they have two meanings

Modals in past: They are used to express a situation in the past

Single Concept Modal		
Will	future	Ram will travel to Kolkata next week.
Might	Small probability	He might move to London some day.
Should	recommendation	you should go to a doctor.
Ought to	Formal recommendation	We ought to know about first aids.
Had better	Warning	I had better study or I will fail the test.
Double Concept Modals		
Shall	Educated expression Offer	Excuse me, I shall go now Shall I clean it?
Shall	Contractual obligation	The company shall pay on January 1st.
Could	Unreal Ability	I could go if I had time
Could	Past Ability	She could play the piano (but she can't anymore)
Can	Present Ability	We can speak English
Can	Permission	Can I have a candy?

MODALS IN THE PAST

They are modals referred to actions that happened in the past
MODAL + HAVE + verb in past participle
It must have been a difficult decision
They should have invited her to their wedding.

MODAL PERFECT		
MUST HAVE + P.P.	Logical conclusion on a past event	Peter has arrived late. He must have been in a traffic jam
MAY / MIGHT HAVE + P.P.	Deduction on a past event	Joe may / might have taken the wrong train
COULD HAVE + P.P.	Possibility to do something, gone unfulfilled	You could have played better
COULDN'T HAVE + P.P.	Certainty that something couldn't have happened	He couldn't have passed because you hadn't studied enough.
WOULD HAVE + P.P.	Desire to do something, but impossibility to do it for external causes	I would have visited you, but I forgot your address.
SHOULD / OUGHT TO HAVE + P.P.	Lament on something that should have been done	You should / ought to have warned me earlier.
SHOULDN'T HAVE + P.P.	Critique on something that shouldn't have happened	He shouldn't have told them
NEEDN'T HAVE + P.P.	Something that wasn't necessary doing	You needn't have bought it .

Modals-like verbs		
Modal	Concept	Example
Like	To enjoy	I like to watch TV
Want	To desire	John wants to buy a car
Need to	Necessity	We really needed to talk to you
Have to	Obligation	Susan had to pay the rent
Have got to	Have to	I've got to go now.
Look forward to	Future plan	I look forward to seeing you again

Direct / Indirect Speech:

In a Direct Speech, the actual words of the speaker are reproduced, whereas in an indirect speech the main idea of the speaker is reported by another person. Note that whereas quotation marks or inverted commas are used for direct speech, it is not done for the indirect speech. For example –

Direct: Asin said, "I wanted to go to a mall."

Indirect: Asin said that she wanted to go to a mall.

The following are some of the points that you should keep in mind while converting Direct speech to indirect speech.

- Question marks or exclamation marks are used in direct speech, but not in reported or indirect speech. For example –

  Direct: The teacher asked the student, "Do you want to go for the excursion?"

  Indirect: The teacher asked the student if he/she wants to go for the excrusion.
- In indirect speech, the reporting verbs such as asked / enquired, commanded, ordered, requested, exclaimed, shouted etc are often used to state the mood of the spoken words, in place of the word "said" or "told" in the direct speech.
- The tense of the verb in the indirect speech is always in past tense, though if the indirect speech is stating a universal truth or a habit or a situation which is constant then the verb does not changed from the direct speech. While converting direct speech into indirect speech, the tense of the verb must change accordingly –
  - Simple present — Simple past
  - Simple past — Past perfect
  - Present continuous — Past continuous
  - Present perfect — Past perfect
  - Can — Could
  - Shall — Would
  - Will — Would
  - May — Might
- In indirect speech the auxiliary verb "do" is not used. For example,

  Direct: "Rahul asked, "What time did the train leave?"

  Indirect: Rahul enquired what time the train left."
- Pronouns of first person are changed in the reported speech from "I" to "he or she"; "We" to "they", "my" to "her" or "his". Pronouns of Second person are changed in the reported speech from "you" to "he", "she", "they" depending on whom it is addressed to, whereas for the third person it is not changed. For example –

  Direct: Geeta said "I am bored."

  Indirect: Geeta stated that she was bored.

  Direct: Ram told her, "You are cute."

  Indirect: Ram told her that she is cute.
- Words indicating 'nearness' of time and place are changed to words indicating 'distance' of time and place:
  - This — that
  - These — those
  - Here — there
  - Now — then
  - Today — that day
  - Tomorrow — the next/following day
  - yesterday — the day before/the previous day
- When reporting a question, an order or a request, the connector 'that' is not used.
- In reported speech, the word/words or the sound used by the speaker to express an emotion is usually omitted. For example —

  Direct: Ram said to Shyam, "Hello!, How are you doing?"

  Indirect: Ram greeting Shyam and asked how he was doing?"

Some Questions for Practice:

1. Change the Following from Direct to Indirect Speech.
   (a) Ashok asked Asin, "Will you like to go for a movie tomorrow?"
   (b) The teacher asked, "Have you submitted your assignment?"
   (c) Robin stated, "The rich are not always dishonest."
   (d) She said to her friend, "Hello! How are you doing?"
   (e) Vijay announced, "Hurrah! We have won the match!"
2. Change the following from Indirect to Direct Speech.
   (a) Ram said to Jadu that they should have a party.
   (b) Riya greeted her grandparents and asked them to sit down.
   (c) The teacher greeted the students and asked them to open page ten.
   (d) The Police inspector asked the accused if he has any proof of innocence.

Aryan asked her if she had taken the examination the day before.

Active and Passive voice

(a) In the Active voice, the subject does the action.

e.g.: Ram cooked the food. (Active voice)

(b) In the Passive voice, the action is done to the subject.

e.g.: The food was cooked by Ram. (Passive voice)

Passive verbs can be formed in different ways.

(a) By using the Past participle.

Example:

(a) He will drive the car. (Active)

The car will be driven by him. (Passive)

(b) He may drive the car. (Active)

The car may be driven by him. (Passive)

(c) He was driving the car. (Active)

The car was being driven by him.

Direct and Indirect objects

Verbs like 'give' and 'bring' which can have two objects can have two passive forms.

He gave her (indirect object) a bouquet. (direct object) (Active)

She was given a bouquet by him (Passive)

A bouquet was given to her by him. (Passive)

Adjectives and Adverbs:

Nouns are proper names and verbs are all about actions. Nouns and Verbs consist the major part of language; but beauty of a language lies in the adjectives and adverbs. If there would have been no adjectives and adverbs then language would be very dry and the poetic quality of language would not have been there. Adjectives and adverbs describe – whereas adjectives describe the noun, adverbs describes or modifies a verb or an adjective or an adverb. Thus beautification, modification of a language is only possible when one has a good command over adjectives and adverbs. Therefore it is essential that a person has a good command over adjectives and adverbs if he or she wants to use language effectively.

Adjectives:

Words used with nouns to describe or point out the person, animal, place or thing which the nouns name, or to tell the number or quantity, are called adjectives. In other words, it can be said that an adjective adds something to the meaning of the noun. For example, when we say "it was a *lazy* afternoon" we are adding something to the quality of the afternoon which is a noun. The word lazy describes the nature of afternoon. It may not be that the afternoon is lazy; but the person who is saying it is feeling lazy during afternoon. So to speak about his laziness during afternoon, he may modify the afternoon as "lazy" making his utterance sound descriptive and at the same time beautiful. It is the role of an adjective. An adjective when added to a noun, it gives a description the noun.

Let us take another example – "In my childhood days, I was in *great* care of my grandparents." If we ponder over the phrase "good care", we will figure out that the word "care" would have been enough but when the speaker says "good care" we come to know how much the person values the care of his or her grandparents. The word "good" as an adjective provides the quality / nature of the care the person received when he or she was a child.

There can be numerous examples to talk about the beauty of adjectives and how they help us in our description. But let us for now consider the different kinds of adjectives and as we do so we will come to know the variety and the depth of the role adjectives play in our language.

Kinds of adjectives:

- **Adjectives of Quality or Descriptive Adjectives:**
  Adjectives of Quality of Descriptive Adjectives indicate / point out the kind or quality of a person or a thing. For example, the phrase "large city" indicates the kind of city.
- **Adjectives of Quantity:**
  Adjectives of Quantity indicate how much of a thing is meant. For example, the phrases "some rice", "enough exercise", "no sense", "great care", "half share", "whole sum", etc. are all dealing with adjectives which are indicating the quantity in each case.
- **Adjectives of Number or Numeral Adjectives:**
  Adjectives of Number or Numeral Adjectives are ones which indicate how many persons or things are meant or in what order a person or thing stands. For example, "five fingers", "first day of the week", "some mangoes", "several mistakes", "no pictures", etc.
- **Demonstrative Adjectives:**
  Demonstrative Adjectives point out which person or thing is meant. For example, "this boy", "that person", "these cars", "such things", etc.

Formation of Adjective:

The question that arises next is that how does one form an adjective. Adjectives are usually formed by adding something to nouns, verbs or other adjectives. In most cases, to make an adjective we add a "suffix." For example, to make an adjective from the noun "man" we may use the suffix "-ly" and the adjective would be "manly."

Noun – Adjective	Verb – adjective	Adjectives – Adjectives
• Boy – boyish • Fool – foolish • Hope – hopeful • Shame – shameful / shameless • Gold – golden • Laugh – laughable • Man –manly • Gift – gifted	• Tire – tireless, tiresome • Talk – talkative • Move – moveable	• Tragic – tragical • Black – bleakish • Sick – Sickly

Comparison of Adjectives:

Positive – Comparative – Superlative

- The Positive degree of an adjective is the adjective in its simple form. It is used denote the mere existence of some quality if what we speak about. It is used when no comparison is made.
- The comparative degree of an adjective denotes a higher degree of the quality and is used when two things are compared.
- The superlative Degree of an Adjective denotes the highest degree of the quality and us used when more than two things (or set of things) are compared.

Adverb:

An Adverb is a word which modifies the meaning of a verb, an adjective or another adverb. The difference between an adjective and an adverb is that what while an adjective modifies a noun, an adverb modifies a verb, an adjective or another adverb. For example, in the sentence "Sunil runs quickly" the word "quickly" modifies the verb "run" and therefore it is an adverb. Other examples can be "This is a *very* sour mango", where the word "very" is an adverb modifiying the adjective "sour"; and in the sentence "Nishant reads quite clearly" the word "quite" is an adverb modifying another adverb "clearly."

Kinds of Adverbs

Adverbs of Time: Adverbs of time shows "when"

I have heard this before
We shall now begin to work
Wasted time never returns

Adverbs of Frequency (which show how often)

I have told you twice
He often makes mistakes
He seldom comes here
He always tries to be best

Adverbs of Place (which show where)

- Stand here
- Is he within?
- Walk backward

Adverbs of Manner: (which show how or in what manner)

- Govind reads clearly
- The Indians fought bravely
- I was agreeably disappointed

Adverbs of Degree or Quantity (which show how much or in what degree or to what extent)

- He was too careless
- Is that any better?
- You are partly right
- She sings pretty well

Adverbs or Affirmation and Negation

- Surely you are mistaken
- He certainly went.

Adverbs of Reason

- He is hence unable to refute the charge.
- He therefore left the school.

Prepositions:

Prepositions of Time

Prepositions are words that are used to show the relationship between a noun or a pronoun and some other word.

1. 'For' and 'Since'
   'For' is used over a certain period of time (past till now)
   e.g.: 'I have been staying here for two years.'
   'Since' is used from a certain point of time (past till now)
   e.g.: She is learning dancing since six months.
2. 'At', 'in' and 'on'
   'At' is used for night, for weekend, a certain point of time (when?)
   e.g.: at night, at the weekend, at half past nine.
   'In' Room, building, street, town, country, book, paper, car, taxi, picture, world
   e.g. in the drawing room, in India, in the break in the car, in a taxi, in the picture, in the world
3. By
   'By' is used by indicate the sense of 'at the latest' and also to indicate up to a certain time
   e.g.: I will reach home by seven o'clock
   By two o'clock, I had completed the work.
4. 'In' and 'Within'
   'In' is used with months, season, country times of the day, city or town names in Venice.
   'Within' is used to say that something is inside something else.
   e.g.: There was a whole story within the book.
5. 'In' and 'After'
   'In' means at the end at a period of time in the future.
   e.g.: He will be back in a weak (Future)
   'After' means at the end of a period of time in the past.
   e.g.: He returned after a weak. (Past)
6. 'Before' and 'For'
   'Before' means a point of future time.
   e.g.: She will be there before 7 o'clock. (a point of time in future)
   'For' shows a period of future time.
   e.g.: She will be there for an hour. (period of time)

Prepositions of Place

Preposition of place shows the position or location of one thing with another. It answers the questions 'where'?

'At'

'At' is used in the description of a place, position or address.

e.g.: At home, at the station, at the top of the page, at the back of the book.

'On'

'On' is used will surface, roads or things that can be thought of as surfaces.

e.g.: On my desk, on the first floor, on the east coast.

'In'

'In' is used with geographical regions, cities, larger areas

e.g.: in the mountain, in India, in the park, in the church.

Some Simple rules of Joining sentences:

(a) Two sentences can be joined and made into one by using a present participle.
   e.g.: We found the door open. We went inside.
   Ans.: Finding the door open, we went inside.

(b) Two sentences can be joined and made into one using a past participle.
   e.g.: The car was damaged in an accident. It needed a new door.
   Ans.: Damaged in an accident, the car needed a new door.

(a) Two sentences can be joined using preposition and whom or whose.
e.g.: The boy was very friendly. I played with him.
Ans.: The boy with whom I played was very friendly.
(b) Two sentences can be joined using infinitives.
e.g : She works very hard. She wants to pass the test.
Ans.: She works very hard to pass the test.

Transformation of sentences

It describes how the sentences can be transformed in their nature without changing the meaning and the sense.

1. Changing from interrogative to assertive sentence and vice-versa.
   e.g. (a) When can their glory fade? (Interrogative sentence)
   Their glory can never fade. (Assertive sentence)
   (b) He was a villain to do such a deed. (Assertive sentence)
   Was he not a villain to do such a deed? (Interrogative sentence)
2. Changing from Exclamatory sentence to Assertive sentence and vice-versa.
   e.g. (a) How beautiful is this night! (Exclamatory sentence)
   This night is very beautiful.
   (b) I wish I were a bird (Assertive sentence)
   If only I were a bird! (Exclamatory sentence)
3. By changing the verb into the noun form.
   e.g.: It costs twelve rupees. (Verb form)
   Its cost is twelve rupees. (Noun form)
4. By changing the adjective into an adverb.
   e.g : He gave a curt reply. (Adjective)
   He replied curtly. (Adverb)
5. Changing from imperative to an interrogative sentence.
   e.g.: Please have something to drink. (Imperative)
   Will, you have something to drink, please. (Interrogative)

Transformation of sentences:

1. Using conjunction 'if' or 'unless'
   e.g.: (a) You will pass if you study well.
   (b) Unless you study well, you will not pass
2. Using conjunction 'as', 'though' and 'although'
   e.g.: (a) Poor as he is, he is honest.
   (b) Though he worked hard, he failed.
   (c) Although he is rich, he is not contented.
3. Using 'but', 'not withstanding', 'never the less' and 'however'.
   e.g.: (a) But for his help, she would not have passed.
   (b) Notwithstanding, that the works got knows, they called for a stoke.
   (c) He is my friend, nevertheless, he does not trust me.
   (d) However deep the river is, we will swim across it.
4. Using 'so' and 'too'
   e.g.: (a) He is too tired to walk.
   (b) He is so tired that he cannot walk.

EXERCISES

Re-write the following sentences according to the instructions given after each question. Make necessary changes, but do not change the meaning.

(i) Rana fought bravely.
(Begin: Rana put up .....)

(ii) Amazing facts were revealed by our teacher.
(End: ..... revelations)

(iii) He is discontented notwithstanding that he gets more salary then he deserves.
(Begin: Although .....)

(iv) You will win if you study well.
(Use: Unless)

(v) If I were a rich man, I would have helped him.
(Begin: Were .....)

(vi) She was beautiful and humble.
(Begin: Not only .....)

(vii) A fragrant flower is the loveliest creation of nature.
(Begin: No other .....)

(viii) Jasdeep has probably forgotten his mother's birthday.
(Begin: In .....)

(ix) As soon as Ram narrated the story, he was praised.
(Begin: No sooner .....)

(x) The elephant is more intelligent than all other creatures.
(Use: Most intelligent .....)

(xi) The police was too slow to catch the thief.
(Begin: The police was so .....)

(xii) Mr. Sharma said, "Children, do not go out in the cold."
(Rewrite in indirect speech .....)

(xiii) Though I warned Dev, he ignored me.
(Begin: I would rather .....)

(xiv) I prefer singing to dancing.
(Begin: I would rather .....)

(xv) I have never seen Raman behave rudely.
(Begin: Never .....)

(xvi) She found your purse in the cupboard.
(Begin: The purse .....

(xvii) He said, "I was dejected thinking I would never be able to take my exam."
Change to reported speech .....

(xviii) He was convicted by the Jury.
(Begin: The Jury .....)

(xix) He is too qualified to be a peon.
Rewrite removing 'too' .....

(xx) I started learning French a year ago.
(Begin: I have been .....)

(xxi) As soon as she received the message, she rushed to the office.
(Begin: No sooner .....)

(xxii) Rishi plays basketball better than Karan.
(Begin: Karan does not .....)

(xxiii) If Maya does not study hard, she will not succeed.
(Begin: Unless .....)

(xxiv) They have renovated their house.
(Begin: Their house .....)

Transformation of Sentences

From _____ Sentence to _____ Sentence	RULE	Example
Affirmative sentence to Negative Sentence	RULE 1: Only / alone / merely ? Replaced by ? None but (person)/ nothing but(things) / not more than or not less than (number)	Aff: Only God can help us. Neg: None but God can help us. Aff: He has only a ball. Neg: He has nothing but a ball. Aff: He has only ten dollars. Neg: He has not more than ten dollars.
	RULE 2: Must ? Replaced by ? Cannot but/ Cannot help+ (v+ing).	Aff: We must obey our parents. Neg: we cannot but obey our parents/ we cannot help obeying our parents.
	RULE 3: Both---and ? Replaced by ? not only --- but also.	Aff: Both Asha and Lata were excited. Neg: Not only Asha but also Lata were present.
	RULE 4: and (if join two words) ? Replaced by ? Not only ----- but also.	Aff: He was obedient and gentle. Neg: He was not only obedient but also gentle.
	RULE 5: Everyone/ everybody/every person/ (every + common noun)/all ? Replaced by ? There is no + attached word + but.	Aff: Every mother loves her child. Neg: There is no mother but loves her child.
	RULE 6: As soon as ? Replaced by ? No sooner had ----- Than.	Aff: As soon as the thief saw the police, he ran away. Neg: No sooner had the thief saw the police he ran away.
	RULE 7: Absolute Superlative degree ? Replaced by ? No other+ attached word+so+ positive form+ as+subject.	Aff: Mumbai is the biggest city in India. Neg: No other city is as big as Mumbai in India.

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	RULE8: Sometimes affirmative sentences are changed into negative by using opposite words. Before the word, off course 'not' is used.	Aff: I shall remember you. Neg: I shall not forget you.
	RULE 9: Always ? Replaced by ? Never.	Aff: Nitin always attends the class. Neg: Nitin never misses the class.
	RULE 10: Too ---- to ? Replaced by ? so ---that+ cannot/could not(in past).	Aff: He is too weak to walk. Neg: He is so weak that he cannot walk.
	RULE 11: As – as ? Replaced by ? Not less – than.	Aff: Simi was as wise as Rimi. Neg: Simi was not less wise than Rimi.
	RULE 12: Universal truth are change by making them negative interrogative.	Aff: The Sun sets in the west. Neg: Doesn't the Sun set in the west.
	RULE 13: Sometimes ? Replaced by ? Not + always.	Aff: Gogol sometimes visits me. Neg: Gogol doesn't always visit me.
	RULE 14: Many ? Replaced by ? Not a few.	Aff: I have many friends. Neg: I do not have few friends.
	RULE 15: A few ? Replaced by ? not many.	Aff: Nepal has a few scholars. Neg: Nepal doesn't have many scholars.
	RULE 16: Much ? Replaced by ? A little.	Aff: He has much money. Neg: He doesn't have a little money.
	RULE 17: A little ? Replaced by ? not much.	Aff: Madan has a little riches. Neg: Madan doesn't have much riches.

Assertive Sentence to Interrogative Sentence	RULE 1: If the sentence is in the affirmative you have to change it into negative interrogative. If it is in negative then you have to change it into bare interrogative	Ass: He was very gentle. Int: Wasn't he very gentle? Aff: He is not a good person. Int: Is he a good person?
	RULE 2: No auxiliary verb in sentence ? ? Change it by using ? ? Do/does/did Or Don't/doesn't/didn't.	Ass: He plays Football. Int: Does he play football? Ass: They did not play football yesterday. Int: Did they play football yesterday?
	RULE3: Never ? Replaced by ? Ever.	Ass: I never drink tea. Int: Do I ever drink tea?
	RULE 4: Everybody/everyone/ All ? Replaced by ? Who + Don't/ Doesn't/ Didn't	Ex: Everybody wishes to be happy. Int : Who doesn't wish to be happy?
	RULE 5: Every + noun ? Replaced by ? Is there any + noun+ Who don't/doesn't/didn't.	Ex: Ass: Every man wishes to be happy. Int: Is there any man who doesn't wish to be happy?
	RULE 6: No body/ no one / None ? Replaced by ? Who.	Nobody could count my love for you. Int: Who could ever count my love for you?

	RULE 7 : There is no ? Replaced by ? Is there any/ Who(person)/ What(thing).	Ex: Ass: There is no use of this law. Int: What is the use of this law.
	RULE 8: It Is no ? Replaced by ? Is there any/Why.	Ex: Ass: It is no use of taking unfair means in the exam. Int: Why take unfair means in the exam? Or, Is there any use of this law?
	RULE 9: It Doesn't matter ? Replaced by ? what though/ Does it matter.	: Ass: It does not matter if you fail in the exam. Int: What though if you fail in the exam?
Interrogative sentence to Assertive sentence is to be done doing Vice versa.		
Exclamatory sentence to Assertive sentence	RULE1: Subject and Verb of exclamatory sentence are to be used as the subject and verb of assertive sentence at the outset of the sentence. How/what ? Replace by ? Very(before adjective)/ Great(before noun)	Ex: How fortunate you are! Ass: You are very fortunate. Exc: What a fool you are! Ass: You are a great fool
	RULE 2: Sometimes the subject and verb may be eclipsed.	Ex: What a beautiful scenery! Ass: It is a very beautiful scenery. Ex: What a pity! Ass: It is a great pity.
	RULE 3: Hurrah/ Bravo ? Replace by ? I/we rejoice that/ It is a matter of joy that.	Ex: Hurrah! We have own the game. Ass: It is a matter of joy that we have won the game.
	RULE 4: Alas ? Replace by ? I/we Mourn that/ It is a matter of sorrow or grief that.	Ex: Alas! He has failed. Ass: We mourn that he has failed.
	RULE 5: Had/were/If /Would that(at the out set) ? Replaced by ? I wish + subject again + were/ had+ rest part.	Ex: Had I the wings of a bird! Ass: I wish I had the wings of a bird. Ex: Were I a bird! Ass: I wish I were a bird. Ex: If I were young again

Assertive sentence to exclamatory sentence is to be done doing Vice versa.		
Imperative sentence to Assertive sentence	RULE 1: Add subject + should in doing assertive. RULE 2: Please/kindly ? Replaced by ? you are requested to.	Ex: Do the work. Ass: You should do the work. Ex: Please, help me. Ass: You are requested to help me.
	RULE 3: Do not ? Replaced by ? You should not.	Ex: Do not run in the sun. Ass: You should not run in the sun.
	RULE 4: Never ? Replaced by ? you should never. RULE 5: Let us ? Replaced by ? We should.	Ex: Never tell a lie. Ass: You should never tell a lie. Ex: Let us go out for a walk. Ass: We should go out for a walk.
	RULE 6: Let + noun/pronoun ? Replaced by ? Subject + might.	Ex: Let him play football. Ass: He might play football
CHANGE of DEGREE While Transforming Sentence		
Change of degree	RULE1: If the superlative degree says about the best thing then the RULE is:- For comparative, use – subject +verb + adjective/adverb(comp. form) + Than any other + rest part For positive, use- No other + rest part after supr. Degree + verb + so/as + positive form of adj/adv + as + sub.	Ex: Su: Suman is the tallest boy in the class. Com: Suman is taller than any other boy in the class. Pos: No other boy in the class is as tall as Suman.

	RULE 2: If In superlative degree 'One of the' is transformed in this way: Comparative: Sub+verb +comp. form +than most other+ Rest part. Positive: Very few+ rest part after supr. Degree + verb + so/as + positive form of adj/adv + as + sub.	Ex: Nazrul was one of the greatest poets in Bangladesh. Comp.: Nazrul was greater than most other poets in Bangladesh. Positive: Very few poets in Bangladesh were so great as Nazrul. Note: Superlative: Of all/ of any Comparative: Than all other/than any other. Positive: It does not exist. Ex: Sup: Mr. khan is the oldest of all men in the village. Com: Mr. Khan is older than all other men in the village. Pos: No other man is as old as Mr. Khan.
	RULE 3: Simple comparative is transformed into positive by using (not so + adj/adv+as)/ (so+adj/adv+as)if negative. Second noun or pronoun is used first.	Ex: 1. com: Rina is wiser than Mina. Pos: Mina is not so wise as Rina. 2. Com: Mina is not wiser than Rina.' Pos: Rina is as wise as Mina.
	RULE 4: No/not less --- than is transformed into positive by using as +adj/adv+ as	Ex: com: Karim is not less meritorious than Suman. Pos: Karim is so meritorious as Suman.

Changing from Complex to Simple or Compound

Complex ? ? ? Simple ? ? ? compound	RULE 1: Since/As/When Change is to be made in the subordinate clause . When subjects are same. Simple: 1) Omit since/as/when. 2) (Verb+ing) of the subordinate clause. 3) then write the rest part. 4) use subject with the principal clause. 5) Principal clause remains unchanged. Compound: 1) Omit since /as /when 2) write down the rest part. 3) join clauses by using and, and so, and therefore 4) write the main clause unchanged.	Ex: Since the boy worked hard, he made a good result. Simple: Working hard, the boy made a good result. Ex: The boy worked hard and made a good result.
	Rue2: In case of Be verb in subordinate clause: 1) Use being/ Because of + Pronoun/noun(possessive form)+ being.	Ex: Since he was weak, he could not work hard. Simple: Because of his being weak, he could not work hard. Compound: He was weak and therefore could not work hard. Note: and therefore, is used for showing reasons.
	RULE3: When the subject of clauses are different: Simple: 1)Subject of s.c. 2) Verb+ing (be verb ? being; Have verb ? having) Compound: use 'and therefore'to join two clauses.	Ex: Since the weather was foul, we did not go out. Sim: The weather being foul, we did not go out. Ex: The weather was foul and therefore we did not go out.

	RULE 4: If,' is Replaced by – 'by + (verb+ing) Compound: 1) Omit if+subject. 2. use 'and' to join two clauses.	Ex: If you work hard, you will succeed in life. Simple: By working hard, you will succeed in life. Ex: Compound: Work hard and you will succeed in life.
	RULE 5: Simple: If-not/unless, is replaced by, - without+(verb+ing) Compound: Use or'/otherwise to join two clauses.	Ex: Complex: If you donot work hard, you will fail in the examination. Sim: Without working hard, you will fail in the examination. Ex: Work hard or you will fail in the examination.
	RULE 6: Simple: Though' is replaced by In spite of+ Possessive form of the subject+ (verb+ing) Compound: Use 'but' to join two clauses.	Ex: Com: Though he tried heart and soul, he could not succeed in life. Sim: In spite of his trying heart and soul he could not succeed in life.
	RULE 7: Simple: So that is replaced by to/in order to. Compound: "and want/wants to" is used to join two clauses.	Ex: Comp: He works hard so that he may prosper in life. Sim: he works hard to/in order to prosper in life. Ex: He works hard and wants to prosper in life.
	RULE 8: Simple: 'so + adjective + that' is replaced by 'Too + adjective + to'. Compound: Use 'And Therefore' to make it a compound sentence.	Ex: The boy is so foolish that he cannot understand it. Sim: The boy is too foolish to understand it. Ex: He is so foolish and therefore cannot understand it.

	RULE 9: When(if mentions time) is replaced by For short time – At For month or Season – In For age--- at the age of. Compound: Use and to join clauses.	Ex: She woke up when it was midnight. Simple: She woke up at midnight. Com: When it is spring, the cuckoo sings. Sim: In Spring the cuckoo sings. Con: When Samira was four she went to school. Sim: At the age of four, Samira went to school. Ex: She woke up and it was midnight.
	RULE 10: Simple: If the clause says a bout a continuous fact then use- At the time of instead of 'When' Compound: Use 'And'.	Ex: When I was eating the phone rang. Sim: At the time of my eating, the phone rang. Ex: I was eating and the phone rang.
	RULE 11: Simple: Noun clause can be replaced by noun. Compound: Use 'And'	Ex: Com: He admitted that he was guilty. Sim: He admitted his guilt. Com: That he is honest is known to all. Sim: his honesty is known to all. Ex: He is honest and it is known to all.
	RULE 12: Simple: If Complex sentence is made with relative pronoun (who, what, which, that), omit it and make (verb+ing). Note: If the verb is in the past participle it remains unchanged.	Com: The Doctor who is working in the hospital is known to all. Sim: The doctor working in the hospital is known to all. Ex: The picture which was drawn by Liza is very fine. Sim: The picture drawn by

	RULE 13: Simple: Adjective Clause is changed into - Adjective, Past participle Phrase, Noun in apposition, infinitive. Adjective: ex: A man who is drowning catches at a straw. Sim: A drowning man catches at a straw. Compound: A man is drowning and so catches at a straw. Past participle phrase: Ex: The answer that he wrote was not correct. Sim: The answer written by him was not correct. Compound: He wrote the answer and it was not correct. Gerundial Infinitive: Ex: I have no money that I can lend you. Sim: I have no money to lend you. Compound: I have no money and I cannot lend you.	
	RULE 14: In the compound "not only -- But also" is Changed by "Besides + (Verb +ing)" In the simple.	Ex: Mr. Khan not only teaches us English but also writes novels. Sim: Besides teaching us English, Mr. Khan writes novels.

Certain passages provided underneath with some mistakes in them. Figure out the mistakes and rewrite the passage. It is given as a practice for sentence correction, based on the grammar we have been discussing

Figure out the errors in the passages:

Passage 1:

A long time ago there live a eight year old boy in Holland whose name was Hans. He is the son of a sluicer. A sluicer is an man whose job is open and closed the sluices located near the face of a dams. A big parts of Holland is lower than the level of seas. The sea water is keep from flooding the land only by means with strong dams and sluices.

Passage 2:

I decided at last to wrote out the confession, to submit it to my father and ask his forgiveness. I wrote it in a slip of paper and handed it to him myself. In this note not only did I confessed my guilt, but I asked adequte punishment for it, and closed with a request to him not to punish himself for my offence. I also pledged myself not to steal in future. I was trembling as I handed the confession to my father. He was then suffering from a fistula and was confine to bed. His bed was a plain wood plank. I handed him the note and sat opposite the plank. He read it through, and pearl drops trickled by his cheeks, wetted the paper. For a moment he closed his eyes on thought and then tore up a note. He had sit up to read it. He again lay down. I also cried.

Passage 3:

The Industrial Revolution is a long train of changes starting 1760. It is not alone: it forms one of a triad of revolution of which the other two were the American Revolution that started in 1775, and the French Revolution that started in 1789. It may seems strange to put into the same bracket an industrial revolution and two political revolutions. The Industrial revolution is simply the English ways of make those social changes. It can be referred to as English Revolution.

Passage 4:

Long back the beaver thought to be fish. Its meat and especially, tail was value highly as a delicacy. A beaver's tail is flat as if pressed down. Such a tail makes a good rudder oar when the beaver swim. Its hind feet are also good swimming because it have webs between the toes. The forefeet is not web since they would make it difficult for the animal to walk. It not easy pick up a branch with a wooded paw.

Passage 5:

Ever since British shifted his capital from Kolkata to the Delhi, the nature of Delhi changed and it became significantly more cosmopolitan. From being a virtually Muslim town, as reflects in the literary and cultural traditions of the city, Delhi began to assume the tone of a Punjabi city with the massive flow of Hindu and Sikh refugees from Pakistan. The road signs in Delhi is now written in Hindi, English, Urdu and Gurumuhki, highlighting the city's ethno-linguistic mosaic. With the passage of times and as Delhi's economy grown rapidly, accompanied by a construction boom, large number of enterprising Indian from all over the country, especially the less-developed states such as Bihar, Uttar Pradesh (UP) and Rajasthan flocks to the city. The lure of Delhi spread even beyond India, particularly to Bangladesh. Delhi is now the eighth –large metropolis in the world.

Passage 6:

Aromatherapy involve the extraction of essential oils by plants. These oils be very potent and to understand their benefits you need to know what they are and how they works in aromatherapy. An essential oil is the delicate fluids which form the fragrance of a plant or flower. It can be stored in the stem, leaves, roots or bark, depending on the plant. Although called oil, they doesn't actually resemble one in tekschure, but was a highly scented and extremely potent liquid. The powerful aroma of essential oils can affect the way we felt.

Passage 7:

Developmental activities such construction, transportation and manufacturing not only depletes the natural resources but also produced large amount of wastes that leads to pollution of air, water, soil, and oceans; global warming and acid rains. Illtreated or improperly treated waste is major cause of pollution in rivers and environmental degradation. When left unchecked, it causes ill health and loss of crop productivity. It is high time that we start addressing the major causes of pollution, there effects in our environment and a various measures that can be taken to control those activities who cause pollution.

Passage 8:

My mother have a catering business. When people have parties at their homes, my mother cooked for them. She is a fabulous cook. Many customers contacts her before they have a party. She asks they a lot ofquestions about the upcoming party. She finds out how people will be there. Then, her and the customer decide on the menu. She hires helpers to serve the food and to weight on the people. She usually will cook the food in her own home, but sometimes people ask her to preparing the food at their houses. She enjoy running her business and is really good at it.

Passage 9:

Human activities directly or indirectly effect the environment adversely. A stone crusher add a lot of suspended particulate matter and noise unto the atmosphere. Automobiles emits from there tail pipes oxides ofnitrogen, sulphur dioxide carbon dioxide, carbon monoxide and the complex mixture of ilburnt hydrocarbons and black soot who pollute an atmosphere. Domestic sewage and run off form agricultural fields, laden with pesticides and fertilizers, pollute water bodies.

Passage 10:

The students of Delhi University Lady Shri Ram College has conducted a safety audit of the area in and out the campus. The audit has been conduct by a groups of ten student with the help for some of their faculty members. The audit, which was kick-start in January this year, use an mobile appliacation called Safetipin.

GRAMMAR EXERCISES

Transformation of Sentences

1. When he was presented with the trophy, he started crying. (Use 'No sooner .... than')
2. If you read, you will learn. (Use "Unless .... not")
3. It is too hot. (Remove "too")
4. Rakesh went to the market. (Add a question tag)
5. I have a piece of bread please? (Use the modal auxiliary May or Might)
6. What is your name? (Change to assertive sentence)
7. Sam went home early today. (Change to interrogative sentence)
8. I wish I were young again. (Change to exclamatory sentence)
9. It is impossible that he is dead. (Change to negative sentence)
10. Avoid making silly mistakes. (Change to negative sentence)
11. I am doubtful of the outcome. (Change to negative sentence)
12. This is one of his more readable books. (Use the superlative degree)
13. Avoid eating junk food. (Change to assertive sentence)
14. When can their glory fade? (Change to assertive sentence)
15. What a delicious meal! (Change to assertive sentence)
16. Only they can do the work. (change to negative sentence)
17. They have not more than two cars. (Change to affirmative sentence)
18. Rana is an honest boy. (Change to negative sentence)
19. I never tell lie. (Change to affirmative sentence)
20. Hurrah! We have own the game. (Change to assertive sentence)
21. You are requested to help me. (Change to exclamatory sentence)
22. Suman is the tallest boy in the class. (Change to comparative)
23. The boy is so foolish that he cannot understand it. (Change to simple sentence)
24. Only god can help us. (Start with "None but ....")
25. I am sure Madhu had left when they arrived. (Start with "Madhu must.....")
26. Speaking French is difficult. (Start with "It is .....)
27. I am the tallest member of my group. (Change into Comparative)
28. I saw a house and it was beautiful. (Change into a simple sentence)
29. They escaped the fire and it was amazing. (Change into a simple sentence)
30. He saw his wife and ran. (change into complex sentence)
31. I breathe all right. At least I think so. (Join the sentence)
32. I won and managed to shock everybody. (Make a complex sentence)
33. It was only when I got home that I realized that I had forgotten my keys. (Begin: Not until ...)
34. He went to the bank and to the ATM. (Begin: Not only .....)
35. Raman is an intelligent boy. The other boys are not so intelligent. (join the sentence to make a single one)
36. Rita is too short to reach the top shelf. (Begin: Rita is so ...)
37. They were very afraid and so they could not speak. (Begin With "Being")
38. She was the only student capable of being the monitor. (Use capability)
39. If you had not supported me, I would have fallen. (Begin: But for ...)
40. It was cloudy but she did not take the umbrella. (Begin with Although)
41. There is very little difference between the two brothers. (Begin: The two brothers ...)

42. It was warm. We left the window open. (Make a Compound Sentence)
43. Kathmandu is the biggest city I Nepal. (Use comparative degree)
44. I saw the fox. It was chasing a rabbit. (Make a simple sentence joining two)
45. He knocked at the door. He demanded admission.
    (Combine the sentences using present participate)
46. It is not necessary for you to work on Sundays. (Rewrite using 'need')
47. Since the weather was foul, we did not go out. (Begin: The weather ...)
48. He disguised himself as a cleaner. He entered the bank.
49. There was almost nothing to eat. (Rewrite using 'hardly')
50. Why don't you listen to me? (Rewrite using ever in the appropriate place)

Voice Change – change from active to passive or vice versa.

1. Ram was caught by his friend doing yoga.
2. Hari was chased by some criminals.
3. Mr. De teaches us grammar.
4. He was praised by his teacher.
5. The injured were taken to the hospital by the firemen.
6. The building was damaged by the earthquake.
7. Shyam will give you a ticket.
8. The streets were thronged by the spectators.
9. Everyone will praise us.
10. The letter was posted by me.
11. The hostess received us.
12. The snake was killed by him with a stick.
13. They found him guilty of murder.
14. I built the house in 2009.
15. The minister was greeted by the crowd.

Narration – Change from Direct to Indirect or vice versa.

1. John said, "Hurrah! We have won the game!"
2. The teacher said, "Do your assignments properly."
3. "Go back to the room," said the father to his sons.
4. "Make a mark with your deeds," said the Yogi.
5. Govind said to me, "I am sick."
6. He said to her, "I shall help you."
7. They said, "We came, worked and returned."
8. I said, "I will do it today."
9. Gopal said, "I will work tonight."
10. Kapil said, "These are not for us."
11. They said to me that I had done well.
12. They told us, "He will meet us."
13. He said, "I was reading a book."
14. Naveen ordered Rahim to read a magazine.
15. He said that rain had been falling the previous day.
16. The boy said, "If I could win this game."

ANSWERS

Transformation of Sentences

1. No sooner was he presented with a trophy than he started crying.
2. Unless you read you will not learn.
3. It is extremely hot.
4. Rakesh went to the market, didn't he ?
5. May I have a piece of bread please!
6. I would like to know your name.
7. Didn't Sam go home early today ?
8. If only I were young again!
9. It is not possible that he is dead.
10. Do not make silly mistakes.
11. I am not sure of the outcome.
12. This is his most readable book.
13. You should avoid eating junk food.
14. Their glory can never fade.
15. This meal is delicious one.
16. None but they can do the work.
17. They have only two cars.
18. Rana is not a dishonest boy.
19. I always tell truth.
20. It is a matter of joy that we have won the game.
21. Please, help me!
22. Suman is taller than any other boy in the class.
23. The boy is too foolish to understand it.
24. None but God can help us.
25. Madhu must have left when they arrived.
26. It is difficult to speak French.
27. I am more taller than any other member of my group.
28. I saw a beautiful house.
29. Amazingly they escaped the fire.
30. He saw his wife which made him run.
31. I think atleast I breathe all right.
32. I won which shocked everybody. (Make a complex sentence)
33. Not until I reached home that I realized that I had forgotten my keys.
34. Not only he went to the bank but also to the ATM.
35. Raman is more intelligent among the other not so intelligent boys.
36. Rita is so short that she cannot reach the top shelf.
37. Being very afraid, they could not speak.
38. She was the only student who had the capability of being the monitor.
39. But for your support, I haven't fallen.
40. Although it was cloudy, she did not take the umbrella.
41. The two brothers had little difference.
42. It was warm and we left the window open.
43. No other city in Nepal is as bigger as Kathmandu.
44. I saw the fox chasing a rabbit.

45. Demanding admission, he knocked at the door.
46. Is it a need that you to work on Sundays?
47. The weather was foul and so we did not go out.
48. Disguised himself as a cleaner, he entered the bank.
49. There was hardly anything to eat.
50. Why don't you ever listen to me?

Voice Change – change from active to passive or vice versa.

1. His friends caught Ram doing yoga.
2. Some criminals chased Hari
3. Grammar is taught to us by Mr. De. / We are taught grammar by Mr. De.
4. His teacher praised him.
5. The firemen took the injured to the hospital.
6. The earthquake damaged the building.
7. A ticket will be given to you by Shyam..
8. The spectators thronged the streets.
9. We will be praised by everyone.
10. I posted the letter.
11. We were received by the hostess.
12. He killed the snake with a stick.
13. He was found guilty of murder by them.
14. Tthe house was built by me in 2009.
15. The crowd greeted the minister.

Narration – Change from Direct to Indirect or vice versa.

1. John exclaimed in joy that they had won the game."
2. The teacher asked to do our assignments properly."
3. Father ordered sons to go back to the room.
4. The Yogi remarked that we should make a mark with our deeds.
5. Govind told me that he was sick.
6. He told her that he should help her.
7. They told that they came, worked and returned.
8. I told that i will do it the same day.
9. Gopal told that he would work the same night.
10. Kapil informed that those were not for them.
11. They said to me, " I had done well."
12. They told us that he would meet us.
13. He said that he was reading a book.
14. Naveen said, "Rahim, read a magazine."
15. He said, "Rain has been falling yesterday."
51. The boy wished that if he could win that game.

CHAPTER

3

Idioms and Phrases

IDIOMS AND PHRASES:

A bird of passage	one that stays for a short time
A bird's eye view	a general view
A blind date	meeting with someone you don't know
A bolt from the blue	a sudden mishap
A bone of contention	subject of dispute
A bookworm	one who loves reading books and is always hooked on to them
A brazen-faced fellow	impudent fellow
A cold reception	a welcome-lacking affection
A fair weather/feather friend	a friend in prosperous time
A fly in the ointment	two problem in a situation
A forlorn hop	a plan which has remote chances of success
A grass widow	a woman whose husband is temporarily away from her
A march hare	as mad as a hare in spring
A great hand at	to be an expert
A hard nut to crack	a hard work to do
A man of straw	a man of no substance
A red letter day	a glorious day
A stepping stone	source of advancement
A storm in a tea cup	used derisively to indicate a great fuss about a trifle
A swan song	statement made just before death or retirement
A white elephant	unprofitable possession
A wild goose chase	fruitless task/endeavour
Achilles' heel	one's vulnerable or susceptible spot
All and sundry	everyone without distinction
All moonshine	far from reality
All the go	in fashion
An old flame	an old sweetheart
An old head on young shoulders	to be wise beyond one's age
An olive branch	an offer of peace
At death's door	about to die
At one's beck and call	to be always at one's service or command
Backstairs influence	influence exerted secretly

Bad blood	a feeling of enmity
Bag of bones	an extremely weak person
Bats in the belfry	to be mad
Bear away	win
Between Scylla and carbides	between two dangers
Black ox	misfortune
Black sheep	a bad fellow
Blind alley	unprofitable action
Blow over	pass off
Brown study	deep in thought
By fits and starts	irregularly
Crocodile tears	hypocritical tears
Donkey's years	after a long time
Eagle-eyed	keen sighted
Elbow room	to give enough space to move or work in
Fabians policy	policy of cautious persistence
Falling foul of	quarrelling with
Fling away	to throw aside violently
Gate crasher	uninvited intruder
Gift of the gab	power of talking
Go to roost	retire for the night
Grist to one's mill	useful for one's purpose
Halcyon days	happy and peaceful days
Harp on the same string	to dwell tediously on the same subject
Hear someone out	to hear up to the end
Heart to heart	frank and free
Hobson's choice	option of taking the one offered or nothing
In merry pin	in a merry humour
In sackcloth and ashes	in a state of great mourning
In the swim	knowing latest current things
Jack of all trades	one who knows a little about all things
Jog on	to continue at a slow pace
Judas' kiss	false love
Kangaroo court	unofficial court
Keep a good table	to provide luxurious food
Kith and kin	relatives
Knit one's brows	to frown
Lame excuse	false excuse
Led up to	culminated in
Lion's share	major part
Long for	desire
Long in the tooth	rather old
Losing ground	becoming less powerful or acceptable
Lump in the throat	a highly emotional state
Mad as a march hare	as mad as a hare in spring
Maiden speech	first speech of individual
Mother wit	common sense
Much ado about nothing	to make a fuss
My hands are full	I am very busy
Nail to the counter	to expose publicly as false

Neck and crop	completely
No love lost between them	to be as friendly as ever
Not cricket	unfair
Not worth a rap	worth nothing
Not worth his salt	he is not worth his salt if he fails at this juncture.
Of the deepest dye	of the worst type
On one's mettle	roused to do one's best
On the Greek calendars	something which is impossible
On the knees of God	yet uncertain
Once in a blue moon	rarely
Open question	a question without any generally agreed answer
Other fish to fry	more important work to attend to
Pen and ink	in writing
Printer's devil	an apprentice in a printing office
Put one's foot down	to be firm about something
Raw deal	unjust treatment
Spick and span	neat and clean
Splitting hairs	to dispute over petty points
Stave off	prevent; avert
Stick by	to adhere closely
Swollen headed	to be conceited
Take up the cudgels	to defend someone vigorously
Thanks to	be grateful
Three R's	reading, writing, arithmetic
Through thick and thin	under all circumstances
Tied to the apron strings of	to be dominated by
To add fuel to the fire	to aggravate the matter
To back up	to support
To be all eyes	watching closely and attentively
To be at sea	to be perplexed
To be at sixes and sevens	to be undecided
To be at the zenith of	on the peak of
To be hand and glove with someone	to be intimate
To be in the doldrums	to be in low spirits
To be like a fish out of water	to be in a strange situation
To be no chicken	to be no longer in use
To be on the carpet	under consideration
To be on the wane	to be diminishing
To be well off	in comfortable circumstances
To bear out	substantiate
To beat black and blue	to beat mercilessly
To beat the air	efforts that is vain, useless
To beggar description	something beyond description
To bite one's lips	to be angry
To blow one's own trumpet	to boast
To break the ice	to break the awkward silence
To bring the house down	to appeal greatly to the audience
To burn a hole in the pocket	money that is spent quickly
To bury the hatchet	to make peace
To call a spade a spade	to speak plainly without mincing matters

To cast an aspersion	bring discredit to
To change hands	to pass to a different owner
To clean the Augean stables	to purge the administration of its abuses
To close with	accept
To commit to memory	to learn by heart
To cool one's heels	to be kept waiting for sometime
To cry over spilt milk	to feel sorry for what has already happened
To cut a sorry figure	to make a poor impression
To cut someone short	to interrupt someone
To cut the Gordian knot	to solve the difficult problem
To die hard	to change or disappear very slowly
To dispose off	to sell
To draw a bed on	to take careful aim at
To draw a line	to fix a limit
To draw the long bow	to exaggerate
To eat away	to corrode
To egg on	to instigate, to proceed further
To eke out	to supplement
To end in smoke	to fail
To feather one's nest	to enrich oneself when opportunity knocks
To fight shy of	to keep aloof from
To get one's back up	to become irritated
To gild the pill	to cover a disagreeable thing with something pleasant
To give a person the cold shoulder	to treat coldly
To give the devil his due	to be just to a person even though he does not deserve
To go leeward	to move in the direction the wind blows
To go to the dogs	to be ruined
To grease the palm	to bribe
To grow upon	to have stronger and stronger hold over
To hang in the balance	to be undecided
To haul over the coals	to scold for something done wrong
To have a dig at	to criticize someone
To have a jaundiced eye	to be prejudiced
To have an edge on	to be slightly better than
To have not truck with	to have no dealing with
To have the upper hand	to have more influence
To have too many irons in the fire	to have too many things in hand
To hit it off	to agree or be congenial
To hit the nail on the head	to do the right thing at the right time
To hit upon	to find
To hold water	to sound logical
To keep abreast of	to keep in touch
To keep an open mind	to come to no decision on a subject of discussion
To keep hanging about	to loiter about
To keep the pot boiling	to maintain interest
To know the ropes	to know the procedure of doing a job
To know which side the bread is buttered on	to know where one's interest lies
To knuckle under	to submit
To leap the pale	to get into debt, to spend more than one's income
To leave no stone unturned	to make all possible effort

To live from hand to mouth	to live in scarcity
To lose heart	to be hopeless
To make a pile	to make a fortune
To make both ends meet	to manage
To make up one's mind	to be determined
To move heaven and earth	to make all possible effort
To nip in the bud	to eradicate in early stage
To pick a quarrel	to seek fight
To play ducks and drakes	to waste
To play fast and loose	to say one thing and do another
To play possum	to feign ignorance
To play to the gallery	to try to get the appreciation from the least intelligent people
To play up	to try to make something appear more important than it is
To plough the sands	to busy oneself in an unprofitable proposition
To provide against a rainy day	to provide for a possible future in time of difficulty or need
To put a spoke in one's wheel	to obstruct progress
To put the cart before the horse	to do a thing in a wrong way
To put two and two together	to conclude keeping in view the merits and demerits
To rate soundly	to censure strongly
To read between the lines	to try to understand the hidden meaning
To rip up old sores	to reopen a quarrel
To rule the roost	to dominate
To run in the same groove	to advance in harmony
To save one's skin	to avoid harm or injury
To see eye to eye with	to agree
To see pink elephant	to show courage
To set stores by	to value highly
To set Thames on fire	to do something extraordinary or brilliant
To shake off	to get rid off
To shake the dust off one's feet	to leave a place with a sense of resentment
To show a clean pair of heels	to run away
To snap one's fingers at	to be careless
To sow one's wild oats	to engage in youthful excesses
To sow the wind and reap the whirlwind	to suffer for one's foolish conduct
To stand on ceremony	to treat with cold rigid civility
To stick to one's colours	to refuse to yield
To stir up a hornet's nest	to excite the hostility of people
To strike oil	to have lucky success
To take French leave	to take leave without permission
To take people by storm	to captivate
To take stock of	to survey
To take the bull by the horns	to face difficulty boldly
To take to one's bed	to lie in bed due to illness
To talk shop	to use phrase peculiar to one's employment
To tax one's patience	to test one's patience
To test on one's laurels	to cease to strive for further glory
To throw up the sponge	to admit defeat
To turn over a new leaf	to change for the better
Under someone's wing	to be under someone's protection
Wild cat strike	strike not approved by the trade union

Wink at	to pretend not to see
With a string	gift or aid with condition
Within an ace of	very close
Without fear and favour	impartially
Wolf in sheep's clothing	to be a hypocrite
Wool gathering	to engage in idle or aimless day dreaming
Yearn for	to crave for
Yeoman's service	excellent work done

CHAPTER

Confusing Words

CONFUSING WORDS

1. Abdicate / abrogate – Abdicate – abandon, give up-Abrogate – abolish
2. Accede / exceed – Accede - to agree or to allow- Exceed - to go beyond or to surpass
3. Accept (receive) except (to leave without)
4. Access (approach) excess (too much)
5. Adapt (change) adept (skilled) adopt (choose). Adapt means to adjust. Adept means skilled, Skilful, Expert
   Adopt means to take as your own
6. Adverse /averse. Adverse means inauspicious or hostile
   Averse means disinclined or repelled. Unwilling, Hesitant
7. Advice (suggestion) advise (recommend)
8. Affluence / effluence. Affluence is wealth. Effluence is waste or useless product
9. Aisle / isle. An aisle is a narrow passageway, especially in a church or store. An isle is an island.
   Aisle (space, between rows) advise (recommend)
10. All right/ alright. All right is the correct form. Alright is grammatically incorrect
11. Allot (assign, distribute) a lot (a large amount)
12. Allude (suggest) elude (escape)
13. Allusion (suggestion) illusion (deception, fantasy)
14. Aloud / allowed
    Aloud means 'out loud' or 'speaking so that someone else can hear you'
    Allowed means 'permitted'
15. Already (previously) / all ready (completely prepared)
16. Altar (church table) alter (change)
    I was married at the altar of church.
17. Altogether (entirely) all together (complete group)
18. Always (at all times) all ways (all methods)
19. Amoral (unrelated to morality) immoral (to denounce some one's behaviour)
20. Amount (when things are lumped together) / number (when things are counted)
21. Annual (yearly) / annul (to make void or invalid)
22. Anxious / eager
    Any one (any one person) anyone (anybody, any person at all)
23. Appraise (to estimate/ assess) apprise (to inform or notify)
24. Ascent (climb) assent (agree)
25. Ascribe / subscribe
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26. Assume (thing in the absence of evidence)
Presume (things when it is reasonable to do so and there is no evidence to the contrary)
27. Assure (guarantee) ensure (to make sure) insure (to protect against loss or damage)
28. Attain / obtain
Attain means reach
Obtain means get
29. Auger (tool) augur (to predict or a sort of an omen)
30. Aid / aide
We aid people when we help them.
An aide is a special assistant
31. Bail: Security, Surety, Payment, Financial guarantee
Bale-a large bund bound for storage or transport
32. Baited / bated
33. Bemuse (confuse) amuse (to entertain)
34. Biannual (happening twice a year) biennial (every two years)
35. Birth (childbearing) berth (place of rest)
36. Blonde (beautiful lady) blond (handsome man)
37. Blunt / brunt - Blunt in the sense of not sharp and brunt means to receive the effects of something unpleasant.
38. Bore (as a noun, boaring or tiresome person)
Boar (male pig)
Boor (a vulgar person)
39. Braise / braze
Braise – to cook slowly in liquid
Braze – solder together
40. Breach/breeches
Breach is to break and breeches are worn by horse riders on their legs.
41. Break (smash, split)
Brake (stopping device)
42. Bridal / bridle
Bridal has to do with brides and weddings
Bridle and a noun means a halter or restraint and as a verb it means to restrain or to draw oneselfup in anger.
43. Broach / brooch
Broach – a decorative pin
Broach – to touch upon or start especially a topic
44. Canon (principles or rules) cannon (a large piece of artillery)
45. Canvas (fabric) canvass (examine, campaign for election)
46. Capital (city, wealth) capitol (building)
47. Choose (select) chose (past tense of choose)
48. Chunk (a big piece) chuck (throw)
49. Collaborate (to work with someone) corroborate (to establish the truth of something.)
50. Complement (to complete) compliment (praise)
51. Concentrate / concentrated
52. Connote / denote
53. Conscience (moral sense) conscious (aware)
54. Core/ corps / corpse
Apples have cores
A corps is an organisation, like the Peace Corps
A corpse is a dead body, a carcass
55. Creak/ creek, Creak- scream/cry , Creek –waterway/stream
56. Credible (believable or trustworthy) credulous (gullible)

57. Curb (to control) kerb (edge of a footpath or sidewalk) 94
58. Current (contemporary) currant (fruit usually dried) 95
59. Cursor (computer marker) curser (sweater)
60. Decent (good) descent (movement downwards or your ancestry) 96
61. Definite (certain) definitive (conclusive and unamendable) 97
62. Dessert (sweet food) desert (dry land) 98
63. Device (a mechanism) devise (to arrange) 99
64. Discreet (respectful) discrete (separate) 10
65. Dribble (to drool) drivel (when you mean to criticise someone else's speech as stupid or pointless, the word you want is drivel. 10
66. Emigrant (a person who moves out of a country) 10
Immigrant (a person who moves into a country) 10
67. Eminent (noteworthy/ distinguished/ famous) 10
Immanent (inherent) 10
Imminent (near or close at hand) 10
68. Exhausting (tiring). Exhaustive (considering all aspects) 10
69. Empathy /sympathy 10
If you think you feel just like another person, you are feeling empathy. 10
If you feel just sorry for another person. You are feeling sympathy. 11
70. Enormous (a big thing) enormity (a heinous or atrocious thing has enormity) 11
71. Envelop (to wrap something up in a covering) envelope (the specific wrapping you put around a letter is an envelope) 11
72. Etymology (the study of origins of words) 11
Entomology (the study of insect) 11
73 Evoke (suggest) invoke (to appeal or cite)
74. Exasperate (irritate) exacerbate (to make worse) 11
75. Exercise (activity) exorcise (drive out) 11
76. Faze (disturb) phase (stage) 11
77. Fearful (afraid) fearsome (dangerous) 11
78. Fiancé (male) fiancée (a girl to whom you plan to marry) 11
79. Flare (to flash or blaze) flair (ability or skill) 12
80. Flaunt / flout
Flaunt – when you show off something or boast about it, you flaunt it. 12
Flout – when you flout something you show your contempt for it.
81. Forbear (to refrain from) forebear (ancestor or forefather)
82. Foreword (the preface in a book, usually written by someone who is not author.)
83. Formerly (at an earlier time) formally (according to a pattern)
84. Gild / guild
You gild an object by covering it with gold.
A guild is a group or an organisation of people doing similar things.
85. Hanged (a criminal is always hanged) hung (a picture is hung)
86. Heel (part of foot) heal (to make better)
87. Heroin (drug) heroine (principle female character)
88. Historic (an event is historic) historical (a place is historical)
89. Hoard (to stock pile) horde (large group)
90. Home
Hone (to sharpen)
91. Human (people) humane (merciful)
92. Illusion (image of nonexistent thing) delusion (delusions are misguided belief due to mental imbalance)
93. Immigrate (to come in) emigrate (leave)

94. Imply (to suggest something) infer (conclusions from evidence by reasoning from data to claims)
95. Incredulous (unbelieving or sceptical)
    Incredible (unbelievably of good)
96. Install / instil
97. Myth (traditional story) legend (famous deed)
98. Licence (noun) license (verb)
99. Luxurious (you enjoy luxuries) luxuriant (growing abundantly)
100. Moral (good, lesson or conduct)
    Morale (mental condition or spirit)
101. Mute (to quieten) moot (debatable)
102. Pair (two) pare (peel or reduce)
103. Paramount (best or top) tantamount (equivalent)
104. Partake (consume)
105. Peak (climax) peek (peep or snoop)
106. Pique (excite or irritate)
107. Persecute (to treat badly) prosecute (a legal officer can prosecute against a criminal)
108. Perspective (angle of view) prospective (in the future)
109. Podium / lectern
110. Perpetrate / perpetuate
111. Progeny (your kids) prodigy (admirable) protégé (a person you take under your wing in order to promote or his or her career is your protage)
112. Prophecy (noun) prophesy (verb)
113. Raise (to lift) raze (to pull down)
114. Reality (real life)
    Realty (real estate)
115. Rebut (to argue) refute (to prove something incorrect)
116. Reference (something referred to) reverence (respect)
117. Retch (to gag or try to vomit) wretch (a creep)
118. Root (to cheer up) rout (plant rout) route (road)
119. Serf (servant / slave) surf (wave or surfing the internet)
120. Soar /sore
    Sore refers to aches, pains and wounds – sore feet, sore backs,
121. Sever / severe
    Sever means to separate or to detach
    Severe means grim or stern

CHAPTER

5

Previous year GATE questions

GATE-2010

1. Which of the following options is the closest in meaning to the word below-"Circutious"
(a) Cyclic (b) Indirect (c) Confusing (d) Croocked.

Soln. Circutious means indirect.
(a) Cyclic means recuring cycle
(b) Indirect means net leading to straight line.
(c) Confusing means lacking clarity.
(d) Crooked means irregular in shape.
Answer is (b).

2. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Unemployed: Worker
(a) Fallow: Land (b) Unaware: Sleeper (c) Wit:Jester (d) Renovated:House

Soln. Unemployed:Worker. Here one is opposite to other.
(a) Fallow:Land, follow means undeveloped land.
(b) Unware:Sleeper, Both are same unaware or sleeper
(c) Wit:Jester, wit means ability to make Jokes and jester is a joker.
(d) Renovated:House, renovate means to make better and house can be renovate.
Answer is (a)

3. Choose the most appropriate word from the options given below to complete the following sentence:
If we manage to ..... our natural resources, we would leave a better planet for our children.
(a) Uphold (b) Restrain (c) Cherish (d) Conserve.

Soln. (a) Uphold means cause to remain.
(b) Restrain means keep under control
(c) Cherish means be fond of.
(d) Conserve means to keep in safely and protect from harm, decay loss or destruction.
Answer is (d).

4. Choose the most appropriate word from the options given below to complete the following sentence.
His rather casual remarks on politics ..... his lack of seriousness about the subject.
(a) Masked (b) Belied (c) Betrayed (d) Suppressed.

Soln. Masked:Hind under a false appearance.

(b) Blied:Be in contradiction with.
(c) Betrayed:Reveal unintetionally
(d) Suppressed:To put down by force or authority.
Answer is (c).

5. Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warefare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.
Which of the following statements best sums up the meaning of the above passage?
(a) Modern warfare has resulted in civil strife.
(b) Chemical agents are useful in modern warfare.
(c) Use of chemical agents in warfare would be undesirable.
(d) People in military establishments like to use chemical agents in war.

Soln. (a) Modern warfare has resulted in civil stride: there is no direct consequence of warfare given, so it is not appropriate.
(b) Chemical agents are useful in modern warfare: passage does not say whether chemical agents are useful or not, so not appropriate.
(c) Use of chemical agents in warfare would be undesirable: given that people in military think there are useful, undesirable is wrong.
(d) People in military establishments like to use chemical agents in war. It is correct choice cause military people think that chemical agents are useful tools for their cause.
Answer is (d).

GATE-2011

6. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair :
Gladiator : Arena
(a) Dancer:stage (b) Commuter:train (c) Teacher:classroom (d) Lawyer:courtroom

Soln. Gladiator performs his action in arena.
(a) Dancer : stage, Dancer is a person who dance professionally and stage is a raised platform where the actor/dancer performed.
(b) Commuter : train, Commuter is a person one who travels regularly from one place to another and train in a vechicle which used for mass transportation of goods and peoples.
(c) Teacher : Classroom teacher is one who teaches, who is especially hired to teach and classroom is a place especially for educational purpose in school and institute.
(d) Lawyer : courtroom: lawyer is the person who gives legal advice and assists to clients and represents them in court and courtroom is place where proceedings of a court are held.
Answer is (a).

7. Choose the most appropriate word from the options given below to complete the following sentence :
It was her view that the country's problems had been ..... by foreign technocrats, so that to invite them to come back would be counterproductive.
(a) Identified (b) Ascertained (c) Exacerbated (d) Analysed.

Soln. (a) Identified means to establish the identity of.
(b) Ascertained means to discover with certainty, as through examination or experimentation.
(c) Exacerbated means to increase the reverity, violence or bitterness or aggravate.
(d) Analysed means to examine detail in order to discover meaning essential feature, etc.
Answer is (c).

8. Choose the word from the options given below that is most nearly opposite in meaning to the given word :
Frequency
(a) Periodicity (b) Rarity (c) Gradualness (d) Persistency

Soln. Frequency means occuring at frequent intervals.
(a) Periodicity means the equality or state of being periodic, recurrence at regular intervals.
(b) Rarity means something which is very rare or say infrequent.
(c) Gradualness means the quality of being gradual or of coming about by gradual stages.
(d) Persistancy means determination, the quality of being determined to do or achieve something.
Answer is (b)

9. Choose the most appropriate word from options given below to complete the following sentence:
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which ..... treatments are unsatisfactory.
(a) Similar (b) Most (c) Uncommon (d) Available.

Soln. (a) Similar means resemblance in qualities, characteristic or appearance alike but not identical.
(b) Most means in very big amount or number.
(c) Uncommon means rare or remarkable.
(d) Available means present and ready for use.
Answer is (d)

10. The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way. It can be inferred from the passage that horses were.
(a) Given immunity to diseases (b) Generally quite immune to diseases
(c) Given mediciens to fight toxins. (d) Given diphtheria and tetanus serums.

Soln. The author in the paragraph wants to show the role of horses in the field of medicines for providing the immunity to various diseases, so the answer is (a).

GATE-2012 (SET-1)

11. Choose the most appropriate alternative from the options given below to complete the following sentence:
Despite serveral .....the mission succeeded in its attempt to resolve the conflict.
(a) Attempts (b) Setbacks (c) Meetings (d) Delegations.

Soln. The word 'despite' indicates that there has to be a contrast in the sentence, use of the word 'setbacks' in the blank indicates that despite many problems the mission was successful.

12. Which one of the following options is the closest in meanings to the word given below?
Mitigate:
(a) Diminish (b) Divulge (c) Dedicate (d) Denote

Soln. Mitigate means to reduce, to lessen, etc, so the word diminish is close in meaning to mitigate. Rest all choices have no link with the given word. Divulge means to disclose or reveal which has no link with the given word.
Answer is (a)

13. Choose the grammatically **INCORRECT** sentence :
(a) They gave us the money back less the service charges to three hundred rupees.
(b) This country's expenditure is not less than that of Bangladesh.
(c) The committee initially asked for a funding of fifty lakh rupees, but later settled for a lesser sum.
(d) This country's expenditure on educational reforms is very less.

Soln. The country's expenditure is not less than that of Bangladesh.
The correct statement should be: the country's expenditure is not less than that of Bangladesh. Because the country's expenditure cannot be composed with Bangladesh. It should be compared with Bangladesh expenditure.
Answer is (b).

14. Choose the most appropriate alternative from the options given below to complete the following sentence:
Suresh's dog is the one ..... was hurt in the stampede.
(a) That (b) Which (c) Who (d) Whom

Soln. Suresh's dog is the one that was hurt in the stampede.
"That" is used with restrictive clauses.
Answer is (a)

15. Wanted Temporary, Part-time persons for the post of field interviewer to conduct personal interviews to collect and collate economic data. Requirements: High school-pass, must be available for day, evening and saturday work. Transportation paid, expenses reimbursed.
Which one of the following is the best inference from the above advertisement?
(a) Gender-discriminatory (b) Xenophobic
(c) Not designed to make the post attractive (d) Not gender discriminatory

Soln. (a) Cannot be considered since there is no gender discrimination mentioned in the argument.
(b) Xenophobic is are who has fear of foreigners, no link with the given argument.
(c) It is wrong to say that the profile has not been designed to make the post attractive, since there are certain features which have been added to make the profile lucrative (which are given towards the end of the advertisement, like transportation paid, expenses reimbursed)
Answer is (d).

GATE-2012 (SET-II)

16. Choose the most appropriate word from the options given below to complete the following sentence:
Given the seriousness of the situation that the had to face, his ..... was impressive.
(a) Beggary (b) Nomenclature (c) Jealously (d) Nonchalance

Soln. (a) Beggary means poverty, won't suit in the given situation.
(b) Nomenclature is 'process of naming' therefore, is irrelevant
(c) Jealously means a feeling which is also not fit on the situation.
(d) Nonchlance means indifferent calm which is most suitable for the given situation.
Answer is (d)

17. Which one of the following options is the closest in meaning to the word given below?
Latitude
(a) Eligibility (b) Freedom (c) Coercion (d) Meticulousness

Soln. Latitude means angular distance or freedom from narrowness or liberality of interpretation.
(a) Eligibility means desirable or suitable.
(b) Freedom means being unrestricted or civic liberty.
(c) Coercion means the act or process of coercing or government by force.
(d) Meticulousness means the great or excessive attention to details
Answer is (b)

18. Choose the most appropriate alternative from the options given below to complete the following sentence:
If the tired soldier wanted to lie down, he ..... the mattress out on the balcony.
(a) Should take (b) Shall take (c) Should have taken (d) Will have taken.

Soln. In the given problem the first sentence 'If the tired soldier wanted to lie down' is in past simple tense thus 'he ..... the mattress out on the balcony' most be in past participle tense, thus 'should have taken' is the answer.
Answer is (c).

19. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is INCORRECT?
I requested that he should be given the driving test today instead of tomorrow.
(a) Requested that (b) Should be given (c) the driving test (d) Instead of tomorrow.

Soln. The sentence is indirect speech so we don't use tomorrow. Thus 'instead of tomorrow' is wrong.
Answer is (d).

20. One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.
Which one of the following statements best sums up the meaning of the above passage?
(a) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(b) The legions were treated inhumanly as if the men were animals.
(c) Discipline was the armies' inheritance from their seniors.
(d) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

Soln. Only the statement (a) sums us the meaning of the above passage in best way
Answer is (a).

GATE-2013 (I)

21. The professor ordered to the students to go out of the class.
I II III IV
Which of the above underlined parts of the sentence is grammatically incorrect?
(a) I (b) II (c) III (d) IV

Soln. II
Correct answer is (b)

22. Which of the following option is the closest in meaning to the word given below:
Primeval
(a) Modem (b) Historic (c) Primitive (d) Antique

Soln. Primitive
Correct answer is (c)

23. Friendship, no matter how _____ it is, has its limitations.
(a) cordial (b) intimate (c) secret (d) pleasant

Soln. Intimate
Correct answer is (b)

24. Select the pair that best expresses a relationship similar to that expressed in the pair:
Medicine : Health
(a) Science : Experiment (b) Wealth : Peace (c) Education : Knowledge
(d) Money : Happiness

Soln. Education : Knowledge
Correct answer is (c)

GATE-2013 (II)

25. The Headmaster _____ to speak to you.
Which of the following options is incorrect to complete the above sentence?
(a) Is wanting (b) Wants (c) Want (d) Was wanting

Soln. Wants
Correct answer is (b)

26. Mahatma Gandhi was known for his humility as
(a) He played an important role in humiliating exit of British from India
(b) He worked for humanitarian causes.
(c) He displayed modesty in his interactions.
(d) He was a fine human being.

Soln. He displayed modesty in his interactions.
Correct answer is (c)

27. All engineering students should learn mechanics, mathematics and how to do computation
I II III IV
Which of the above underlined parts of the sentence is not appropriate?
(a) I (b) II (c) III (d) IV

Soln. IV
Correct answer is (d)

28. Select the pair that best expresses a relationship similar to that expressed in the pair:
water : pipe : :
(a) cart : road (b) electricity : wire (c) sea : beach (d) music : instrument

Soln. electricity : wire
Correct answer is (b)

GATE-2014 (I)

29. A student is required to demonstrate a high level of ***comprehension*** of the subject, especially in the social sciences:
The word closest in meaning to comprehension is
(a) understanding (b) meaning (c) concentration (d) stability

Soln. Comprehension means to understand which is not clear i.e. understanding
Correct answer is (a)

30. Choose the most appropriate word from the options given below to complete the following sentence.
One of his biggest _____ was his ability to forgive
(a) vice (b) virtues (c) choices (d) strength

Soln. Virtues means moral character.
Correct answer is (b)

GATE-2014 (II)

31. Which of the following options is the closest in meaning to the phrase underlined in the sentence below
It is fascinating to see life forms *cope with* varied environmental conditions
(a) Adopt to (b) Adapt to (c) Adept in (d) Accept with

Soln. **Cope with:** To succeed indealing with a difficult problem or situation
Adapt to : To gradually changes your behaviour and attitudes in order to be successful in new situation.
Correct answer is (b).

32. Choose the most appropriate word from the options given below to complete the following sentence.
He could not understand the judges awarding her the first prize, because he thought that her performance was quite _____
(a) superb (b) medium (c) mediocore (d) exhilarating

Soln. **Superb :** Extremely good, excellant
Medium : Middle size
Medicore : Not very good
Exhilarating : Making you feel happy, Excited.
Correct answer is (c).

33. In a press meet on the recent scam, the minister said, "The buck stops here", what did minister convey by the statement?
(a) He wants all the money (b) He will return the money
(c) He will assume final responsibility (d) He will resist all enquiries.

Soln. **The Bucks stop here :** used to say that a particular person is responsible for something.
Correct answer is (c).

GATE 2015 : PHYSICS-PH

1. Fill in the blank with the correct idiom/phrase.
That boy from the town was a _____ in the sleepy village.
(a) dog out of herd (b) sheep from the heap
(c) fish out of water (d) bird from the flock

Soln. **(c)** fish out of water. The idiom "fish out of water" means someone who does not fit the place or the occasion. It is the right idiom in this context.

2. Choose the statement where underlined word is used correctly.
(a) When the teacher eludes to different authors, he is being elusive.
(b) When the thief keeps eluding the police, he is being elusive.
(c) Matters that are difficult to understand, identify or remember are allusive.
(d) Mirages can be allusive but a better way to express them is illusory.

Soln. **(b)**

3. Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:
Apparent lifelessness _____ dormant life.
(a) harbours (b) leads to (c) supports (d) affects

Soln. **(b)** leads to

4. Select the appropriate option in place of underlined part of the sentence.
Increased productivity necessary reflects greater efforts made by the employees.
(a) Increase in productivity necessary (b) Increase productivity is necessary
(c) Increase in productivity necessarily (d) No improvement required

Soln. **(c)** Increase in productivity necessarily

GATE 2015 : CHEMISTRY-CY

5. Choose the word most similar in meaning to the given word :
Educe
(a) Exert (b) Educate (c) Extract (d) Extend

Soln. **(c)** Extract–an idea taken out from any short-passage, place or things.
Educe means to draw or bring out or elicit.
Educate – a literate person
Exert – to use power or influence to affect.

6. Choose the appropriate word/phase, out of the four options given below to complete the following sentence
Frogs _____
(a) croak (b) roar (c) hiss (d) patter

Soln. **(a)** croak
Sound that frog makes is called "croak."
roar – sound of Lion.
hiss – sound made by snake.

7. Choose the most appropriate word from the options given below to complete the following sentence
The principal presented the chief guest with a _____ as token of appreciation
(a) momento (b) memento (c) momentum (d) moment

Soln. **(b)** memento
momento – very important or serious
memento – things that remind you something.
momentum – the ability to keep increasing or developing.
moment – a very short period of time.

GATE 2015 : COMPUTER SCIENCE & IT

8. Didn't you buy _____ when you went shopping?
(a) any paper (b) much paper (c) no paper (d) a few paper

Soln. **(a)** The option "any paper" is the right one as it fits the sentence structure. Any is always used in negative and interrogative sense.

9. Which of the following options is the closest in meaning to the sentence below?
She enjoyed herself immernsely at the party
(a) she had a terrible time at the party (b) she had a horrible time at the party
(c) she had a horrible time at the party (d) she had a terrifying time at the party

Soln. **(d)** The option "terrific" has positive connotation; whereas the other options have negative connotations.

10. Which one of the following combinations is incorrect?
(a) Acquiescence – Submission (b) Wheedle – Roundabout
(c) Flippancy – Lightness (d) Profligate – Extravagant

Soln. **(b)** Wheedle and Roundabout does not have any connection. Wheedle means to flatter someone for something.

11. Select the alternative meaning of the underlined part of the sentence.
The chain snatchers ***took to their heels*** when the police party arrived.
(a) took shelter in a thick jungle (b) open indiscriminate fire
(c) took to flight (d) unconditionally surrendered

Soln. **(c)** "to take to one's heels" means "to run away."

12. We _____ our friend's birthday and we _____ how to make it up to him.
(a) completely forgot - - - don't just know
(b) forgot completely - - - don't just know
(c) completely forgot - - - just don't know
(d) forgot completely - - - just don't know

Soln. **(d)** The first part of the sentence has already happened, therefore it should be in past tense and the word "completely" comes after the verb; whereas the second part of the sentence should be there in present tense.

13. Choose the statement where underlined word is used correctly
(a) the industrialist had a personnel jet (b) I write my experience in my personnel diary
(c) all personnel are being given the day off (d) being religious is a personnel aspect

Soln. **(c)** "personnel" means "staff"; therefore option three is the right option. All other options should have the word "personal" instead of "personnel."

14. A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is
(a) fabric (b) textile (c) fibre (d) apparel

Soln. **(d)** Apparel means any dress.

15. Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything _____ to the requirements of the exam.
(a) related (b) extraneous (c) outside (d) useful

Soln. **(b)** Useful and related cannot be the options as they do not make sense in the context. The term "outside" is not used in such context, therefore "extraneous" is the right option.

16. Select the pair that best expresses a relationship similar to that expressed in the pair
Children : Pediatrician
(a) Adult : Orthopaedist (b) Females : Gyanecologist
(c) Kidney : Nephrologist (d) Skin : Dermatologist

Soln. **(b)** A Pediatrician is a doctor for children; similarly, a Gynecologist is a doctor for females.

17. The Tamil version of _____ John Abraham-starrer Madras Cafe _____ cleared by the Censor Board with no cuts last week, but the film's distributors _____ no takers among the exhibitors for a release in Tamil Nadu _____ this Friday.
(a) Mr., was, found, on (b) a, was, found, at (c) the, was, found, on (d) a, being, find at

Soln. **(c)** the only grammatically right option.
Madras cafe is proper noun. So, correct article is 'the'. Also it is singular so, use 'was' and generally day contain 'on' preposition before them.

18. The man who is now Municipal Commissioner worked as _____
(a) the security guardat a university
(b) a security guard at the university
(c) a security guard at university
(d) the security gurad at the university

Soln. Correct option is (b)

19. Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia.
Choose the option which is meaning to the underlined phrase in the above sentence.
(a) put up with (b) put in with (c) put down to (d) put up against

Soln. Correct option is (a)

20. Find the odd one in the following group of words:
mock, deride, praise, jeer
(a) mock (b) deride (c) praise (d) jeer

Soln. Correct option is (c)